Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' handbook for

JSS Mathematics

JSS 2

Term 2

STRICTLY NOT FOR SALE

FOREWORD

The production of Teachers' Guides and Pupils' handbooks in respect of English and Mathematics for Junior Secondary Schools (JSSs) in Sierra Leone is an innovation. This would undoubtedly lead to improvement in the performance of pupils in the Basic Education Certificate Examination in these subjects. As Minister of Basic and Senior Secondary Education, I am pleased with this development in the educational sector.

The Teachers' Guides give teachers the support they need to utilize appropriate pedagogical skills to teach; and the Pupils' Handbooks are designed to support self-study by the pupils, and to give them additional opportunities to learn independently.

These Teachers' Guides and Pupils' Handbooks had been written by experienced Sierra Leonean and international educators. They have been reviewed by officials of my Ministry to ensure that they meet specific needs of the Sierra Leonean population.

I call on the teachers and pupils across the country to make the best use of these educational resources.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd. Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank the Department for International Development (DFID) for their continued support. Finally, I also thank the teachers of our country - for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo
Minister of Basic and Senior Secondary Education

The Ministry of Basic and Senior Secondary Education, Sierra Leone, policy stipulates that every printed book should have a lifespan of 3 years.

To achieve this DO NOT WRITE IN THE BOOKS.

Table of contents

Introduction

to the Pupils' Handbook

These practice activities are aligned to the lesson plans in the Teachers' Guide, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Education, Science and Technology.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

Lesson Title: Personal Expenditure	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-056	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the percentage of a person's income spent on a certain type of expense.

Overview

In this lesson you will make calculations on the money that people spend.

Income is the money you receive, usually when you are paid to do work. **Expenditure** means money spent. Personal expenditure means the amount of money you spend yourself. Some personal expenditures are food, clothing, and entertainment.

You can find the percentage of a person's income that they spend on a certain item. This can help people to understand how their income is spent, and make smart decisions with their money.

Remember that to express a quantity as a percentage of another (both in the same unit) write the given quantity as a fraction of the total. Multiply by 100% and simplify. This was covered in lesson 32.

We use the same idea to calculate personal expenditure. This is the formula we will use:

$$\text{percentage of income} = \frac{\text{expenditure}}{\text{income}} \times 100\%$$

Solved Examples

1. Ama's income is Le 4,000,000.00 each month. She spends Le 200,000.00 on transportation each month. What percentage of her income does she spend on transportation?

   Solution
   Apply the formula for personal expenditure:

$$\begin{aligned}\text{Percentage of income} &= \frac{\text{expenditure}}{\text{income}} \times 100\% \\ &= \frac{200{,}000}{4{,}000{,}000} \times 100\% \\ &= \frac{2}{40} \times 100\% \\ &= \frac{20}{4}\% \\ &= 5\%\end{aligned}$$

2. Sia sells goods in the market. This week she earned Le 1,000,000.00 She spent Le 400,000.00 to buy a new table for selling goods. What percentage of her weekly income did she spend on the table?

Solution

Apply the formula for personal expenditure:

$$\begin{aligned} \text{Percentage of income} &= \frac{\text{expenditure}}{\text{income}} \times 100\% \\ &= \frac{400{,}000}{1{,}000{,}000} \times 100\% \\ &= \frac{4}{10} \times 100\% \\ &= \frac{400}{10}\% \\ &= 40\% \end{aligned}$$

3. Abass earned Le 800,000.00 this week. He kept track of his expenses in the table below. Help him calculate the percentage of his income that he spent on each item. Write your answers in the table.

Expense	Amount (Le)	% of income
New clothes	200,000.00	
Food	160,000.00	
Transportation	100,000.00	

Solution

The completed table is below. Find the working below the table.

Expense	Amount (Le)	% of income
New clothes	200,000.00	25%
Food	160,000.00	20%
Transportation	100,000.00	12.5%

Percentage spent on new clothes:

$$\begin{aligned} \text{New clothes} &= \frac{200{,}000}{800{,}000} \times 100\% \\ &= \frac{1}{4} \times 100\% \\ &= 25\% \end{aligned}$$

Percentage spent on food:

$$\begin{aligned} \text{Food} &= \frac{160{,}000}{800{,}000} \times 100\% \\ &= \frac{16}{80} \times 100\% \\ &= 20\% \end{aligned}$$

Percentage spent on transportation:

$$
\begin{aligned}
\textbf{Transportation} &= \frac{\text{expenditure}}{\text{income}} \times 100\% \\
&= \frac{100{,}000}{800{,}000} \times 100\% \\
&= \frac{1}{8} \times 100\% \\
&= 12.5\%
\end{aligned}
$$

Practice

1. Michael earns Le 50,000.00 each day. He spends Le 5,000.00 on lunch at his work place. What percent of his daily income does he spend on lunch?
2. Mohamed earns 8,000,000.00 each month. He spends 400,000.00 each month on electricity. What percentage of his income does he spend on electricity?
3. Juliet earned Le 2,000,000.00 this week. She kept track of her expenses in the table below. Help her calculate the percentage of her income that she spent on each item. Write your answers in the table.

Expense	Amount (Le)	% of income
New clothes	200,000.00	
Food	400,000.00	
Transportation	250,000.00	
Mobile phone use	100,000.00	
Health care	500,000.00	

Lesson Title: Income Tax	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-057	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate tax on a person's income.

Overview

Taxes are how a government raises money to cover public costs. For example, tax money pays for hospitals, roads, and schools. Governments have different ways of raising money through taxes. Income tax is an amount that people pay from the money they earn working.

Income tax is taken from the money that people earn from their employers. It is sent to the government, and the employee gets the rest of the money.

The formula for calculating the income tax a person pays is:

$$\text{Income tax} = \text{income} \times \text{tax rate}$$

Tax rate is given as a percentage, but should be written as a fraction in the formula. For example, $10\% = \frac{10}{100}$

You can find the amount of money that a person earns after paying taxes. Subtract the tax they pay from the total income.

Solved Examples

1. Alpha's income is Le 3,500,000.00 per year. His income tax rate is 10%. How much tax will he pay?

   Solution
   Calculate his income tax using the formula:

$$\begin{aligned} \text{Income tax} &= \text{income} \times \text{tax rate} \\ &= 3{,}500{,}000 \times \tfrac{10}{100} \\ &= 35{,}000 \times 10 \\ &= \text{Le } 350{,}000.00 \end{aligned}$$

   Alpha will pay Le 350,000.00 in income tax for 1 year.

2. Fatu has an annual income of Le 7,000,000.00. Calculate how much she must pay in income tax each year if the rate is 9%.

Solution

Calculate her income tax using the formula:

$$\begin{aligned} \text{Income tax} &= \text{income} \times \text{tax rate} \\ &= 7{,}000{,}000 \times \tfrac{9}{100} \\ &= 70{,}000 \times 9 \\ &= \text{Le } 630{,}000.00 \end{aligned}$$

Fatu will pay Le 630,000.00 in income tax for 1 year.

3. Martin's income is Le 14,500,000.00 per year. His income tax rate is 12%. Calculate the following:
    a. The amount he must pay in income tax.
    b. His remaining income after taxes are paid.

Solution

a. Calculate his income tax using the formula:

$$\begin{aligned} \text{Income tax} &= \text{income} \times \text{tax rate} \\ &= 14{,}500{,}000 \times \tfrac{12}{100} \\ &= 145{,}000 \times 12 \\ &= \text{Le } 1{,}740{,}000.00 \end{aligned}$$

b. Subtract his taxes from his income to find the amount he receives:
$14{,}500{,}000.00 - 1{,}740{,}000 = \text{Le } 12{,}760{,}000.00$

Practice

1. Bendu's income is Le 8,650,000.00 per year. Her income tax rate is 15%. How much tax will she pay?
2. Sia has an annual income of Le 90,000,000.00. Calculate how much she must pay in income tax each year if the rate is 18%.
3. Michael's income is Le 15,860,000.00 per year. His income tax rate is 10%. Calculate the following:
    a. The amount he must pay in income tax.
    b. His remaining income after taxes are paid.

Lesson Title: Sales Tax	Theme: Everyday Arithmetic
Practice Activity: PHM-08-058	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the sales tax on a transaction.

Overview

Taxes are how a government raises money to cover public costs. For example, tax money pays for hospitals, roads, and schools. Governments have different ways of raising money through taxes. Sales tax is an amount that people pay when they buy something from a store. It can also be called "goods and services tax" (GST).

In this lesson, you will calculate sales tax. We don't always pay sales tax at every small shop. However, more and more shops are adding sales tax to the cost of items. This money goes to the government to help Sierra Leone pay for things we need.

The formula for calculating sales tax on an item is:

$$\text{Sales tax} = \text{cost of the item} \times \text{tax rate}$$

As in the previous lesson, rate is given as a percentage, but should be written as a fraction in the formula. For example, $10\% = \frac{10}{100}$

You can find the total amount you will pay for an item by adding sales tax to its cost.

Solved Examples

1. Ama is buying a new maths textbook. The textbook costs Le 30,000.00. If the sales tax rate is 6%, find:
   a. The sales tax
   b. The total amount she will pay

   Solutions
   a. Calculate the sales tax using the formula:
   $$\begin{aligned} \text{Sales tax} &= \text{cost of the item} \times \text{tax rate} \\ &= 30{,}000 \times \frac{6}{100} \\ &= 300 \times 6 \\ &= \text{Le } 1{,}800.00 \end{aligned}$$
   b. Add the sales tax to the price to find the total amount she will pay:
   $30{,}000 + 1{,}800 = \text{Le } 31{,}800.00$

2. A mobile phone costs Le 350,000.00. A 5% goods and services tax (GST) is now imposed. What is the new price of the mobile phone?

Solution

We are not asked to find the amount of the tax in this problem. However, we will find the tax first and use it to find the new price.

