The New Senior Secondary Curriculum for Sierra Leone

Subject Syllabus for Calculus
Subject stream: Mathematics and Numeracy

This subject syllabus is based on the National Curriculum Framework for Senior Secondary Education. It was prepared by national curriculum specialists and subject experts.

SSSC
Senior Secondary School Curriculum

Curriculum elements for Calculus – a core subject

Subject Description

Calculus is a branch of mathematics involving or leading to calculations dealing with continuously varying functions – such as velocity and acceleration, rates of change and maximum and minimum values of curves. Calculus has widespread applications in science, engineering and social science. and is used to solve complicated problems for which algebra alone is insufficient.

General Learning Outcomes (Broad Goals)

At the end of the course, students will be able to:

- demonstrate conceptual knowledge and understanding and manipulative skills in calculus
- analyse data and draw valid conclusions
- apply logical, abstract and precise reasoning skills.

Subject Content Outline by Broad Themes & Specific Topics

Inequalities

- Drawing graphs of equations involving inequalities
- Finding the area inside a shaded region

Coordinate Geometry

- Rectangular coordinates called Cartesian coordinate system
- Equations for two intersect Lines (Perpendicular)
- Distance between two points
- The Midpoint formula

Equation of circle

- Equation of circle with a given radius r
- Centre of a circle with a given equation

Ellipse

- Equation of an ellipse with a given Centre

Indices

- Rules of indices
- Expressing a number as a product of its' prime factors

Standard Form

- Expressing numbers in standard form (including positive and negative powers)
- Operations on standard form
- Conversion of standard form

Surds

- Operation on surds
- Rationalising denominators

Functions

- Definition and Types of functions
- The Domain and Range of a function
- Graphs of functions

Composite Functions

- Combining function and another function

Polynomial functions

- A polynomial function has a degree more than 2

Limits

- Definition of Limit of a function
- Limit properties

Introduction to Derivatives

Differentiation

- Methods of Differentiation
- Implicit Differentiation
- Derivative of Trigonometric Functions

Applications of Differentiation

Integration
The general solution of:
- Indefinite integral
- Definite integral

Techniques of integration
- Integrating Trigonometric Functions
- Integration by substitution
- Integration of Logarithmic functions
- Integration of Exponential Functions

Applications of Integration
- Area under curves and volumes of revolution of solids
- Numerical integration (Trapezoidal Rule)

Differential Equations
- First order differential equations
- Applications of first order differential equation

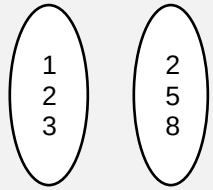
Structure of the Syllabus Over the Three Year Senior Secondary Cycle

	SSS 1	SSS 2	SSS 3
Term 1	**Inequalities** Drawing graphs of equations involving inequalities. Finding the area inside a shaded region **Indices** Rules of indices. Expressing a number as a product of its' prime factors. **Standard Form** Expressing numbers in standard form (including positive and negative powers). Operations on standard form. Conversion of standard form	**Coordinate Geometry** Rectangular coordinates called Cartesian coordinate system. Equations for two intersect Lines (Perpendicular). Distance between two points. The Midpoint formula. **Equation of circle** Equation of circle with a given radius r. Centre of a circle with a given equation. **Ellipse** Equation of an ellipse with a given Centre.	**Techniques of integration** Integrating Trigonometric Functions. Integration by substitution Integration of Logarithmic Functions Integrating Exponential Functions.
Term 2	**Surds** Operation on surds Rationalising denominators **Functions** Definition and Types of functions. The Domain and Range of a function. Graphs of functions **Composite Functions** Combining function and another function	**Differentiation** Methods of Differentiation Implicit Differentiation Derivative of Trigonometric Functions	**Applications of Integration** Area under curves Numerical integration (Trapezoidal Rule) **Differential Equations** First order differential equations **Applications of first order differential equation**
Term 3	**Polynomial functions** A polynomial function has a degree more than 2. **Limits** Definition of Limit of a function Limit properties **Introduction to Derivatives**	**Applications of Differentiation** **Integration** The general solution of (Indefinite integral and Definite integral)	

