The New Senior Secondary Curriculum for Sierra Leone

Subject syllabus for Mathematics for Business and Enterprise
Subject stream: Mathematics and Numeracy

This subject syllabus is based on the National Curriculum Framework for Senior Secondary Education. It was prepared by national curriculum specialists and subject experts.

SSSC
Senior Secondary School Curriculum

Curriculum elements for Mathematics for Business and Enterprise – an applied subject

Rationale for the Business mathematics in the Senior Secondary School Curriculum

In the business world, everyone—employees and managers alike—needs knowledge of and skill in business mathematics. While computers and calculators are used for many calculations, it is important to understand the concepts behind mechanical computations. The purpose of this business mathematics subject is to increase your math knowledge and skill as it applies to many aspects of business and to help make you a more valuable player in the business arena.

Business mathematics provides a solid preparation and foundation for pursuing courses and careers in accounting, marketing, retailing, banking, finance and business administration. It will provide the student skills in using specific business mathematics applications. It also teaches the mathematical skills required for problem solving and decision making in the business world through the use of mathematical models and specialized techniques. Topics include functions as mathematical models, equation-solving techniques, differential and integral calculus, exponential growth and time-value of money, partial derivatives and their applications in economic functions, and simple matrix algebra.

General Learning Outcomes

At the end of the three years, students will be able to:

- Demonstrate understanding of the concepts from the branches of mathematics.
- Use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts.
- Understand how to process and interpret information to arrive at logical conclusions to common business math applications.
- Develop proficiency in the application to solve business math problems and the important role math plays in all facets of the business world.

Structure of the Syllabus over the 3-Year Senior Secondary Cycle

	SSS 1	SSS 2	SSS 3
Term 1	**Sets** Describe set and the various types. Apply the algebra of sets. Solve two and three set problems (including use of Venn diagrams **Indices**	**Sequences and Series** Finite and Infinite sequences • Recurrence sequence • Arithmetic sequence and Geometric sequence	**Matrices** Operations on matrices Finding the determinant and inverse of a matrix (limited to 2 x 2 matrices) Application of matrices (Cramer's rule) to solve simultaneous linear equations in two variables

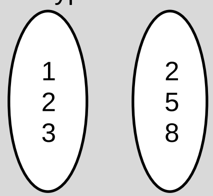
	• Rules of indices • Combination of two rules of indices • Equation of indices **Number Base and Modular Arithmetic** • Express based ten numbers to any other based number. • Express any other based number to a ten number. • Operations on number base • Number base leading to simple line equation • Operation on modulo (limited to 5 elements in the set)	Finite and infinite series • Arithmetic series and Geometric series • Sum of Arithmetic sequences and Geometric sequences **Mathematics of finance 1** • Ratio • Rate • Profit and loss • percentage change (increase/decrease) • Bills and Tariffs • Taxation and Wages	
Term 2	**Basic algebras** • Algebra and real numbers • Operations on polynomials • Factoring polynomials • Linear equations and inequalities in one variable **Relations and Functions** • Relations, Mappings and Functions • Function Notation. • Types of functions • Representing functions. • Inverse Functions and Composite Functions • Graphs and roots of Functions	**Mathematics of finance 11** • Simple interest • Compound interest • Future value of an annuity (Sinking fund) • Present value of an annuity (amortization) **Linear inequalities and linear programing** • Systems of linear inequalities in two variables • Linear programming in two variables • Business application of linear programming • Introduction to simplex method	**Statistics 1** Data types and data collection procedure Data Representation ***Data Analysis*:** (Measure of Central Tendency and Dispersion for both Ungrouped and Grouped Data) **Statistics 11** Correlation (types and coefficient by spearman's ranking method **only**) Regression (least square method **only** for prediction equation).

			Introduction to time series and price index
Term 3	**Polynomial Functions** • General Characteristics • Linear function • Quadratic Function • Exponential function • Logarithmic Function	**Calculus** • Differentiation • Applications of Differentiation • Integration • Applications of Integration	**Probability** • Permutation and Combination • Introductory to probability concepts • Probability events • Bayes' formula • Normal distribution