Step 1. Calculate the sales tax:

$$\begin{aligned} \text{Sales tax} &= \text{cost of the item} \times \text{tax rate} \\ &= 350{,}000 \times \tfrac{5}{100} \\ &= 3500 \times 5 \\ &= \text{Le } 17{,}500.00 \end{aligned}$$

Step 2. Calculate the total cost:

$$350{,}000 + 17{,}500 = \text{Le } 367{,}500.00$$

3. George buys a car with a selling price of Le 5,000,000.00. If the sales tax rate is 4%, how much will he pay for the car?

Solution

Step 1. Calculate the sales tax:

$$\begin{aligned} \text{Sales tax} &= \text{cost of the item} \times \text{tax rate} \\ &= 5{,}000{,}000 \times \tfrac{4}{100} \\ &= 50{,}000 \times 4 \\ &= \text{Le } 200{,}000.00 \end{aligned}$$

Step 2. Calculate the total cost:

$$5{,}000{,}000 + 200{,}000 = \text{Le } 5{,}200{,}000.00$$

Practice

1. Hawa is buying a new phone. The phone costs Le 450,000.00. If the sales tax rate is 5%, find:
   a. The sales tax
   b. The total amount she will pay
2. A fan costs Le 500,000.00. A 6% goods and services tax (GST) is now imposed. What is the new price of the fan?
3. Michael buys a new stove for Le 3,000,000.00. If the sales tax rate is 4%, how much will he pay for the stove?

Lesson Title: Time and Duration	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-059	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify and use language for 12- and 24-hour time.
2. Solve simple problems involving duration.

Overview

In this lesson, you will practise using both 12- and 24-hour time and solve problems involving duration. There are two ways to count time. You can count to 12 two times or count to 24 one time. We start counting the hours in a day at midnight.

If you count to 12 two times we call times using 'AM' after the number the first time or in the morning, and 'PM' after the number the second time around or in the afternoon and evening.

This is how we write hours in both the 12-hour and 24-hour systems:

12-hour	**24-hour**
12 am midnight	00:00
1 am	01:00
2 am	02:00
3 am	03:00
4 am	04:00
5 am	05:00
6 am	06:00
7 am	07:00
8 am	08:00
9 am	09:00
10 am	10:00
11 am	11:00

12-hour	**24-hour**
12 pm noon	12:00
1 pm	13:00
2 pm	14:00
3 pm	15:00
4 pm	16:00
5 pm	17:00
6 pm	18:00
7 pm	19:00
8 pm	20:00
9 pm	21:00
10 pm	22:00
11 pm	23:00

12-hour times are sometimes given with :00 after the number. For example, 8:00 pm and 8 pm are the same. The 12-hour times are usually read with "o'clock". For example, 5 pm is often stated "5 o'clock pm". The 24-hour times are usually read with "hundred hours". For example, 17:00 is often stated "17 hundred hours".

You can convert between the 2 systems. To convert from the 12-hour clock to the 24-hour clock:

- To convert an **am** time, use the same number and write :00 after it. The exception is 12 am midnight, which always converts to 00:00.

- To convert a **pm** time from the 12-hour clock to the 24-hour clock, add 12 and remove the pm label. The exception is 12 pm noon, which always converts to 12:00.

To convert a time from the 24-hour clock to the 12-hour clock:

- To convert an **am** time, use the same number and write am.
- To convert a **pm** time, subtract 12 and write pm.

If you are given 2 times and are asked to find a duration of time, **subtract**. Give your answer in units of hours and minutes. See Solved Examples 3 and 4 for example problems.

You may also be asked to find the time when an event ends, where you are given a time and the duration of the event. **Add** the duration to the time. Your answer will be a time. Use the same clock system in the answer that is given in the problem. See Solved Examples 5 and 6 for example problems.

Solved Examples

1. Convert the following times to the 24-hour clock:

   a. 3:05 am b. 3 pm c. 10 pm d. 7:30 am

   Solutions

   a. 3:05 am = 03:05
   b. Add 12 to change 3 pm to the 24-hour clock: 3 + 12 = 15:00
   c. Add 12 to change 10 pm to the 24-hour clock: 10 + 12 = 22:00
   d. 7:30 = 07:30

2. Convert the following times to the 12-hour clock:

   a. 05:00 b. 12:35 c. 16:00 d. 00:15

   Solutions

   a. 05:00 = 5 am
   b. 12:35 = 12:35 pm
   c. Subtract 12 from the hours: 16 – 12 = 4 pm
   d. Remember that the hour 00 is 12 midnight. This gives 00:15 = 12:15 am.

3. Foday started working on his maths assignment at 5:00 pm. He finished working at 6:45 pm. How long did it take him to finish his assignment?

   Solution

   Subtract the hours and minutes to find how long he worked on the assignment:
   6:45 – 5:00 = 1:45

   It took him 1 hour and 45 minutes to finish the assignment.

4. Bendu drove from Freetown to Bo. She started at 12:15, and arrived in Bo at 16:45. How long did it take her to drive to Bo?

   Solution

   Subtract the hours and minutes to find how long it took her: 16:45 – 12:15 = 4:30

   It took her 4 hours and 30 minutes to drive to Bo.

5. Fatu started working at 9:00 am. She worked for 3 hours and 30 minutes. At what time did she finish working?

   Solution

   Add the hours and minutes separately: 9:00 am + 3:30 = 12:30 pm

   Note that the answer is **pm**. If you count up 3 hours from 9 am, you reach 12 pm noon.

6. Michael is in a choir. Choir practice is on Tuesdays at 18:30. If practice lasts for 1 hour and 30 minutes, at what time does it finish?

   Solution

   Add the duration to the time: 18:30 + 1:30 = 19:60

   We cannot have a time of 19:60! Remember that there are 60 minutes in an hour. If you add the minutes and they are 60 or more, convert 60 minutes to an hour and add 1 hour.

   This gives us 18:30 + 1:30 = 20:00

Practice

1. Convert the following times to the 24-hour clock:
   a. 1 am b. 12:45 am c. 8 pm d. 12:30 pm
2. Convert the following times to the 12-hour clock:
   a. 02:00 b. 10:15 c. 19:00 d. 23:30
3. Mustapha has a small backyard garden. He spends 45 minutes each day working in his garden. If he started working at 9 am, what time did he finish?
4. Hawa has an important maths exam. She will spend 2 hours and 30 minutes studying for the exam. If she starts studying at 17:00, at what time will she finish?
5. Abass plays the drums. This morning he practised playing the drums from 08:00 to 12:15. How much time did he spend practising?
6. Sia walks to school each day. She starts at 07:30 and reaches school at 08:30. How long does it take her to walk to school?

Lesson Title: Problem Solving with Time	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-060	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to solve story problems involving time and duration.

Overview

In this lesson, you will continue solving problems involving time and duration. The problems in this lesson have multiple steps.

Solved Examples

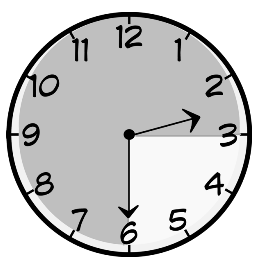
1. Ama has a maths exam tomorrow. She studied in the morning from 7 am to 8:30 am. She studied again in the afternoon from 2:30 to 3:15. How much time did she spend studying all together?

   Solution
   Ama studied twice. To find how much time she studied all together, find the duration she spent studying each time, and add the times together.

   Time spent studying in the morning: 8:30 – 7:00 = 1:30. She spent 1 hour and 30 minutes.

   Time spent studying in the afternoon: 3:15 – 2:30 = 0:45. She spent 45 minutes.

   - In this problem, the minutes do not easily subtract as in the previous problems. It can be helpful to draw or imagine a clock face. On the face of the clock, you can see that from 2:30 to 3:15, 45 minutes pass:

   Total time spent studying: 1 hour 30 minutes + 45 minutes = 1 hour 75 minutes = 2 hours 15 minutes

   Answer: Ama spent 2 hours and 15 minutes studying.

2. Martin drove from Kailahun to Freetown. He started at 07:00 hours, and took 4 hours to drive to Kenema. He spent 30 minutes resting in Kenema, then spent 6 and a half more hours to finish driving to Freetown?
   a. How much time did he spend driving all together?
   b. At what time did he reach Freetown?

Solutions

a. We know that Martin drove twice: 4 hours from Kailahun to Kenema, and 6.5 hours from Kenema to Freetown.

   Add the durations of time that he spent driving: 4 + 6.5 = 10.5 hours

   Answer: Martin drove for 10 hours and 30 minutes.

b. Add Martin's rest time to find the total amount of time he spent going from Kailahun to Freetown:

   10 hours 30 minutes + 30 minutes = 10 hours + 1 hour = 11 hours

   Martin spent 11 hours. Add 11 hours to his starting time to find the time he reached Freetown:

   07:00 + 11 = 18:00

   Answer: He reached Freetown at 18:00 hours.

3. Aminata left her house at 7:45 am and took 25 minutes to walk to the market. She stayed there 15 minutes and spent 30 minutes on her way back to her house. At what time did she arrive at her house?

   Solution
   You can solve this problem in 2 ways. You can add the 3 durations together and then add them onto 7:45 am. You could also add them 1 by 1, so add 25 minutes to 7:45 am, then 15 minutes, then 30 minutes. The first method is shown as follows:

   Total time spent outside: 25 + 15 + 30 = 70 minutes = 1 hour 10 minutes

   Add the total time she spent to her departure time: 7:45 + 1:10 = 8:55

   Answer: She arrived home at 8:55 am.

4. Abdul goes to work at 8:45 am and returns at 4:37 pm every day. How long does Abdul spend at his place of work?

   Solution
   You can find the amount of time he spends by breaking the total duration into parts.

   First, count the hours. From 8:45 to 3:45 is 7 hours.

   Next, count the minutes. From 3:45 to 4:00 is 15 minutes. From 4:00 to 4:37 is 37 minutes.

   Add the total time he spent at his place of work: 7 hours + 15 mins + 37 mins =
   7 hours 52 minutes.