Teaching Syllabus

Topic/Theme/Unit	Expected learning outcomes	Recommended teaching methods	Suggested resources	Assessment of learning outcomes
YEAR 1/TERM 1				
Inequalities Drawing graphs of equations involving inequalities. Finding the area inside a shaded region.	Students should be able to: Draw graphs of equations involving inequalities. Calculate the area inside a shaded region.	Teacher to explain symbols involve in inequalities ($<, >, \leq, \geq$) Demonstrate to the students on the graph board how to illustrate inequalities Teacher to work with the students to do more work examples Teacher to work with the students calculate work examples on the area of the shaded region	Graph paper Graph board Foot rule Pencil	Students to draw graphs of inequalities on a graph paper E.g. Draw the graph of $y \geq 2x$ Ask students to calculate the area of the shaded region for a given equation
Indices Rules of indices Expressing a number as a product of its' prime factors.	Students should be able to: Apply the Rules of indices to simplify expressions. Express a number as a product of its' prime factors.	Discuss with the students the rules or laws of indices Solve problems with the students involving two or more rules to simplify expression. Demonstrate to the students how to Express a number as a product of its' prime factors.		Short answer quiz Ask students to state the rules of indices. Simple drills with students Ask students to simplify expressions using the rule of indices.

Standard Form Expressing numbers in standard form (including positive and negative powers) Operations on standard form. Conversion of standard form	Students should be able to: Express decimals in standard form $a \times 10^n$ if $1 \leq a < 10$ and n is an integer E.g. 41460 = 4.146×10^4 0.000075 = 7.5×10^{-5} Add and subtract standard form. Multiply and divide standard form. Convert standard form to ordinary number.	Review the concept of decimal places Discuss with the students the principle of standard form including positive and negative powers. Solve problems with the students involving addition, subtraction, multiplication and division of standard form. Demonstrate to the students how to convert standard form to ordinary number		Give students to practice work exercises and some homework to students for practice E.g. Write these numbers in standard form. i. 548144 ii. 0.000005123 E.g. Evaluate i). $3.9 \times 10^4 + 8.5 \times 10^4$ ii) $9.7 \times 10^6 - 4.1 \times 10^6$ E.g. Change the following standard form to ordinary number. a. 7.13×10^{-3} 3.2456×10^3
YEAR 1/TERM 2				
Surds Operation on surds Rationalising denominators	Students should be able to: Add and Subtract Surds. Multiply and Divide Surds. Expand and simplify surds	Review the concept of perfect squares Discuss with the students how a multiple number is simplified into two factors E.g. $\sqrt{50}=\sqrt{25 \times 2}$ $= \sqrt{25} \times \sqrt{2}$ $= 5\sqrt{2}$ Solve problems with the students involving addition, subtraction, multiplication and division of surds Demonstrate to the students how to Rationalise denominators		Short answer quiz on perfect squares Class exercises or test on operations on surds E.g. Evaluate the following

[H] Functions Definition and Types of functions. The Domain and Range of a function. Graphs of functions	Students should be able to: Define the term function. Describe the Types of functions • One to one • One to many • Many to one Determine the Domain and Range of a function. Draw graphs of functions	Teacher to Review the concept of mapping and relation. Demonstrate various functions to the students using the diagram Discuss with the students the domain and range of the given functions. Illustrate graphs of functions using the graph board. If f is a function with domain D, then the graph of 'f' is the set of all points P(x, f(x)) in the plane. That is the graph of '*f*' is the graph of y = f(x).	Diagrams showing the domain and image (range) of various types of functions: • One to one • One to many • Many to one Graph paper Graph board Foot rule Pencil Blackboard rule	Ask students to draw the various types of functions  E.g. Find the images of the elements of the domain [-2,-1 0, 1, 2] define by the function $f : x \rightarrow \frac{3x-1}{x-3}$ E.g. Draw the graph of the function f(x) = 2x + 1 in the interval $-2 \leq x \leq 4$
Composite Functions Combining functions	Students should be able to: • Evaluate function of a function {Composite function) i.e., (fog)(x) = f(g(x))	Solve problems with the students involving function of a function i.e., (fog)(x) = f(g(x))		E.g. Find fg given that $f(x) = 2x - 7$ *and* $g(x) = x^2 - 3$
YEAR 1/TERM 3				
Polynomial functions A polynomial function has a degree more than 2.	Students should be able to: Describe and simplify polynomial function (the highest power is more than 2)	Solve problems with the students involving polynomial function including power of more than 2.		Eg. Expand and simplify $(3x - 2)(1 - x)(2x + 1)$
Limits Definition of Limit of a function	Students should be able to:	Teacher to explain the concept of limits	White board	Evaluate 1. $\lim_{x \to 2} x^3 = 2^3$