Detailed teaching syllabus outline

Topic/Theme/Unit	Expected learning outcomes	Recommended teaching methods	Suggested resources	Assessment of learning outcomes
		YEAR 1 FIRST TERM		
Sets Understanding and applying the algebra of sets. Solving two- and three-set problems	Students will be able to: Describe set and the various types. Apply the algebra of sets. Solve two and three set problems (including use of Venn diagrams)	Introduce set as very important concept in mathematics in everyday life collection. Discuss with the students the definition of set as a well-defined collection of objects of the same kind. Explain the various types of sets Universal set, empty set, subset. Teacher to Describe *set notation, members/elements by* • Listing its members T = (2,3,4,5,7,11) • Given word description of its members A = (prime numbers less than 12). • Using a set-builder notation B = $(x{:}\, 1 < x < 12)$ $where\ x\ is\ a\ prime\ numbers$	Diagram of various set type on vanguard Illustrated Venn diagram on vanguard	Ask student short answer questions. E.g. Name any 3 types of set. Write two sets and ask students to illustrate union, intersect and complement of set. Write a three sets word problem on the board and asked the students to calculate i). One only ii) Both iii). All the three

		Explain the various types of sets Teacher to explain the illustrated Venn diagram on the vanguard Discuss operation of sets ∪ : Union ∩ : Intersection ! : Complement disjoint sets Solve two and three set problems (including use of Venn diagrams)		
Indices Rules of indices Combination of two rules of indices Equation of indices	Students will be able to: State the rules of indices and solve problems Solve problems on exponential equations using the laws of indices	Explain the meaning of indices (as the plural of an index). Try give synonyms to index. E.g., Exponent, power etc. State each rule/law with cited examples. Example: $a^m \times a^n = a^{m+n}$. Solve problems related to exponential equations		Organize students in pairs/groups and give the tasks to solve. Move round helping struggling students. Example: Simplify $125^{-4} \div 25^3$ Solve the equation $8^{2x-3} =$ 0.5
Number Base and Modular Arithmetic Express based ten numbers to any other based number. Express any other based number to a ten number. Operations on number base Number base leading to simple line equation	Students will be able to: Explain and define number base and modular arithmetic Convert between bases Solve simple equations on number base Difference between number base and modular arithmetic	Ask the students to count in some of our local dialects like mende, themne or limba. Relate that to number base that some count in base 5 some in base 10 etc. as the case maybe. Solve some problems on conversion between bases. Like $base_3\ to\ base_4$ Use the clock to introduce modular arithmetic (i.e., GMT and the normal time).		Give pupils class exercises for them to try' Example: Convert 522_{seven} to a number in base two. If the numbers are in base three solve the equation $121x + 11 = 1100$ Simplify 4×3 in $mod\ 5$ Find $(3-4)mod5$

Operation on modulo (limited to 5 elements in the set)	Solve problems on modular arithmetic	Solve simple problems on modular arithmetic.		
		YEAR 1 SECOND TERM		
Basic algebras Algebra and real numbers Operations on polynomials Factoring polynomials Linear equations and inequalities in one variable	Students will be able to: Explain basic terminologies like variable, coefficient etc. Use the properties of the real number system to algebra Simplify algebraic expressions Factorize a trinomial Solve simple problems on linear equations in one variable and two variables (simultaneous equations) Solve inequality problems.	Explain and discuss the various terminologies with cited examples (E.g. Numerical and literal coefficients, variables etc.) Solve problems using the associative or distributive rules of real numbers Solve problems of factorization and simultaneous linear equations not more than two variables.		Ask students explain certain terms in algebra. Organize students in groups/pairs and give exercises to try in class Example: Expand and simplify $\{2(x+3)-4(2x-3y)+1\}-3x-2y$ factorize$x^2-8x-20$ solve the equation $\frac{2(z-1)}{3}-\frac{3(2z+3)}{4}=\frac{2}{12}$
Relations and Functions Describing Relations, Mappings and Functions Using Function Notation. Specifying the different types of functions Representing functions.	Students will be able to: Describe Relations, Mappings and Functions Apply Function Notation, (domain, range, dependent and independent variables) Identify the different types of functions (one to one, one - many & many to one	Introduce the concept of relation as an association that exits between two sets of objects the domain and co- domain. Further explain that relation can be shown by means of order pairs Discuss with the students that the set of all possible images of the domain is called range.	Illustrated cases of relation, mapping and functions on vanguard Graph board Graph paper Blackboard ruler Foot rule	Ask students to draw the various types of functions  E.g. Find the images of the elements of the domain [-2,-