Practice

1. Mohamed is a football player. Today he practised from 8 am to 9:45 am. He practised again from 3:30 pm to 5 pm. How much time did he spend practising all together?
2. Abu left his house at 09:00. He spent 30 minutes walking to the market. He sold goods in the market for 3 hours. It took him 25 minutes to walk home. At what time did he arrive at his house?
3. Juliet is a tailor. She spent 8 hours making clothes today. She started work at 8:30 am. She took a 15-minute break in the morning, and a 30-minute break in the afternoon. At what time did she finish working?
4. Abass slept from 11:30 am to 2 pm. How long did he sleep?
5. Fatu started studying at 18:00. She spent 30 minutes studying maths, 45 minutes studying English, and 20 minutes studying biology. At what time did she finish studying?
6. John left home at 8:15 am and took 20 minutes to get to school. He discovered that he arrived 5 minutes after the start of the examinations. At what time did the examinations start?

Lesson Title: Perimeter and Area of Rectangles and Squares	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-061	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the perimeter and area of rectangles and squares.

Overview

This is the first of 3 lessons on finding the perimeter and area of quadrilaterals. Quadrilaterals are plane shapes with 4 sides.

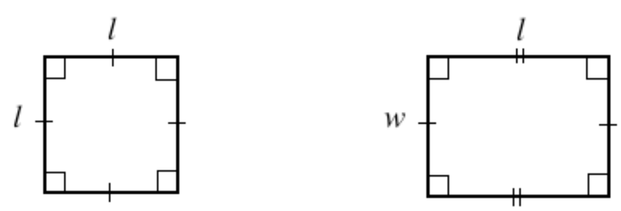
In maths, **perimeter** is the total length around a shape. To find the perimeter of a rectangle or square, add the lengths of all 4 sides together. We have some short cuts for squares and rectangles, because they have sides of the same length. The formulae are in the table below.

In maths, **area** is the size of the space inside of a shape. Imagine you want to cover the floor of your house with mats. Your floor is a certain area. If you measure the area of your floor, you will be able to buy the correct number of mats. To find the area of a square or rectangle, multiply the measurements of the two sides, length and width. For a square, the sides are the same length, so the area will be length squared.

The formulae are:

Shape	Perimeter	Area
Square	$P = l + l + l + l = 4l$	$A = l \times l = l^2$
Rectangle	$P = l + l + w + w = 2l + 2w$	$A = l \times w$

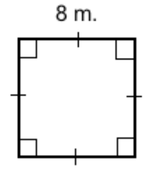
For the shapes:



Always remember to write the unit of measurement with your answer. For example, perimeter can be measured in m or cm. Area is always given in units squared. Remember to write the unit squared with your answer. For example, $64\ \text{m}^2$ is read as "64 square metres" or "64 metres squared".

Solved Examples

1. Find the perimeter and area of the square:



Solution

Find the **perimeter** by adding the length 4 times, or by multiplying the length of 1 side by 4. Both methods are shown:

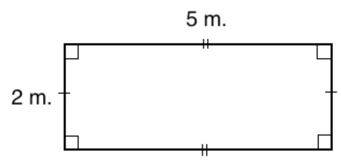
$$\mathrm{P} = l + l + l + l = 8 + 8 + 8 + 8 = 32 \text{ m}$$

$$\mathrm{P} = 4l = 4 \times 8 \text{ m} = 32 \text{ m}$$

Find the **area** by substituting the given length in the formula:

$$\mathrm{A} = l^2 = 8 \text{ m} \times 8 \text{ m} = 64 \text{ m}^2$$

2. Find the perimeter and area of the rectangle:



Solution

Find the perimeter by adding the lengths of the 4 sides. You can also find the perimeter by multiplying the length and width by 2, and adding the results. Both methods are shown below:

$$\mathrm{P} = l + l + w + w = 5 + 5 + 2 + 2 = 14 \text{ m}$$

$$\mathrm{P} = 2l + 2w = 2 \times 5 \text{ m} + 2 \times 2 \text{ m} = 10 \text{ m} + 4 \text{ m} = 14 \text{ m}$$

Find the **area** by substituting the given length in the formula:

$$\mathrm{A} = l \times w = 5 \text{ m} \times 2 \text{ m} = 10 \text{ m}^2$$

3. Find the perimeter and area of a square with sides of length 15 cm.

Solution

Find the **perimeter** by substituting $l = 15$ cm into the formula:

$$\mathrm{P} = 4l = 4 \times 15 \text{ cm} = 60 \text{ cm}$$

Find the **area** by substituting $l = 15$ cm into the formula:

$$\mathrm{A} = l^2 = 15 \text{ cm} \times 15 \text{ cm} = 225 \text{ cm}^2$$

4. Find the perimeter of a rectangle of breadth 4 m and length 12 m.

   Solution
   Breadth is another way of saying width. The rectangle is 4 m by 12 m. Apply the formula:

   $\mathrm{P} = 2l + 2w = 2 \times 12\ \mathrm{m} + 2 \times 4\ \mathrm{m} = 24\ \mathrm{m} + 8\ \mathrm{m} = 32\ \mathrm{m}$

5. A square has the same perimeter as a rectangle which measures 7 cm by 5 cm. Find the area of the square.

   Solution
   This problem requires multiple steps. Find the perimeter of the square. Use the perimeter to find the length of the sides of the square. Then, find the area of the square.

   Perimeter of the rectangle = $2(l + b) = 2(7 + 5) = 2 \times 12 = 24\ \mathrm{cm}$

   Since the perimeters are equal, we have: Perimeter of the square = $24\ \mathrm{cm} = 4l$

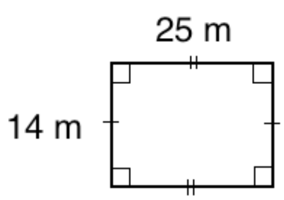
   Solve for l, the side of the square: $l = \frac{24}{4} = 6\ \mathrm{cm}$

   Area of the square = $l^2 = 6^2 = 6 \times 6 = 36\ \mathrm{cm}^2$

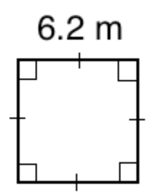
Practice

1. Find the perimeter and area of shapes a. and b.

   a.



   b.



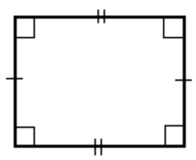
2. Find the perimeter and area of a square with sides of 14 cm.
3. Find the perimeter and area of a rectangle with a length of 5 metres and a width of 3 metres.
4. A square has sides of length 4 cm. A rectangle has a length of 5 cm and a width of 3 cm. Which shape has a greater area?
5. The perimeter of a rectangle is 36 cm. Find the breadth of the rectangle if its length is 10 cm.
6. The perimeter of a rectangular farm is 1,168 metres. If the length of one side is 320 metres, what is the length of the other side?

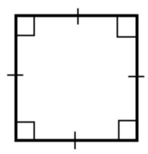
Lesson Title: Perimeter and Area of Parallelograms	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-062	**Class:** JSS 2

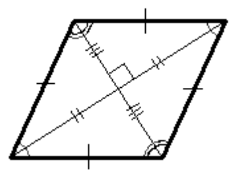
Learning Outcome
By the end of the lesson, you will be able to find the perimeter and area of parallelograms, including rhombuses.

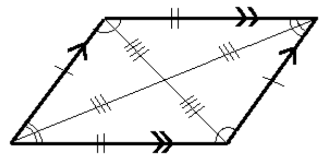
Overview

A parallelogram is a four-sided plane figure with opposite sides parallel. The following shapes are parallelograms:


Rectangle


Square


Rhombus


Parallelogram

Rectangles and squares are special type of parallelograms that were covered in the previous lesson. A rhombus is another special type of parallelogram with four equal sides.

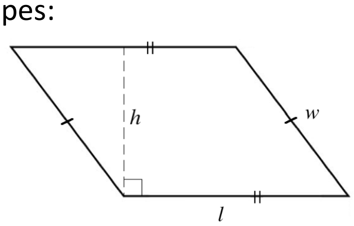
We calculate the **perimeter** in the same way for any shape: by adding all of the sides. For a **parallelogram,** we use the same formula that we used for a rectangle, using its length and width. For a **rhombus**, the perimeter is found using the same formula we used for a square.

We use different, special formulae to find the **area** of parallelograms and rhombuses. To find the area of a **parallelogram**, multiply its base times its height. The height of a parallelogram forms a right angle with its base. To find the area of a **rhombus**, multiply $\frac{1}{2}$ and its 2 diagonals. The diagonals are lines that connect opposite angles.