Limit properties 1. Limits of constant 2. Limits of the function x^k 3. Limits of the function x 4. Limits of the function $k\dot{x}$ 5. Limits of the function $f(x).g(x)$ 6. Limits of rational functions 7. Limits involving infinity	• Define the concept of limits of a function. • Apply the limit property to evaluate given functions i). If $\lim_{x\to a} f(x) = k$ where k is a constant, then $\lim_{x\to a} k = k$ ii). $\lim_{x\to a} x^k = a^k$ iii). $\lim_{x\to a} x = a$ iv). $\lim_{x\to a} kx = ka$ v). $\lim_{x\to a} f(x).g(x) = \lim_{x\to a} f(x) . \lim_{x\to a} g(x)$ = $f(a).g(a)$ vi). $f(x) = \frac{g(x)}{h(x)}$, then $\lim_{x\to a} f(x) = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{g(a)}{h(a)}$ vii). $\lim_{n\to\infty} f(x)$.	Discuss with the students the properties or theorem of limits with given examples Example: Find $\lim_{x\to 2}(x+3)(x^2-5)$ Solve problems with the students involving application of limit properties		2. $\lim_{x\to 2} x = 2$ 3. $\lim_{x\to 5} 3x = 3(5)$ 4. $\lim_{x\to 2}(x^2-4x+2)$ 5. $\lim_{x\to 2}\left\{\frac{x^2-7x+10}{x^2-4}\right\}$ 6. $\lim_{x\to\infty}\left\{\frac{5x^2-1)}{2x^2+1}\right\}$
Introduction to Derivatives Find the derivatives of simple functions.	Students should be able to: • Define the derivative of a function • Find the derivative of simple function.	Ask questions about the meaning of a straight line between two points (x_1, y_1) and (x_2, y_2) Record various responses from pupils on the board. Gradient=$\frac{increase\ y}{increase\ x} = \frac{y_2-y_1}{x_2-x_1}$ Teacher explains that small increments were	Electronics graph board Graph boards Rulers Graph papers	Give class work. E.g. differentiate from first principles the function $y = x^2$. Ask pupils to explain how they arrive at the answer

		added to both x and y then $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$. Write the notations of differentiation $\frac{dy}{dx}$ or $f^1(x)$ all denoting first differentials Solve problems with the students involving derivative of a function.		
YEAR 2/TERM 1				
Coordinate Geometry Rectangular coordinates called Cartesian coordinate system Equations for two intersect Lines (Perpendicular) Distance between two points The Midpoint formula	Students should be able to: Describe Rectangular coordinates. (X – axis and Y – axis) Find the equations for two intersecting Lines (Perpendicular) Calculate the distance between two points d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Use the Midpoint formula $\{\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2)\}$	Teacher to explain the four quadrants on the x,y plane Demonstrate to the students on the graph board how to plot points on graph paper. Discuss with the students the distance between two points and the midpoint formula. Solve problems on distance between two points and the midpoint formula as work examples	Graph paper Graph board Foot rule pencil A4 paper Vanguard Markers Cello tape	Ask students to explain what quadrants are. Allow students to plot points on graph paper using (X,Y) coordinate. E.g. R(3,5) T(6,-10) and H(-5,4) Give class exercises or test to students on calculating the distance between two points. E.g. Find the distance between two points (-2,3) and (1,7) Ask students to find the midpoint of the line segment joining (3,-4) and (7,2)
Equation of a circle	Students should be able to:	Present to the students how to draw circle using pair of compasses on a	Pair of Compasses Graph paper Foot rule	Ask students to use their pair of compasses to draw a