Inverse Functions and Composite Functions Graphs and roots of Functions	etc.) Represent functions using tables, algebraically and graphically Evaluate Inverse and Composite Functions Draw graphs of functions and determine the roots of functions	**Note.** A relation may exist between two sets but not all the elements of the domain may be associated with elements of the co-domain. Types of relation: One –to-one, many- to- one, many –to- many. Describe Mapping as a relation in which each member in the domain maps onto only one member in the co-domain I.e. one to one and many to one relation are mappings. Discuss with the students how to identify functions from these characteristics. a). Each element in A must be matched with an element in B b). Some elements in B may not be matched with any element in A c). Two or more elements in A may be matched with the same element in B d). An element in A cannot be matched with two different elements in B. Discuss with the students the domain and range of the given function. Illustrate Graphs of functions using the graph board. If f is a function with domain D, then the graph of 'f' is the set of all points P(x, f(x)) in the plane. That is the graph of '*f*' is the graph of y = f(x).	Markers Colored chalks Pencils Diagram of the various types of functions • One to one • One to many • Many to one Showing the Domain and the image (Range)	1 0, 1, 2] define by the function $f : x \to \frac{3x-1}{x-3}$ E.g. Draw the graph of the function f(x) = 2x + 1 in the interval $-2 \leq x \leq 4$ **Inverse Function** Given the function i). f (x) = 3x − 2, find its inverse. ii). Given f(x) = 2x + 3, find $f^{-1}(x)$. iii). Find the inverse of the following function g(x) = (x + 4)/ (2x -5) **Composite Function** i). Given the functions $f(x) = x^2 + 6$ and g (x) = 2x – 1, find $(f \circ g)(x)$. ii). Given the functions g (x) = 2x – 1 and f (x) = $x^2 + 6$, find $(g \circ f)(x)$. iii). Find $(g \circ f)(x)$ given that, f (x) = 2x + 3 and g (x) = $-x^2 + 5$

		Solve problems with the students involving function of a function i.e. (fog)(x) = f(g(x)) and inverse function Restrict to simple algebraic functions only. Draw Graphs of Functions and determine the Roots of Functions using the graph board		
		YEAR 1 THIRD TERM		
Polynomial Functions 1 General Characteristics of functions	Students will be able to: Recognise equations of polynomial functions of degree ≤ 4 Simplify the algebra of polynomial functions State and apply the Remainder theorem and the Factor theorem	Write the remainder and factor theorem and demonstrate how to apply them in simplifying polynomial **Remainder Theorem** if a polynomial $f(x)$ is divided by $x - k, the\ remainder\ is$ $r = f(k)$ E.g. Use the remainder theorem to evaluate the function at x = -2 $f(x) = 3x^3 + 8x^2 + 5x - 7$ **Factor Theorem** A polynomial $f(x)$has a factor$(x - k)$ if and only if $f(k) = 0$ E.g. Show that $(x - 2) and (x + 3)$ are factors of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$	Textbooks Chart showing polynomial functions of degree ≤ 4 a). Linear function b). Quadratic function c). Cubic function	i). The remainder after $2x^2-5x-1$ is divided by x−3 ii). the remainder after $2x^2-5x-1$ is divided by x−5 iii). Use the Factor Theorem to find the zeros of $\boldsymbol{f(x) = x^3 + 4x^2 - 4x - 16}$ given that (x-2) is a factor of a polynomial. iv. use the factor theorem to find the zeros of $\boldsymbol{f(x) = x^3 - 6x^2 - x + 30}$. Given that (x+2) is a factor of a polynomial.
Polynomial Functions 11 Linear Functions	Students will be able to: Identify linear function represented by a straight-line graph Sketch graphs of linear equations	Discuss linear function as a graph $f(x) = ax + b$ is a line with slope $m = a$ and y- intercept at $(0, b)$. Use the graph board, Blackboard ruler, colored chalks and allow students to work on graph paper to demonstrate how to sketch linear graph	Graph board Graph paper Blackboard ruler Foot rule Markers Colored chalks Pencils	Plot the points and find the slope of the lie that passes through the pair of points i). $(-3, -)\ and\ (1,6)$ ii). $(2,4) and\ (4, -4)$ Use the point on the line and the slope of the line to