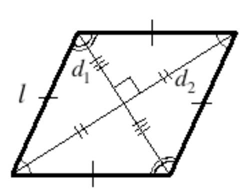
The formulae are:

Shape	Perimeter	Area
Parallelogram	$P = l + l + w + w = 2l + 2w$	$A = b \times h$
Rhombus	$P = l + l + l + l = 4l$	$A = \frac{1}{2} d_1 \times d_2$

For the shapes:

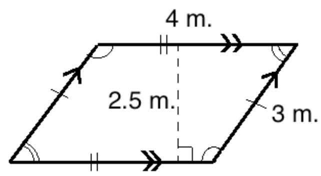


Parallelogram



Rhombus

Solved Examples

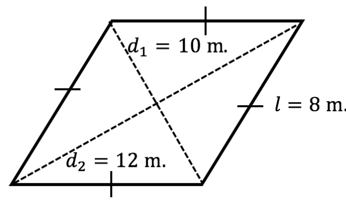


1. Find the perimeter and area of the parallelogram:

   Solution

$$
\begin{aligned}
P &= 2l + 2w \\
&= 2 \times 4\text{ m} + 2 \times 3\text{ m} \\
&= 8\text{ m} + 6\text{ m} \\
&= 14\text{ m}
\end{aligned}
$$

$$
\begin{aligned}
A &= b \times h \\
&= 4\text{ m.} \times 2.5\text{ m} \\
&= 10\text{ m}^2
\end{aligned}
$$

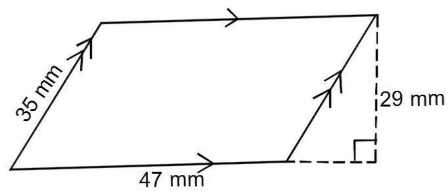


2. Find the perimeter and area of the rhombus:

   Solution

$$
\begin{aligned}
P &= 4l \\
&= 4(8) \\
&= 32\text{ m}
\end{aligned}
$$

$$
\begin{aligned}
A &= \tfrac{1}{2} d_1 \times d_2 \\
&= \tfrac{1}{2}(10 \times 12) \\
&= \tfrac{1}{2}(120) \\
&= 60\text{ m}^2
\end{aligned}
$$



3. Find the perimeter and area of the parallelogram:

   Solution

$$
\begin{aligned}
P &= 2l + 2w \\
&= 2 \times 47\text{ mm} + 2 \times 35\text{ mm} \\
&= 94\text{ mm} + 70\text{ mm} \\
&= 164\text{ mm}
\end{aligned}
$$

$$
\begin{aligned}
A &= b \times h \\
&= 47\text{ mm} \times 29\text{ mm} \\
&= 1{,}363\text{ mm}^2
\end{aligned}
$$

4. Find the area of a parallelogram with base of 0.5 metres and height of 25 centimetres. Give your answer in square centimetres.

Solution

To solve an area problem, measurements should be in the same units. The answer should be in cm^2, so convert 0.5 m to cm. Use the fact 100 cm = 1 m. Remember to multiply to convert larger units to smaller units: $0.5\text{ m} = 0.5 \times 100 = 50\text{ cm}$

Apply the area formula:

$$\begin{aligned} A &= b \times h \\ &= 50\text{ cm} \times 25\text{ cm} \\ &= 1{,}250\text{ cm}^2 \end{aligned}$$

5. A rhombus has sides of 17 cm and diagonals of 28 cm and 10 cm. Find:
   a. The area of the rhombus
   b. The perimeter of the rhombus

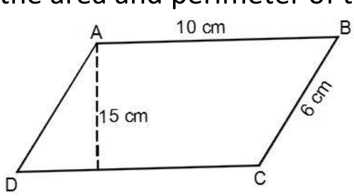
Solutions

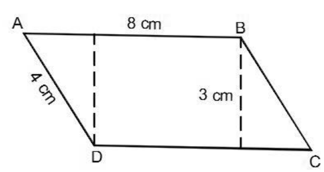
a.
$$\begin{aligned} A &= \tfrac{1}{2} d_1 \times d_2 \\ &= \tfrac{1}{2}(28 \times 10) && \text{Substitute values} \\ &= \tfrac{1}{2}(280) \\ &= 140\text{ cm}^2 \end{aligned}$$

b.
$$\begin{aligned} P &= 4l \\ &= 4 \times 17\text{ cm} \\ &= 68\text{ cm} \end{aligned}$$

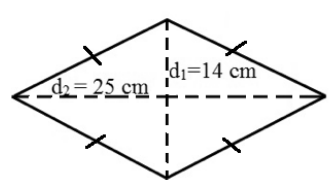
Practice

1. Find the area and perimeter of the parallelograms below:

a. 

b. 

2. Find the area of a parallelogram with a base of 220 cm and height of 2 metres. Give your answer in square metres.
3. A parallelogram has sides of length 20 m and 35 m. Find its perimeter.
4. Calculate the perimeter and area of a rhombus with sides of length 8.3 m, and diagonals of 9 m and 14 m.
5. Find the area of the rhombus shown at right. →



Lesson Title: Perimeter and Area of Trapeziums	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-063	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the perimeter and area of trapeziums.

Overview

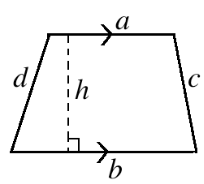
A trapezium is a quadrilateral with 2 parallel sides. The other 2 sides are not parallel.

We calculate the perimeter of a trapezium in the same way for any shape: by adding all of the sides. To calculate the area of a trapezium, we use a special formula. In the area formula below, a and b are the parallel sides. h is the height.

The formulae are:

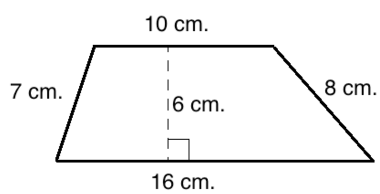
Shape	Perimeter	Area
Trapezium	$P = a + b + c + d$	$A = \frac{1}{2}(a + b)h$

For the shape:



Solved Examples

1. Find the perimeter and area of the trapezium:



Solution

Add the lengths of the sides to find the perimeter:

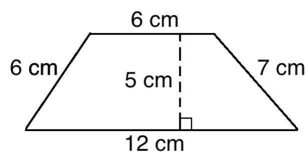
$$\begin{aligned} P &= 10 + 16 + 8 + 7 \\ &= 41 \text{ cm} \end{aligned}$$

To find the area, first take note of the values needed. The parallel sides are $a = 10$ and $b = 16$. The height is $h = 6$. The other measurements are not needed for the area formula.

Apply the area formula:

$$\begin{aligned} A &= \tfrac{1}{2}(a+b)h \\ &= \tfrac{1}{2}(10+16)6 \\ &= \tfrac{1}{2}(26)6 \\ &= 78\ \mathrm{cm}^2 \end{aligned}$$

2. Calculate the perimeter and area of the trapezium below.



Solution
Add to find the perimeter:

$$\begin{aligned} \mathrm{P} &= 6+6+7+12 \\ &= 31\ \mathrm{cm} \end{aligned}$$

Apply the area formula:

$$\begin{aligned} A &= \tfrac{1}{2}(a+b)h \\ &= \tfrac{1}{2}(6+12)5 \\ &= \tfrac{1}{2}(18)5 \\ &= 45\ \mathrm{cm}^2 \end{aligned}$$

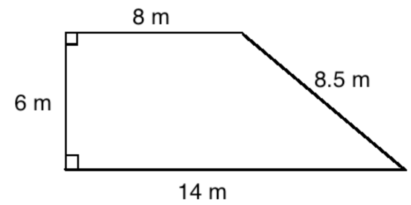
3. Find the area of a trapezium whose parallel sides are $75\ \mathrm{mm}$ and $82\ \mathrm{mm}$ long, and whose vertical height is $39\ \mathrm{mm}$.

Solution
Apply the area formula:

$$\begin{aligned} A &= \tfrac{1}{2}(a+b)h \\ &= \tfrac{1}{2}(75+82)39 \\ &= (78.5)39 \\ &= 3{,}061.5\ \mathrm{mm}^2 \end{aligned}$$

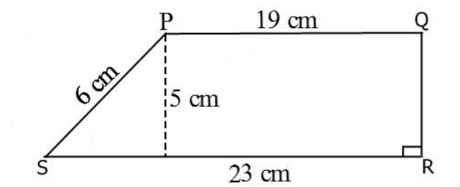
Practice

1. Find the perimeter and area of the trapezium:

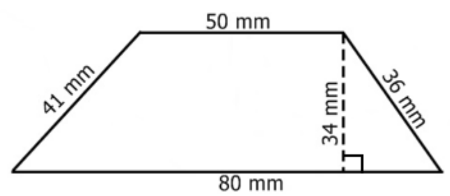


2. The parallel sides PQ and RS of a trapezium are 7.6 and 10 cm. If the distance between them is 6 cm, calculate the area of the trapezium.

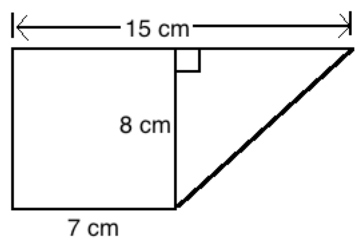
3. Calculate the perimeter and area of the trapezium PQRS.



4. Find the perimeter and area of the trapezium below:



5. Find the area of the figure below.



Lesson Title: Perimeter and Area of Triangles	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-064	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the perimeter and area of triangles.

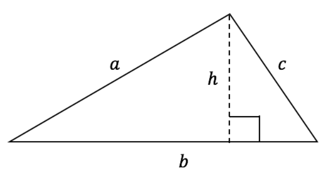
Overview

We calculate the perimeter in the same way for any shape: by adding all of the sides. To find the perimeter of a triangle, simply add the 3 sides. The area is found using a formula that is specifically for triangles. Multiply the base and height by $\frac{1}{2}$. The height of a triangle is a perpendicular line drawn from the base to the opposite angle of the triangle. In a right-angled triangle, the height can also be the length of a side.

The formulae are:

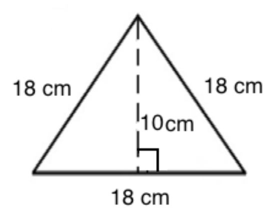
Shape	Perimeter	Area
Triangle	$P = a + b + c$	$A = \frac{1}{2} b \times h$

For the shape:



Solved Examples

1. The equilateral triangle at right has sides of length 18 cm, and a height of 10 cm. Find the area and perimeter of the triangle.



Solution
Apply the area formula. Note that $b = 18$ cm and $h = 10$ cm.

$$\begin{aligned} A &= \frac{1}{2}\, b \times h \\ &= \frac{1}{2} \times 18 \times 10 && \text{Substitute values} \\ &= \frac{1}{2}(180) && \text{Simplify} \\ &= 90 \text{ cm}^2 \end{aligned}$$

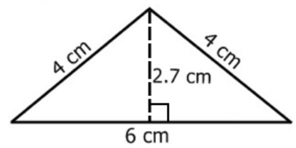
Add the lengths of the sides to find the perimeter:

$$\begin{aligned} P &= 18 + 18 + 18 \\ &= 54 \text{ cm} \end{aligned}$$

2. The height of an isosceles triangle is 2.7 cm, its base is 6 cm, and the equal sides are 4 cm long. Find the area and perimeter of the triangle.