Equation of circle with a given radius r. Centre of a circle with a given equation.	Calculate the equation of circle with a given radius r. $r^2 = (x - x_0)^2 + (y - y_0)^2$ Determine the Centre of a circle with a given equation. $x^2 + y^2 + dx + ey + f = 0$ Where d, e and f are constants.	graph paper using coordinate point (X,Y). Solve problems with the students on i). The equation of circle with a given radius r. ii). The centre of a circle with a given equation.	Pencil Blackboard compass and ruler Graph board	circle and label the center and the radius. Assign class activities to student like the following: Calculate the equation of circle with a given radius r. E.g. Radius = 4 and centered at (-5,3) E.g. Find the Centre of a circle with equation $x^2 + y^2 - 8x + 2y + 8 = 0$
Ellipse Equation of an ellipse with a given Centre.	Students should be able to: Write equation of an ellipse with a given Centre. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Illustrate ellipse on a vanguard as teaching aid to teach the students. Explain the general standard equation of ellipse with a given center. Solve problems with the students on equation of an ellipse with a given center.	Vanguard Markers Cello tape Foot rule	Test the students to write the equation of an ellipse with a given centre. E.g. Find the equation of an ellipse having foci F_1(-4,0) and F_2(4,0) and sum of focal radii 10.
YEAR 2/TERM 2				
Methods of Differentiation Differentiate a function using first principle. Common functions Product rule of differentiation	Students should be able to: • Use the idea of limits to differentiate a function from first principles. • Differentiate common functions • Differentiate a product using product rule. • E.g. If $y = uv$	Teacher explains the method of finding derivative of function by first principles. Teacher discusses with students how to differentiate common functions such as : $y = c, y = x^n$, etc	White board, textbooks	Group pupils and give them class activities on the concepts taught. E.g. Use the quotient rule to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $y = \frac{2x}{x+5}$.

Quotient rule differentiation Chain rule (also known as function of a function) Successive differentiation (higher derivatives)	• $then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ • E.g. If y = $\frac{u}{v}$ $then\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ • Differentiate a function of a function. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ • Differentiate a function successively. E.g. $\frac{d^2y}{dx^2}$	Teacher can further discuss with pupils through questioning the meanings of product and quotient of numbers. Apply the product and quotient rule to Differentiate functions E.g. If $y = (2x-2)(2x^3)$ (Product rule) E.g. If y= $\frac{(2x-2)}{(2x^3)}$ (Quotient rule) Solve problems on Differentiating function of a function. Teacher to introduce higher or successive differentiation.		
Implicit Differentiation How to differentiate a function of another function	Student should be able to: • Use the chain rule to differentiate implicitly • Find the slope of a curve at a given point. Apply the concept of implicit differentiation to find the equation of a tangent to a curve at a given point.	Explain the meaning of implicit functions. Eg $x^2 - 3xy^2 - y = 6$ Explain to pupils how to differentiate implicitly Solve problems on implicit Differentiating as work examples		Group pupils in pairs and ask them to solve some problems E.g., Find $\frac{dy}{dx}$ for the function$2x^2 - 3xy = 7$.
Derivative of Trigonometric Functions How to determine the derivative of a	Student should be able to: • Compute the differentials of trigonometric functions	Discuss with pupils the three basic trigonometric ratios $(\sin x\,.\cos x\ and\ \tan x)$ with their corresponding reciprocals		Ask pupils to list the trigonometric ratios. Record their responses on the board. Ask pupils to find the differential coefficient of $y =$