	Derive equations of linear equations using slope-intercept, slope point, two points Find equations of parallel and perpendicular lines to a given line Solve simultaneous linear equations graphically or algebraically	Help the students Derive equations of linear equations using **a) slope-intercept** $m = \frac{y_2 - y_1}{x_2 - x_1}$ **b) point - slope** $y - y_1 = m(x - x_1)$ **c) two points** $D(x, y)$ and $R(x, y)$ **For Parallel lines the gradients are equal i.e.,** $m_1 = m_2$ **For Perpendicular lines, the product of their gradients is minus one i.e.** $m_1m_2 = -1$ Where m_1 and m_2 are gradients of the two lines? Teacher Use the graph board, Blackboard ruler, Colored chalks and allow students to work on graph paper to demonstrate how to sketch simultaneous linear equations graphically and algebraically (including methods of elimination and substitution		determine the general equation of the line. 1. Point (2,1) and slope m =1 2. Point (-5,4) and slope m =2 Determine whether the lines L_1 and L_2 are parallel or perpendicular i). L_1(0,-1),(5,9) $L_2(0,3), (4,1)$ .L_1(3,6),(-6,0) $L_2(0, -1), (5, \frac{7}{3})$ **Simultaneous Linear** Solve the following pair of simultaneous linear equations: 2*x* + 3*y* = 8 3*x* + 2*y* = 7 Using elimination, substitution and graphical methods.
Quadratic Functions	Students will be able to: Recognise quadratic functions represented by a parabola Sketch graphs of quadratic functions using turning	Teacher to define quadratic function. Let a, b, and c be real numbers with $a \neq 0$. The function $f(x) = ax^2 + bx + c$ Use the graph board, blackboard ruler to illustrate quadratic graph		Solve the quadratic equation by completing the square $x^2 + 4x + 1 = 0$ Solve the quadratic equation by formula method

	points, intercepts and axis of symmetry Determine the nature of the roots of a quadratic equation Use the discriminant Solve quadratic equations by the graphical method, factorizing method, completing the square and Quadratic formula Derive quadratic equations given sufficient information Solve simultaneous equations for one linear, one quadratic	turning points, intercepts, and axis of symmetry. Solve problem on quadratic equation by a) Graphical method b) Factorizing method c) Completing the square d) Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Demonstrate the Roots of quadratic equations – equal roots ($b^2 - 4ac = 0$), real and unequal roots ($b^2 – 4ac > 0$), imaginary roots ($b^2 – 4ac < 0$); sum and product of roots of a quadratic equation e.g., if the roots of the equation $3x^2 + 5x + 2 = 0$ are α and β, form the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.		$-4x^2 + x + 3 = 0$. Solve the quadratic equation by graphical method. $-4x^2 + x + 3 = 0$.
Polynomial Functions III Rational Functions Exponential and Logarithmic Functions	Students will be able to: Recognize rational function as a quotient of two polynomial functions Apply the four operations on rational functions Students will be able to: Decompose rational functions into partial	Teacher to explain to the students that rational function can be written in the form $f(x) = \frac{N(x)}{D(x)}$ Where N(x) and D(x) are polynomials and D(x) is not zero. Solve problems as work examples with the students involving rational functions E.g. Find the domain of the function	Charts of laws of indices Chart of laws of logarithms Graph board Graph paper Blackboard ruler Foot rule	1). If $f: x \rightarrow \frac{1}{2+x}$,find the range if the domain is the set $[x: 1 \leq x \leq 5]$ 2). Simplify the following rational functions $\frac{1}{x-2} + \frac{3}{x+1}$ $\frac{4}{x+2} - \frac{3}{x+3}$

	fractions: – Linear factors in the denominator – Repeated linear factors in the denominator – Quadratic factors in the denominator Students will be able to: Apply the laws of indices Solve equations involving indices Apply the laws of logarithms Solve equations involving logarithm and change of base Draw and interpret graphs of exponential relations	$f(x) = \frac{4(x+1)}{x(x-4)}$ Discuss with the students relation between exponential and indices. i.e. Exponential function $\boldsymbol{f}$ with base $\boldsymbol{a}$ is denoted by $f(x) = a^x$ Where a $>$ 0, a $\neq$ 1 and x is any real number. *Note to the students that in many applications the most convenient choice for a base is the irrational number $e = 2.718281828$ Discuss the definition of logarithms function with base a. I.e. for $x > 0$ $and 0 < a \neq 1$ $y = log_a x$ $if\ and\ only\ if$ $x = a^y$ Hence $f(x) = log_a x$ is the logarithms function with base a. E.g., Simplify $log_5 5^x$ Solve problems with students involving exponential (indices) and logarithm equations E.g., Solve $2(3^{2x-5}) - 4 = 11$ Solve $log_3(5x-1) = log_3(x+7)$ Demonstrate the properties of logarithms. $log_a(UV) = log_a U + log_a V$	Markers Colored chalks Pencils	$\frac{2x}{x^2-1} \div \frac{x^2-2x}{x^2-2x+1}$ Without using mathematical table simplify the following $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$ ii). $16^{-\frac{3}{2}}$ Find the value of x in the following i). $3^{x^{2-1}} = 9^4$ ii) $3^{2x} - 4(3^x) + 3 = 0$ Simplify the following i). $log_5 10 + log_5 12$ ii) $log_3 24 + log_3 15 - log_3 10$ Solve the following equation $log_{10}(5x+6) = log_{10}(5x - 6)$ $log_{10}(x^2 1) - 2log_{10} x = 1$