Solution

First, draw a picture of the triangle described. Your drawing does not need to be to scale:



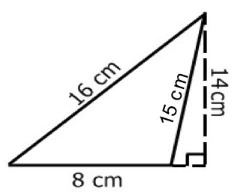
Apply the area formula. Note that $b = 6$ cm and $h = 2.7$ cm.

$$\begin{aligned} A &= \frac{1}{2}\, b \times h \\ &= \frac{1}{2} \times 6 \times 2.7 \\ &= \frac{1}{2}(16.2) \\ &= 8.1 \text{ cm}^2 \end{aligned}$$

Add the lengths of the sides to find the perimeter:

$$\begin{aligned} P &= 4 + 4 + 6 \\ &= 14 \text{ cm} \end{aligned}$$

3. Find the area and perimeter of the triangle:



Solution

This triangle is shaped different than the previous examples. The height of this triangle is drawn on the outside. The same formula is used.

$$\begin{aligned} A &= \frac{1}{2}\, b \times h \\ &= \frac{1}{2} \times 8 \times 14 && \text{Substitute values} \\ &= \frac{1}{2} \times (112) && \text{Simplify} \\ &= 56 \text{ cm}^2 \end{aligned}$$

Add the lengths of the sides to find the perimeter:

$$\begin{aligned} P &= 16 + 8 + 15 \\ &= 39 \text{ cm} \end{aligned}$$

4. The area of a triangle is 12 m^2, and its base is 8 m. Calculate its height.

Solution

Substitute the given values in the formula $A = \frac{1}{2}\ b \times h$, then solve for the height.

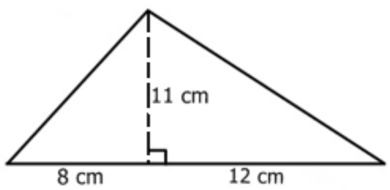
A	$= \frac{1}{2}\ b \times h$		
12	$= \frac{1}{2} \times 8 \times h$		Substitute values
12	$= 4 \times h$		Simplify
$\frac{12}{4}$	$= \frac{4 \times h}{4}$		Divide both sides by 4
3 m	$= h$		

The height of the triangle is 3 m.

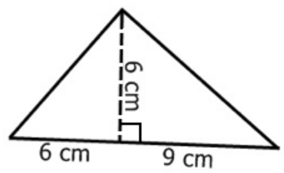
Practice

1. Find the area of the triangles below:

a.

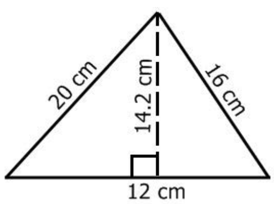


b.

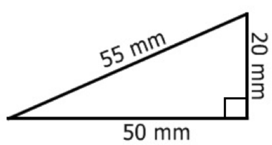


2. Calculate the perimeter and area of the following triangles:

a.



b.



3. A plot of land to be cultivated is in the shape of a triangle which has sides of length $24\ \mathrm{m}$, $20\ \mathrm{m}$ and $18\ \mathrm{m}$. Find the perimeter of the plot of land.

4. The area of a triangle is $126\ \mathrm{m}^2$. Calculate its base if the height is 7 m.

Lesson Title: Perimeter and Area of Circles	Theme: Measurement and Estimation
Practice Activity: PHM-08-065	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the circumference and area of circles.

Overview

The circumference of a circle is the same as the perimeter of the circle. It is the distance around a circle. To calculate the circumference or area of a circle, we have special formulae.

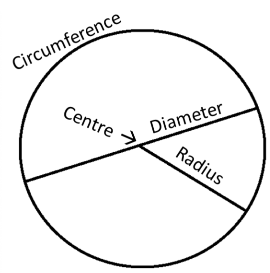
Each formula involves radius, which is the distance from the centre of the circle to its side. In some problems, the diameter is given. The diameter is the distance across the circle, passing through its centre. The diameter is twice the radius ($d = 2r$), and the radius is half of the diameter ($r = \frac{1}{2}d$).

The formulae also use pi (π). Pi is a decimal number that stretches on forever. It can be estimated with numbers such as 3.14 and $\frac{22}{7}$. We will use these numbers in our calculations.

The formulae for circumference and area of a circle are:

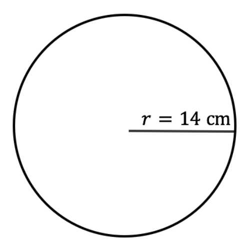
Shape	Circumference	Area
Circle	$C = 2\pi r$	$A = \pi r^2$

For the shape:



Solved Examples

1. Find the circumference and area of the circle. Use $\pi = \frac{22}{7}$.



Solution

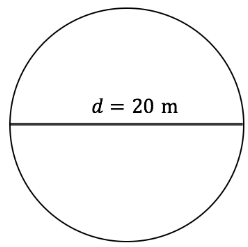
Apply the circumference formula:

$$\begin{aligned} C &= 2\pi r && \text{Formula} \\ &= 2 \times \frac{22}{7} \times 14 && \text{Substitute values} \\ &= 2 \times 22 \times 2 && \text{Cancel 7} \\ &= 88 \text{ cm} \end{aligned}$$

Apply the area formula:

$$\begin{aligned} A &= \pi r^2 && \text{Formula} \\ &= \left(\frac{22}{7}\right) 14^2 && \text{Substitute values} \\ &= \left(\frac{22}{7}\right) 196 && \text{Simplify} \\ &= 22 \times 28 \\ &= 616 \text{ cm}^2 \end{aligned}$$

2. Find the circumference and area of the circle. Use $\pi = 3.14$.



Solution

First find the radius. Divide the diameter by 2:

$$r = \frac{1}{2} d = \frac{1}{2}(20 \text{ m}) = 10 \text{ m}$$

Find the circumference:

$$\begin{aligned} C &= 2\pi r && \text{Formula} \\ &= 2(3.14)(10) && \text{Substitute values} \\ &= 2(31.4) && \text{Multiply 10} \\ &= 62.8 \text{ m} \end{aligned}$$

Find the area:

$$\begin{aligned} A &= \pi r^2 && \text{Formula} \\ &= (3.14)(10^2) && \text{Substitute values} \\ &= (3.14)(100) && \text{Simplify} \\ &= 314 \text{ m}^2 \end{aligned}$$

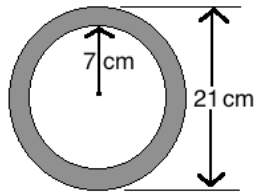
3. Find the circumference of the circle whose radius is 3.5 mm. Use $\pi = \frac{22}{7}$.

Solution

Apply the circumference formula:

$$\begin{aligned} C &= 2\pi r && \text{Formula} \\ &= 2 \times \frac{22}{7} \times 3.5 && \text{Substitute values} \\ &= \frac{154}{7} && \text{Simplify} \\ &= 22 \text{ mm} \end{aligned}$$

4. Find the area of the shaded portion in the figure below, correct to 1 decimal place.



Solution

To find the area of the shaded portion, subtract the area of the smaller circle from the area of the larger circle.

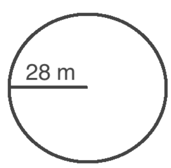
Area of the smaller circle= $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 22 \times 7 = 154 \text{ cm}^2$

Area of the larger circle= $\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} = 346.5 \text{ cm}^2$

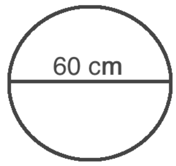
Area of the shaded portion = Area of the larger circle – Area of smaller circle
= $346.5 - 154 = 192.5 \text{ cm}^2$

Practice

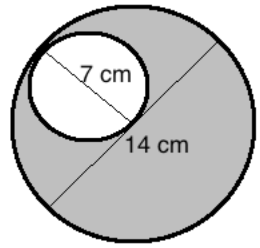
1. Find the circumference and area of a circle with a diameter of 14 cm, using $\pi = \frac{22}{7}$.
2. The radius of a circle is 100 cm. Find its circumference using $\pi = 3.14$.
3. Find the circumference of a circle whose radius is 35 cm. Take $\pi = \frac{22}{7}$.
4. Find the area of a circle with a radius of 3 cm. Give your answer to 1 decimal place. Take $\pi = 3.14$.
5. Find the area of a circle with a diameter of 8 m. Give your answer to 2 decimal places. (Take $\pi = \frac{22}{7}$)
6. Find the circumference of a circle whose diameter is 140 cm. Take $\pi = \frac{22}{7}$.
7. Find the circumference and area of the circle, using $\pi = \frac{22}{7}$:



8. Find the circumference and area of the circle, using $\pi = 3.14$:



9. Find the area of the shaded portion in the figure. Take $\pi = \frac{22}{7}$.



Lesson Title: Perimeter and Area of Composite Shapes	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-066	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of composite shapes.

Overview

This lesson reviews how to calculate the perimeters and areas of composite shapes.

Composite shapes are shapes made up of one or more different types of shapes. They can be made up of a combination of circles, triangles, rectangles and other polygons.

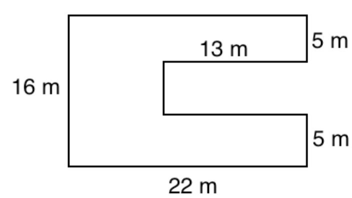
To find the perimeter and area of composite shapes you use the formulae for basic shapes such as squares, rectangles, and circles, which we already know.

To find the perimeter and area of a composite shape, first divide the shape into its individual parts and find any missing lengths of sides.

To find the **perimeter** of a composite shape, add all of its sides. To find the **area** of a composite shape, find the individual areas of its parts using the appropriate formulae. Then, add the areas together.