trigonometric function with a given function. Differentiation of natural log functions and exponential functions	• Apply the techniques of differentiation to calculate the differentials of trigonometric functions • Differentiate composite trigonometric functions. • Differentiate logarithmic functions such as: $y = \log_e(2x - 5)$	$(\csc x . \sec x \ and \ \cot x)$ using the right-triangle. Solve problems on Differentiating trigonometric ratios applying the techniques of differentiation. Solve problems on Differentiating logarithmic and exponential functions applying the techniques of differentiation.		$\sin x$. Ask one or two pupils to try and solve it on the board.
YEAR 2/TERM 3				
Applications of differentiation Increasing and decreasing functions. Rates of change, velocity, and acceleration, turning points (maximum and minimum). Points of inflexion Tangents and normal Practical problems	Students should be able to: • Describe an increasing and decreasing function. • Apply differentiation to determine: I. rates of change II. velocity and acceleration III. (Maximum and minimum) IV. Tangents and normal V. Practical problems	Teacher to discuss with the students the meaning of rate of change, velocity and acceleration, turning points (maximum and minimum). Explain that at a turning point $\frac{dy}{dx} = 0$. Solve problems as work examples on some application of differentiation.		Ask pupils to explain velocity and acceleration. Give pupils some class work for them to try. E.g., Find the maxima and minima points of the function$y = (2x - 1)(4 - x)^2$.
Integration Process of Integration	Students should be able to:	Explain to pupils the meaning of integration		Ask pupils to give the difference between differentiation and integration

The general solution of 1. Indefinite integral 2. Definite integral	• define integration as the reverse of differentiation • determine the integrals of the form x^n and ax^n. Where n is a fractional, zero, or positive or negative integer $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ (indefinite integral) • $[x]_a^b = (b) - (a)$ (definite integral)	and the notation for integration as $\int$ Solve problems on indefinite integrals $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$. C is the arbitrary constant also known as the constant of integration. Explain the concept of definite integral $[x]_a^b = (b) - (a)$. Solve some mathematical problems on the definite and indefinite integrals.		Give pupils (groups) exercises to try in class. E.g., integrate x^2 E.g., find $\int_1^2 (3x - 4)\,dx$
YEAR 3/TERM 1				
Techniques of integration Introduction to integration of trigonometric functions. Integration by substitution Integration of logarithmic functions Integration of exponential functions.	Student should be able to: • Integrate simple trigonometric functions $\int \sin x\,dx$. • Integrate functions by substitution method • Integrate logarithmic functions $(\int \ln x\,dx)$ • Integrate exponential functions ($\int e^x\,dx$)	Ask pupils to state the basic trigonometric ratios. Explain and guide pupils to integrate trigonometric functions. Discuss with pupils the process of substitution in integration. Explain how to integrate logarithmic and exponential functions.		Integrate $\sin x$ and $\cos x$. E.g., Find $\int \frac{1}{2x}\,dx$.
YEAR 3/TERM 2				
Some applications of integration Area under curves	Students should be able to:	Discuss the concept of definite integral to find the area		Give class work to pupils whilst you walk around supervising. E.g., Find the

Numerical integration	• Apply integration to calculate areas under curves • Apply the trapezoidal rule to evaluate the area under a curve • Apply Simpson's rule to evaluate the area of a curve	$\left(\int_a^b f(x)\,dx \text{ or } \int_a^b y\,dx\right)$ and the volume of a solid obtained by rotating the area bounded by the curve $\left(V = \pi \int_a^b (f(x))^2\,dx\right)$ Explain the use of trapezium and Simpson's rule. Solve problems on the applications.		area bounded by the curve $y = 4x^2$, the x-axis and the ordinates x=0 and x=1
Differential Equations First order differential equations Applications of first order differential equation	Students should be able to: • Describe a first-order differential equation • Solve a differential equation of the form $\frac{dy}{dx} = f(x)$ • Solve a differential equation of the form $\frac{dy}{dx} = f(y)$ • Solve a differential equation of the form $\frac{dy}{dx} = f(x) \cdot f(y)$ • Apply first order differential equation	Explain with given examples of differential equations. Discuss with pupils the solution of first order differential equations in different forms. Solve some differential equations		Arrange pupils in groups and give them some problems to solve. E.g., Solve the differential equation$2dy + 1 = dx$. Move around to help those in need of explanations.

Resources

- Graph paper
- Foot rule
- A4 Vanguard
- Permanent markers
- Cellotape
- Compasses
- Graph Board
- Board ruler
- Electronic graph board