		$log_a\left(\frac{U}{V}\right) = log_aU - log_aV$ $log_aU^n = nlog_aU$ *Note to the students that there is a natural logarithmic function defined by $f(x) = log_ax = \ln x \quad x > 0$		

Topic/Theme/Unit	Expected learning outcomes	Recommended teaching methods	Suggested resources	Assessment of learning outcomes
YEAR 2 FIRST TERM				
Sequences and Series	Students will be able to: Differentiate between finite and infinite <u>sequences</u> Describe the properties of Recurrence sequences, Arithmetic sequences, Geometric sequences Differentiate between finite and infinite <u>series</u> Describe the properties of Recurrence series, Arithmetic series, Geometric series Calculate the sum of Arithmetic sequences and Geometric sequences	Teacher to explain finite and infinite <u>sequences.</u> Illustrate how to find terms of a sequence $a_1, a_2, a_3, a_4, \ldots a_n$ Discuss the properties of sequences. **a). Recurrence sequences** Generating the terms of a recurrence series and finding an explicit formula for the sequence e.g. 0.9999 = $\frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + \frac{9}{10^4} + \cdots$ **b). Arithmetic sequences** A sequence whose consecutive term have a **Common Difference** $a_n = dn + c$ Un = U1 + (n-1)d **c). Geometric sequences** A sequence whose consecutive terms have a **Common Ratio.** $a_n = a_1 r^{n-1}$ Solve problems involving finite and infinite sequence. Demonstrate the step by step method of calculating sum of Arithmetic sequences and Geometric sequences • Sum of Arithmetic series (AP)		Class excises **AP** example, the sum of the first seven terms of the series 1, 4, 7, 10, 13,.. Determine the number of the term whose value is 22 in the series $2\frac{1}{2}$, 4, $5\frac{1}{2}$,7,... Find the sum of the first 12 terms of the series 5, 9, 13, 17, ... **GP** example, find the sum of the first eight terms of the GP 1, 2, 4, 8, 16, ... Find the sum to infinity of the series 3, 1, $1\frac{1}{3}$, ... Find the tenth term of the series 5, 10, 20,40, ...

		$S_n = \frac{n}{2}(a_1 + a_n)$ $S_n = \frac{n}{2}[2a + (n-1)d]$ • Sum of Geometric series (GP) $S_n = \sum_{n=1}^{n} a_1 r^{n-1} = a_1\left(\frac{1-r^n}{1-r}\right)$ when r<1 $s_n = \frac{a_1(r^n-1)}{(r-1)}$ when r>1		
YEAR 2 SECOND TERM				
Mathematics for finance I Ratio Rate Proportion Profit and loss Percentage change (increase/decrease) Bills and Tariffs Taxation and Wages	Students will be able to: Explain the concepts of ratio and determine it. Explain the rate and relate it to ratio. Calculate rates Determine the concept of proportion and how to determine it. Discuss the concept of profit and loss and explain how to compute profit and loss. Solve problems on percentage increase and decrease.	Discuss the comparisons we make on everyday life and how they are related. You can introduce the concept ratio in terms of number of males in the class to females in the form $x \text{ to } y, x: y$ (read as x is to y) Explain also that ratios can be expressed as fractions. That is $x: y \Rightarrow \frac{x}{y}$ Introduce rates by comparing quantities that are not alike and ask for examples. Discuss the concepts of increase and decrease. Increase= $new\ value - old\ value$ decrease= $old\ value - new\ value$		Arrange students in groups and give them class work to try. Example: $Le\ 2600$ was raised at the Mabamba village fun day. It was decided to give 45% to save the children, 30% to street child of Sierra Leone and the rest to the village fund society. How much did the village society receive? The pump price for fuel was raised from $Le\ 9500$ to $Le\ 10000$ per liter. Calculate the percent increase in the pump price. Working on his own, a bricklayer building a wall can lay 288 bricks in 4hrs. How