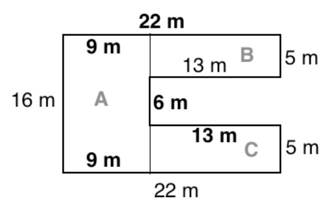
Solved Examples

1. Find the perimeter and area of the shape:



Solution

Divide the shape into its individual parts and find the lengths of the missing sides:



In this diagram, the shape is divided into 3 rectangles: A, B and C. All of the sides needed to find the perimeter and area are labelled. Subtraction is used to find each of these missing sides:

- To find the missing side of A, subtract 22 – 13 = 9 m
- To find the length between B and C, subtract 16 – 5 – 5 = 6 m

Find the perimeter by adding all of the outside edges of the shape:

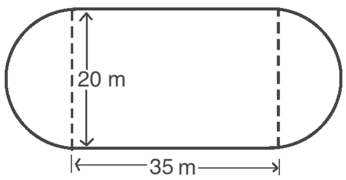
$$P = 16 + 22 + 5 + 13 + 6 + 15 + 5 + 22 = 102 \text{ m}$$

Find the area of each shape and add them to find the total area:

- Area of A: $16 \times 9 = 144 \text{ m}^2$
- Area of B: $13 \times 5 = 65 \text{ m}^2$
- Area of C: $13 \times 5 = 65 \text{ m}^2$

Total area: $A = 144 + 65 + 65 = 274 \text{ m}^2$

2. A farmer's field is in the shape of the figure below. Using $\pi = 3.14$, find the following:
   a. Area of the field
   b. Perimeter of the field



Solutions

Note that this shape is a rectangle with a semi-circle on each side. Together, 2 semi-circles make 1 full circle. This is because each semi-circle is exactly half of the circle.

a. To find the area of the shape, find the area of the circle and the area of the rectangle. Add them together to find the total area.

The diameter of this circle is 20 m, which means the radius is $r = \frac{20}{2} = 10 \text{ m}$.

Area of circle:

A	$=$	πr^2	
	$=$	3.14×10^2	Substitute values
	$=$	3.14×100	Square 10
A	$=$	314 m^2	

Area of the rectangle:

A	$=$	$l \times w$	
	$=$	35×20	Substitute values
	$=$	700 m^2	Multiply

Total area:

A	$=$	Area of circle + Area of rectangle
	$=$	$314 + 700$
	$=$	$1{,}014 \text{ m}^2$

The area of the field is 1,014 m^2.

b. To find the perimeter of the shape, add the circumference of the full circle to the 2 sides of the rectangle that are on the outside of the field, each 35 m.

Circumference of the circle:

$$\begin{aligned} C &= 2\pi r \\ &= 2 \times 3.14 \times 10 \quad \text{Substitute values} \\ A &= 628 \text{ m} \end{aligned}$$

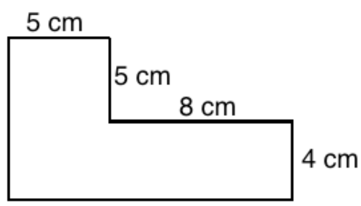
Add the outside edges of the field: $628 + 35 + 35 = 698 \text{ m}$

The perimeter of the field is 698 metres.

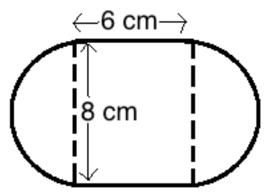
Practice

Find the area and perimeter of the shapes:

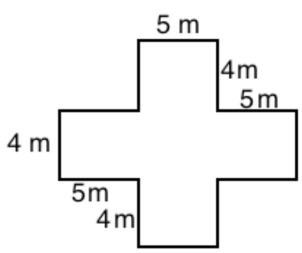
1.



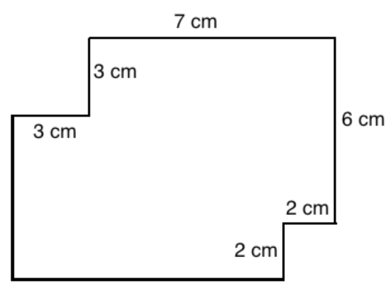
2. Use $\pi = 3.14$



3.



4.



Lesson Title: Perimeter and Area Story Problems	Theme: Measurement and Estimation
Practice Activity: PHM-08-067	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to solve practical problems on perimeter and area.

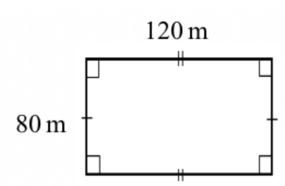
Overview

In this lesson, you will solve perimeter and area story problems. Remember that perimeter is the distance around a shape, and area is the space inside a shape. In this lesson, you will apply perimeter and area formulae to real-life objects. You will also use maths skills that you learned from other lessons to solve problems. For example, you may be asked to calculate the cost of a project, or write a ratio.

It can be helpful to draw a diagram before solving story problems.

Solved Examples

1. Bright Secondary School has a football field that measures 120 metres on one side and 80 metres on the other side. A gardener is hired to plant carpet grass on the field.
   a. Calculate the area of the field.
   b. If the cost of carpet grass is Le 200.00 per square metre, how much will it cost to cover the field?



Solutions

First, draw a diagram. →

a. Calculate the area:

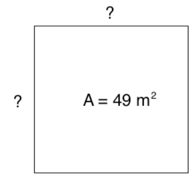
$$\begin{aligned} A &= l \times w \\ &= 120 \text{ m} \times 80 \text{ m} \\ &= 9{,}600 \text{ m}^2 \end{aligned}$$

b. Find the cost. Multiply the cost per square metre by the number of square metres.
$\text{Cost} = 9{,}600 \times \text{Le } 200 = \text{Le } 1{,}920{,}000.00$

2. The area of a square is 49 m^2. Find its perimeter.

Solution
This problem involves 2 steps. First, use the area to find the length l of the square. Then, use l to calculate the perimeter.

Draw a diagram →



Step 1. Find l using the area.

Recall that the area formula for a square is $A = l \times l = l^2$. In this case the area is 49 m^2, so we have $l^2 = 49$ m^2.

You can use the multiplication table or recall perfect squares to identify that $7^2 = 49$. The length of each side of the square is 7 m.

You can also use **square roots** to solve the problem. Square roots are the opposite of squares. Since $l^2 = 49$, then $l = \sqrt{49} = 7$ cm.

Step 2. Find the perimeter: $\mathrm{P} = 4l = 4 \times 7\ \mathrm{cm} = 28\ \mathrm{cm}$

3. The dimensions of the floor of a room are 20 ft by 15 ft. If rectangular tiles of dimensions 1.0 ft by 1.5 ft are used to tile the room, find the number of tiles required.

Solution

This problem involves multiple steps. Find the area of the room, then the area of each tile. Divide the area of the room by the area of 1 tile to find the total number of tiles required.

Step 1. Area of the room: $A = l \times w = 20 \times 15 = 300\ \mathrm{ft}^2$

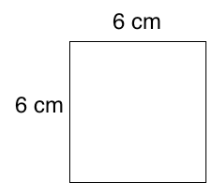
Step 2. Area of 1 tile: $A = l \times w = 1.5 \times 1.0 = 1.5\ \mathrm{ft}^2$

Step 3. Divide the area of the room by the area of 1 tile:

$$300\ \mathrm{ft}^2 \div 1.5\ \mathrm{ft}^2 = 3000 \div 15 = 200$$

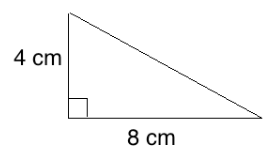
Answer: 200 tiles are needed.

4. A square has a side of 6 cm. A triangle has a base of 8 cm and a height of 4 cm. What is the ratio of the area of the square to that of the triangle? Give your answer in its lowest term.



Solution

You may first draw the shapes, shown on the right.



Find the area of each shape, then write them in ratio form.

Step 1. Area of the square: $A = l^2 = 6^2 = 36\ \mathrm{cm}^2$

Step 2. Area of the triangle: $A = \frac{1}{2}bh = \frac{1}{2} \times 8 \times 4 = 16 \text{ cm}^2$

Step 3. Ratio of the area of the square to that of the triangle:
$36 : 16 = \frac{36}{4} : \frac{16}{4} = 9 : 4$

Practice

1. A plot of land to be cultivated is in the shape of a triangle which has sides of length 24 m, 20 m and 18 m. Find the perimeter of the plot of land.
2. A farmer wants to build a fence around his rectangular field. The field is 30 metres long by 25 metres wide.
    a. What is the perimeter of the field?
    b. If the cost of the fence is Le 4,000.00 per metre, how much will he pay for the fence?
3. A goat is tied to a pole with a rope 15 metres in length. He can eat all the grass around him, the area of which forms a perfect circle. For the questions below, take $\pi = \frac{22}{7}$ and give your answers to the nearest tenth.
    a. What is the area of the grass which the goat can eat?
    b. What is the circumference of the area he can eat?
4. The area of a square is 36 m^2. Find the perimeter of the square.
5. The dimensions of the floor of a rectangular room are 10 metres by 12 metres.
    a. Find the area of the floor of the room.
    b. How many tiles would you need to cover the floor if the tiles are 20 cm by 20 cm?
6. A square has a side of 5 cm. A triangle has a base of 10 cm and a height of 3 cm. What is the ratio of the area of the square to that of the triangle? Give your answer in its lowest term.
7. A pavement of length 50 m and width 15 m is to be laid by using slabs of dimensions of 75 cm by 40 cm. How many slabs are required?

Lesson Title: Volume of Solids	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-068	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

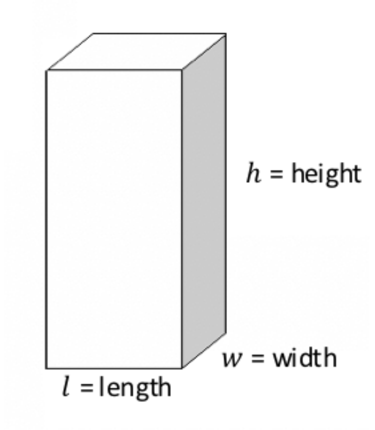
1. Identify the general formula for the volume of prisms and cylinders as cross-sections multiplied by height.
2. Identify and interpret measurements for volume (units cubed).