	Define basic and related terms to taxation like GST. Compute income from salaries, house property, capital gains and income from other sources	Demonstrate using the common rate calculation for speed and work through examples. Explain the concept of proportion and explain means and extremes.		many bricks can he lay in $6\frac{1}{2}$ hrs?
Mathematics for Finance II Simple interest Compound interest Future value of an annuity (sinking fund) Present value of an annuity (amortization)	Students will be able to: Define the concept of interest and show how it relates to the time value of money. Distinguish between simple and compound interest and demonstrate how to calculate each. Outline the process of calculating a repayable schedule for a loan with a blend of interest and principal. Use either formulae or compound interest tables to compute the future and present values of a single payment.	Discuss the terms principal, maturity date, the term of loan, blended payment, amortization, and net present value Give everyday examples of compound interest such as annuity, car loans, mortgages, etc. Solve simple compound interest and use a table to show how it works. Provide the formula for calculation of compound interest. That is $A = P\left(1+\frac{r}{100}\right)^t$ Relay the importance of investing money over time and how the longer the term the greater the reward.		Group students and give exercises for them to try in class. Example: A town borrows $Le\ 2{,}000{,}000$ at 5% per annum and repays $Le500{,}000$at the end of each year. How much still owes after the fourth payment?
Linear inequalities and linear programing Systems of linear inequalities in two variables	Students will be able to: Solve problems on linear inequalities Represent linear inequalities as a regions on	Start by explaining the right and left elbows of your body. That the right elbow is representing greater than($>$) and the left elbow represents the less than ($<$)		**Inequalities** Solve the inequalities and represent your solution on a number line. 1. 2 (x – 4) ≥ 3x – 5

Linear programming in two variables Business application of linear programming Introduction to simplex method	the coordinate plane Graph the feasible region in a linear programming problem Sketch a family of profit lines for a given problem Determine the optimum point or solution to a linear programming problem.	List the other inequality signs and solve simple problems and also try to represent your answers on the number line. Draw graphs of inequalities in two variables and shade required regions of interest.		2. $3y < 1 - 2y < 5 + y$ **Linear programming** A school is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only 9 has drivers available. The rental cost for a large bus is $Le\ 800000\ and\ Le600000$ for the small bus. Calculate how many buses of each type should be used for the trip for the least possible cost. Sketch the graph of the solution of the system of inequalities $x + 2y \leq 160$ $3x + y \leq 180$ $x \geq 0$ $y \geq 0$
YEAR 2 THIRD TERM				
Limits Definition of Limit of a function	Students should be able: Define the concept of limits of a function.	Teacher to explain the concept of limits	White board	Evaluate 1. $\lim_{x \to 2} x^3 = 2^3$ 2. $\lim_{x \to 2} x = 2$ 3. $\lim_{x \to 5} 3x = 3(5)$

Limit properties 1. Limits of constant 2. Limits of the function x^k 3. Limits of the function x 4. Limits of the function kx 5. Limits of the function $f(x).g(x)$ 6. Limits of rational functions 7. Limits involving infinity	Apply the limit property to evaluate given functions i). If $\lim_{x\to a} f(x) = k$ where k is a constant, then $\lim_{x\to a} k = k$ ii). $\lim_{x\to a} x^k = a^k$ iii). $\lim_{x\to a} x = a$ iv). $\lim_{x\to a} kx = ka$ v). $\lim_{x\to a} f(x).g(x) = \lim_{x\to a} f(x) . \lim_{x\to a} g(x) = f(a).g(a)$ vi). $f(x) = \frac{g(x)}{h(x)}$, then $\lim_{x\to a} f(x) = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{g(a)}{h(a)}$ vii). $\lim_{n\to\infty} f(x)$.	Discuss with the students the properties or theorem of limits with given examples Example: Find $\lim_{x\to 2}(x+3)(x^2-5)$ Solve problems with the students involving application of limit properties		4. $\lim_{x\to 2}(x^2-4x+2)$ 5. $\lim_{x\to 2}\left\{\frac{x^2-7x+10}{x^2-4}\right\}$ 6. $\lim_{x\to\infty}\left\{\frac{5x^2-1)}{2x^2+1}\right\}$
Introduction to calculus **Methods of Differentiation** Differentiate a function using first principle. Common functions Product rule of differentiation Quotient rule differentiation Chain rule (also known as function of a function)	Students will be able to: Define the derivative of a function Differentiate common functions Differentiate a product using product rule. E.g. If y=uv then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ Differentiate the quotient using the quotient formula E.g., If y = u/v	Explains the method of finding the derivative of function Discuss with students how to differentiate common functions such as $y = c, y = x^n$, etc and give them the general rules for differentiation. Discuss with pupils through questioning the meanings of product and quotient of numbers. Apply the product and quotient rule to Differentiate functions $y = (2x-2)(2x^3)$ (Product rule)		Organize students in groups and give them class work to try. Example: Find the differential coefficient wrt x for the functions $y = x^2 - 5x$. $f(x) = \frac{x^2-2x}{3-4x^2}$.

Successive differentiation (higher derivatives)	$then\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Differentiate a function of a function. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ Differentiate a function successively. E.g., $\frac{d^2y}{dx^2}$	If y= $\frac{(2x-2)}{(2x^3)}$ (Quotient rule) Solve problems on Differentiating function of a function. Teacher to introduce higher or successive differentiation		
Applications of differentiation Increasing and decreasing functions Stationary points Maximum and minimum points Marginal revenue and total revenue Profit maximization	Students will be able to: Describe an increasing and decreasing function. Explain how to find the stationary point of a function Explain marginal and total revenue Apply the concepts of differentiation to compute the marginal and total revenue Solve problems on profit maximization	Start by explaining that the derivative of a function is positive over the range where it is increasing and negative where it is decreasing. Also tell students that for all stationary points, the necessary condition is that $\frac{dy}{dx} = 0$. Explain the concepts of marginal and total revenue. Solve simple problems on profit maximization.		Organize students in groups and give them exercises to try. Example: X articles are produced at a total cost of $Le\ (x^2 + 40x + 10)$ and each one is sold for $Le\left(\frac{1}{3}x + 200\right)$. Find the value of x which gives the greatest profit and find this profit. Find the profit maximizing output for a firm with the total cost function $TC = 4 + 97q - 8.5q^2 + \frac{1}{3}q^3$ and total revenue function $TR = 58q - 0.5q^2$ A firm faces the nonlinear demand schedule $p = (650 - 0.25q)^{1.5}$. What output should it sell to maximize total revenue?

Topic/Theme/Unit	Expected learning outcomes	Recommended teaching methods	Suggested resources	Assessment of learning outcomes
		YEAR 3 FIRST TERM		
Matrices Operations on matrices Finding the determinant and inverse of a matrix (limited to 2 x 2 matrices) Application of matrices (Cramer's rule) to solve simultaneous linear equations in two variables	**Students will be able to:** Define matrix, the order of matrix (i.e. $2 \times 2, 2 \times 3\ etc$) and recognize the types of matrices Explain the operations (addition, subtraction, and multiplication of matrices up to 3X3 order) and solve problems Explain the determinant and its solution Solve problems on the inverse of a 2x2 matrix.	Define matrix and explain the order of a matrix Discuss and illustrate the operations on matrices. That is addition, subtraction, and multiplication two matrices. Solve problems on determinants and the inverse of a two-by-two matrix. E.g. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ c & a \end{pmatrix}$$		Ask pupils to explain different orders of matrices. Organize students in groups and give the class exercises to solve.
Statistics – Data Representation Grouped Data	Students will be able to: Define statistical terms Data collection procedure Represent statistical data using frequency distribution tables, histograms, cumulative frequency curve	Review the concept of ungrouped data. Introduce the topic by defining statistics as way of collecting, ordering, analyzing and interpreting data for proper decision making. Explain some statistical terms: Discrete data, continuous data, frequency, frequency distribution table, class interval, etc. Discuss how data are represented by: a). Frequency distribution tables b). Histograms	Graph board Graph paper Blackboard ruler Foot rule Markers Colored chalks Pencils	Write a short essay on the origin and development of the science of Statistics. Discuss the utility of Statistics to the state, the economist, the industrialist… in a planned economy. Example Draw histogram for the following frequency distribution. Variable: 10—20, 20—30 ,30—40, 40—50, 50—60 ,60—70, 70—80

		c). Cumulative frequency Illustrate an example on the graph board how to construct histogram and the ogive curve		Frequency: 12 30 35 65 45 25 18 respectively
Statistics 1a. **Data Analysis** Measures of Central Tendency (Grouped Data) Measures of Dispersion (Grouped Data)	Students will be able to: Calculate mean, mode, median, quartiles, and percentiles Estimate mode and modal class for grouped data from a histogram Estimate median and mean from grouped data a histogram Calculate mean and standard deviation, variance, range, inter-quartile range	Explain to the students meaning of mode, mean, median, quartiles, and percentiles Discuss with the students the terminologies and columns used as formula to calculate **Mode for grouped data** $Mode = L + \left[\frac{\Delta_1}{\Delta_1 + \Delta_2}\right]C$ **Median for grouped data** $Median = L + \left[\frac{\frac{1}{2}N - (\sum f)L}{f_{median}}\right]C$ **Mean for grouped data** $\bar{x} = \frac{\sum fx}{\sum f}$ Demonstrate how to estimate mode, median and mean for grouped data using histogram. Solve work examples using the various formulae above. Demonstrate how to Calculate mean and standard deviation, variance, range, inter-quartile range.	Electronic graph board Graph paper Blackboard ruler Foot rule Markers Colored chalks Pencils	Give students class exercises on measures of central tendencies for grouped and ungrouped data. Example: Construct a grouped frequency table and use the table to calculate the mean median and mode for the data below. 10, 20, 22, 67, 45, 43, 20, 14, 34, 54, 76, 43, 32, 21, 22, 12, 23, 34, 54, 67, 77, 56, 66, 54, 43, 76, 66, 54, 34, 32, 23, 43, 23, 25, 32, 12, 21, 23, 35, 34