Overview

This is the first lesson on calculating the volume of a 3-dimensional shape. These are often called **solids**. Volume is the measurement of space taken up by a 3-dimensional solid.

Consider a box of chalk, a balloon and a tin of milk. In the tin of milk, the milk inside is taking up space. In a balloon, air or gas is taken up space. In the chalk box, the chalks are taking up space. This shows that solids, liquids and gases all take up space. This space is called volume.

This lesson is on 2 types of solids, rectangular prisms and cylinders. To find the volume of these solids, we use the area of one of its faces. This is called the **cross-section**. Multiply the cross-section by the height of the solid. For a rectangular prism, you may take any face as the cross-section. For a cylinder, the cross-section is the circular face.

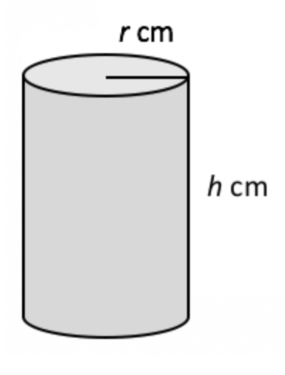


Rectangular Prism

Rectangular prisms (or cuboids) are solids with 6 rectangular faces. They have length, width and height.

The cross-section of a rectangular prism is the area of one of its faces, $A = l \times w$.

To calculate the volume of a rectangular solid, multiply the cross-section by its height:
$V = l \times w \times h$ or $V = A \times h$



Cylinder

A cylinder has 2 circular faces that are equal in size. It also has 1 rectangular face that curves around it.

The cross-section of a cylinder is the area of one of its circular faces, $A = \pi r^2$.

To find the volume of a cylinder, we multiply the cross-section by its height: $V = \pi r^2 h$ or $V = A \times h$.

Since we multiply 3 lengths together, the units for volume are cubed. We use a power of 3 to show 'cubic.' This is the same as the 'cubed' we use for indices. For example, if the size of a rectangular prism is measured in centimetres, its volume is given in cm^3. This is read as "cubic centimetres" or "centimetres cubed". To find volume, it is important that the measurements of the solid are all given in the same units.

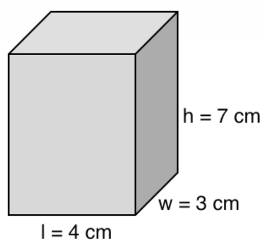
Solved Examples

1. Sketch a rectangular prism with height 7 cm, length 4 cm and width 3 cm. Your measurements do not need to be precise.

   Solution
   This problem asks you to "sketch" and states that the measurements do not need to be precise. You do not need to use a ruler to draw exactly 7 cm, 4 cm and 3 cm.

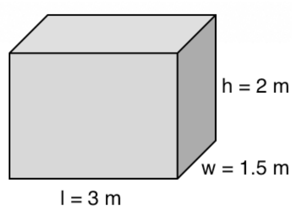
   It is important that sides with longer measures are drawn longer. It is also important that the shape is labelled correctly, so that the height is labelled as 7 cm, and so on.

   Sketch:



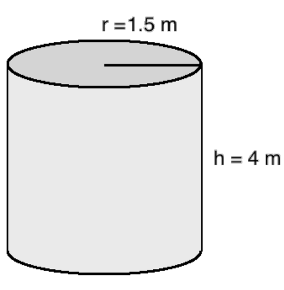
2. A water tank is in the shape of a rectangular prism. It is 2 metres tall, 3 metres long, and 1.5 metres wide. Sketch the water tank.

   Solution



3. A water tank is in the shape of a cylinder. It has radius 1.5 metres and height 4 metres. Sketch the water tank.

   Solution



4. The following statements describe solids. Write down what units the volume will be measured in.
    a. A box of chalk is 10 cm tall, 8 cm long and 2 cm wide.
    b. A shipping container is 3 metres tall, 6 metres long and 2 metres wide.
    c. A can with a radius of 4.5 cm and height of 12 cm.
    d. A box of biscuits is 30 cm tall, 10 cm long and 5 cm wide.
    e. A water tank with a radius of 6 feet and height of 10 feet.

Solutions

Give each answer in units cubed.

a. cm^3 b. m^3 c. cm^3 d. cm^3 e. ft^3

Practice

1. Write down the formula for volume of a rectangular prism.
2. Write down the formula for volume of a cylinder.
3. Sketch a rectangular prism with a height of 5 cm, length of 7 cm, and width of 3 cm.
4. Sketch a cylinder with a radius of 4 metres and height of 12 metres.
5. A dictionary is laying on a table. It has a height of 5 cm, length of 20 cm, and width of 15 cm. Sketch the dictionary.
6. A tin can is in the shape of a cylinder. It has a radius of 5 cm and height of 18 cm.
    a. Sketch the can.
    b. In what units would you measure the can's volume?
7. The following statements describe solids. Write down what units the volume will be measured in.
    a. A box of juice is 25 cm tall, 10 cm long and 6 cm wide.
    b. A can of cola is has a radius of 4 cm and height of 14 cm.
    c. A water tank is 4 metres tall, 3 metres long and 1.75 metres wide.
    d. A container of petrol is 60 cm tall, 40 cm long and 35 cm wide.
    e. A bucket of water has a radius of 1 foot and height of 3 feet.
    f. A building is 14 feet tall, 25 feet long and 10 feet wide.
8. Determine whether each of the following is a measurement of area or volume:
    a. cm^2 b. m^3 c. ft^3 d. km^2 e. m^2

Lesson Title: Volume of Cubes	Theme: Measurement and Estimation
Practice Activity: PHM-08-069	**Class:** JSS 2

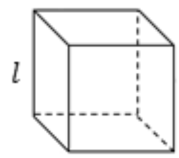
Learning Outcome
By the end of the lesson, you will be able to calculate the volume of a cube using the formula ($V = l^3$).

Overview

A cube is a special type of rectangular prism. The height, length and width are all equal.

Remember that we find volume of a rectangular prism by multiplying the length, width and height: $V = l \times w \times h = lwh$. Since all the sides are of the same length, then one number will represent all the sides.

Volume of a cube is $V = l \times l \times l = l^3$

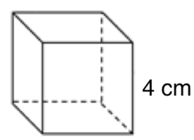


If you only know the length of one side of the cube then you will know its volume.

Solved Examples

1. Find the volume of a cube of side length $l = 4$ cm.

   Solution
   You may sketch the cube to help solve the problem. →

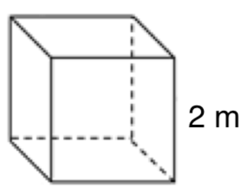
   

   Apply the formula for volume of a cube:

   $$\begin{aligned} V &= l^3 && \text{Formula} \\ &= 4^3 && \text{Substitute } l = 4 \\ &= 4 \times 4 \times 4 && \text{Calculate } 4^3 \\ &= 16 \times 4 \\ &= 64 \text{ cm}^3 \end{aligned}$$

   Remember to give your answer in units cubed.

2. Find the volume of the cube shown:



Solution

Apply the formula for the volume of a cube:

V	$=$	l^3	Formula
	$=$	2^3	Substitute $l = 2$
	$=$	$2 \times 2 \times 2$	Calculate 2^3
	$=$	4×2	
	$=$	$8\ \text{m}^3$	

Remember to give your answer in units cubed.

3. The edges of a cube are 8.5 cm long. Find its volume. Give your answer to the nearest whole number.

Solution

Apply the formula for volume of a cube:

V	$=$	l^3	Formula
	$=$	8.5^3	Substitute $l = 8.5$
	$=$	$8.5 \times 8.5 \times 8.5$	Calculate 8.5^3
	$=$	72.25×8.5	
	$=$	$614.125\ \text{cm}^3$	
	$=$	$614\ \text{cm}^3$	Round to the nearest whole number

Remember to give your answer in units cubed.

4. Determine the length of the side of a cube if its volume is 27 cm^3.

Solution

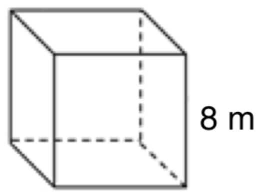
This problem asks us to do the opposite of the problems above. In this case, you are given the volume and asked to find the lengths of the sides of the cube. Since $V = l^3$, we have $27 = l^3$.

Think of a number that can be multiplied by itself 3 times to get 27. You may use a multiplication table to help you brainstorm. Note that $3^3 = 3 \times 3 \times 3 = 27$.

The sides of the cube are 3 cm.

Practice

1. Find the volume of a cube of side 7 cm.
2. Find the volume of a cube with sides 2.5 metres. Give your answer to 1 decimal place.
3. The edges of a cube are 9 cm long. Find its volume.
4. Find the volume of the following cube shown:



5. If the volume of a cube is 8 ft^3, what is the length of its sides?

Lesson Title: Volume of Rectangular Prisms	Theme: Measurement and Estimation
Practice Activity: PHM-08-070	**Class:** JSS 2

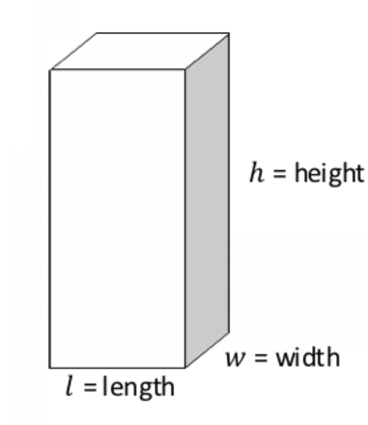
Learning Outcome
By the end of the lesson, you will be able to calculate the volume of a rectangular prism using the formula $V = lwh$.

Overview

Recall that the formula for finding the volume of a cuboid or rectangular prism is $V = l \times w \times h$.