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<td>Statistics II

Correlation

Regression</td>
<td>Students will be able to:

Define correlation

Describe forms of correlation

Draw scatter diagrams

Calculate correlation coefficient by Spearman's rank method

Draw the line and finding the equation of best fit (Regression)</td>
<td>Explain correlation as means of determine the relationship between two variables.

Further discuss with the students that the two variables are independent

Demonstrate using the chart to describe the forms of correlation. (Positive correlation, negative correlation and no correlation)

Illustrate scatter diagram on a graph board with pair values of two variables.

Solve problem with the students to calculate the correlation coefficient using spearman's rank method. *Use data without ties.

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$
$-1 \leq r \leq 1$

Explain the concept of regression line and the equation line of best fit.</td>
<td>Chart showing types of correlation.

Chart -- Line of best fit.2
Chart – illustrating scatter diagram

Electronic Graph board

Graph paper

Blackboard ruler

Foot rule

Markers

Colored chalks

Pencils</td>
<td>Compute the Spearman rank correlation.

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<td colspan="5">YEAR 3 THIRD TERM</td>
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<td>Probability – Introductory concepts</td>
<td>Students will be able to:

Define probability and related terminologies (sample space, outcome, observation, events,</td>
<td>Introduce probability as concept of chance and occurrence of event.

Discuss the related terminologies with cited examples.</td>
<td>Fair coin
Fair dice
Colored balls
Playing cards
Colored marbles</td>
<td>Random experiments, sample spaces, and events

A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5</td>
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	relative frequency, occurrence, not occurrence and experiment) Describe probability events as: – Equally likely events – Mutually exclusive events – Independent events Calculate probability value of experiment (coin flip and dice toss) Apply addition rule or product rule of probability Identify conditional probability Draw objects with and without replacement Construct the tree diagram	*Sample space, Outcome, Observation, Events, relative frequency, occurrence, not occurrence and experiment.* Describe probability events **a). Equally likely events** $P(E) = \frac{n(E)}{n(S)}$ For $0 \leq p(E) \leq 1$ **b). Mutually exclusive events** If A and B are mutually exclusive then. $P(A \cup B) = P(A) + P(B)$ **c). Independent events** If A and B are mutually exclusive, then. $P(A\ and\ B) = P(A) \times P(B)$ Calculate probability value of various events (coin flip and dice toss etc). Apply addition rule or product rule of probability Draw objects with and without replacement Discuss conditional probability and help students to construct the tree diagram.		blue balls. Determine the probability that it is (a) red, (b) white, (c) blue, (d) not red, (e) red or white. Independent events A fair die is tossed twice. Find the probability of getting a 4, 5, or 6 on the first toss and a 1, 2, 3, or 4 on the second toss. One bag contains 4 white balls and 2 black balls; another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that (a) both are white, (b) both are black, (c) one is white, and one is black.
Probability Permutations Combinations	Students will be able to: Define permutation and combinations Count outcome of events using permutation (order of	Introduce the fundamental counting principle Explain the factorial notation (i.e. $n!$ Read as $n\ factorial$)	Chart illustrating permutation and combinations formula	Arrange students in groups and give tasks to do. Examples: Evaluate the value of 7!

	choices considered) Count outcome of events using combination (order of choices not considered)	Discuss what permutation and combinations are: **Permutation** as order of choices considered $P_r^n = \frac{n!}{(n-r)!}$ $= n(n-1)(n-2)(n-3) \ \ldots.(n-r+1)$ **Combinations** order of choices not considered. $C_r^n = \frac{n!}{(n-r)!r!}$ Solve problem on permutation and combination.		Find the permutation and combination if n = 12 and r = 2. In how many ways of 4 girls and 7 boys, can be chosen out of 10 girls and 12 boys to make the team? How many words can be formed by 3 vowels and 6 consonants taken from 5 vowels and 10 consonants? Suppose 30 people are in a room. What is the probability that there is at least one shared birthday among these 30 people?

Resources

- Coloured chalk
- White board
- Coins
- Dice
- Coloured counters
- Coloured marbles
- Vanguard
- A4 coloured paper
- Graph board
- Graph paper
- Board ruler
- Foot rule