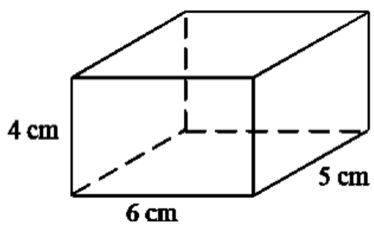
In this lesson, you will find the volume of cuboids. You are able to find the volume of any cuboid if you have its three measurements. Remember that volume is always given in units cubed.

You may sometimes see 'width' called 'breadth'. These two words have the same meaning.



Solved Examples

1. Find the volume of the given cuboid:



Solution
Note that the measurements of the cuboid are $l = 6$ cm, $w = 5$ cm, $h = 4$ cm.
Apply the volume formula:

V	$=$	lwh	Formula
	$=$	$6 \times 5 \times 4$	Substitute the values
	$=$	120 cm^3	Multiply

2. Find the volume of a box with a length of 20 feet, width of 10 feet, and height of 4 feet.

Solution
Apply the volume formula:

V	$=$	lwh	Formula
	$=$	$20 \times 10 \times 4$	Substitute the values
	$=$	800 ft^3	Multiply

3. A rectangular block measures 12 cm by 8 cm by 4 cm. What is its volume?

   Solution
   The 3 dimensions of the cuboid are given. We do not need to know which side is the length, base or height. A rectangular block can be turned in any direction, so that any of the measurements could be the height. Also, the order does not matter in multiplication.

   Multiply the 3 measurements together to find the volume:

V	$=$	lwh	Formula
	$=$	$12 \times 8 \times 4$	Substitute the values
	$=$	384 cm^3	Multiply

4. The length of a cuboid is 1 metre; its breadth is 15 cm and its height is 5 cm. Find its volume in cm^3.

   Solution
   Note that the measurements are given in m, cm, and cm. You are asked to find the answer in cm^3. This means that you must convert all units to cm first.

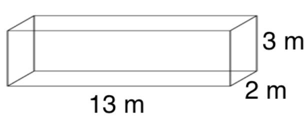
   Convert 1 m to cm: 1 m = 100 cm.

   Apply the volume formula:

V	$=$	lwh	Formula
	$=$	$100 \times 15 \times 5$	Substitute the values
	$=$	7,500 cm^3	Multiply

Practice

1. Find the volume of cuboids with these dimensions:
   a. Length 8 cm, width 5 cm and height 4 cm.
   b. Length 10 cm, breadth 7 cm and height 4 cm.
2. Find the volume of the cuboid on the right:



3. The length of a cuboid is 2 m; its breadth is 30 cm and its height is 20 cm. Find its volume in cm^3.
4. The dimensions of a cuboid are 4 m, 3 m, and 50 cm. Find its volume in m^3.
5. A packet measures 240 mm by 40 mm by 120 mm. How many packets can be packed in a cardboard box measuring 36 cm by 72 cm by 20 cm?

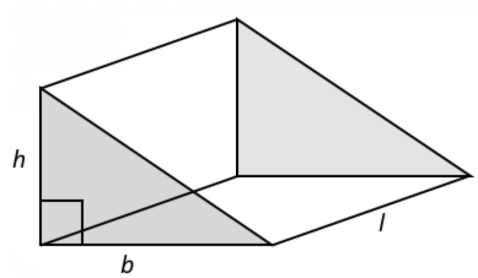
Lesson Title: Volume of Triangular Prisms	Theme: Measurement and Estimation
Practice Activity: PHM-08-071	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the volume of a triangular prism using the formula.

Overview

In this lesson, you will find the volume of triangular prisms using a new formula. A triangular prism is a solid with two parallel surfaces that are triangles. Its volume is measured in cubic units.

The diagram at right shows a triangular prism. When the triangular surfaces are right-angled triangles, like this, it is called a **right** triangular prism.



In the diagram, the shaded faces are parallel. These are also the **cross-sections** of the solid. The volume is found in the same way as a rectangular prism or cylinder. Multiply the cross-section by the length of the solid.

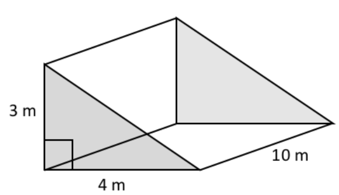
Remember that the formula for the area of a triangle is $A = \frac{1}{2}bh$. Multiplying the area of the triangular cross-section by the length of the solid will give you the formula for its volume:

$$V = \frac{1}{2} \times base \times height \times length = \frac{1}{2}bhl$$

Solved Examples

1. Find the volume of a right triangular prism with base of 4 m, height of 3 m and length of 10 m.

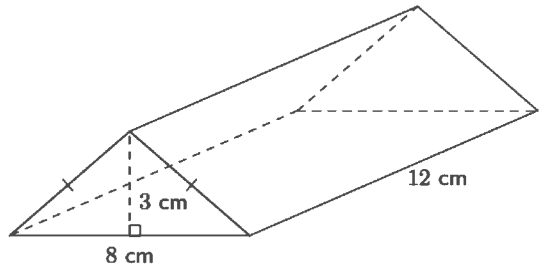
   Solution
   First, draw a diagram:



   Apply the volume formula:

V	=	$\frac{1}{2}bhl$	Formula
	=	$\frac{1}{2} \times 4 \times 3 \times 10$	Substitute the values
	=	$\frac{1}{2} \times 120$	Multiply
	=	60 cm^3	

2. Find the volume of the figure below.



Solution

Identify the base, height, and length of the prism: $b = 8$ cm, $h = 3$ cm, $l = 12$ cm

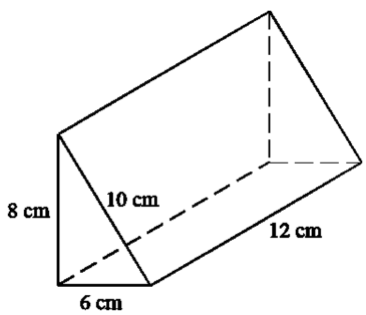
Apply the volume formula:

V	$=$	$\frac{1}{2}bhl$	Formula
	$=$	$\frac{1}{2} \times 8 \times 3 \times 12$	Substitute the values
	$=$	$4 \times 3 \times 12$	Multiply
	$=$	144 cm^3	

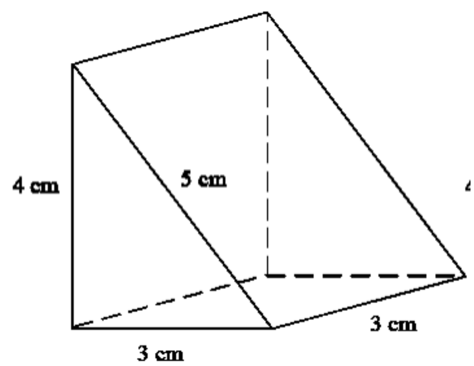
Practice

1. The solids below are right-angled triangular prisms. Find the volume of each.

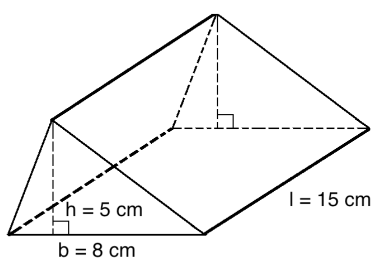
a.



b.



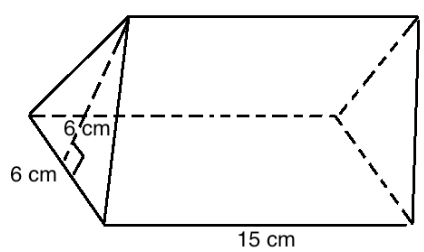
2. Find the volume of a rectangular prism with base 4 m, height 7 m, and length 3 m.
3. Find the volume of the figure:



4. The diagram shows a box in the form of a triangular prism. Calculate:

a. the area of each triangle end

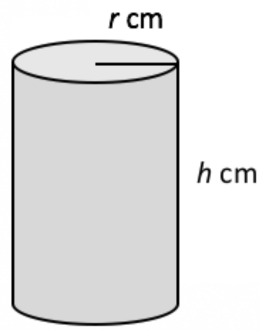
b. the volume of the box.



Lesson Title: Volume of Cylinders	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-072	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the volume of a cylinder using the formula.

Overview



Recall that the formula for finding the volume of a cylinder is $V = \pi r^2 h$.

In this lesson, you will find the volume of cylinders. You are able to find the volume of any cylinder if you have its 2 measurements. Remember that volume is always given in units cubed.

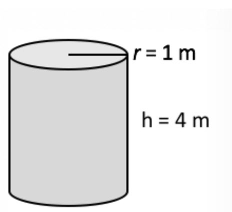
You will use an estimated value for pi, $\pi = 3.14$ or $\pi = \frac{22}{7}$.

In some volume problems, you may be asked to convert the volume from cubic centimetres to litres. You can simply use the conversion factor $1{,}000 \text{ cm}^3 = 1 \text{ litre}$. Divide the volume in cm^3 by 1,000 cm^3 to find the volume in litres.

Solved Examples

1. Find the volume of a cylinder with a radius of 1 m and height of 4 m. Take $\pi = 3.14$.

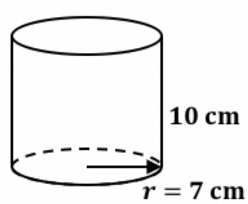
Solution
First, draw a diagram:



Apply the volume formula:

V	=	$\pi r^2 h$	Formula
	=	$3.14 \times (1)^2 \times 4$	Substitute the values
	=	3.14×4	Multiply
	=	12.56 m^3	

2. Find the volume of the figure. Use $\pi = \frac{22}{7}$.



Solution

Identify the value of the radius and height: $r =$ 7 cm, $h = 10$ cm

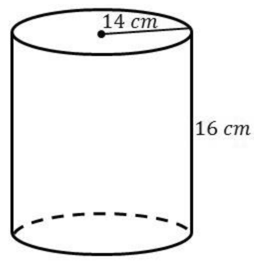
Apply the volume formula:

$$
\begin{aligned}
\text{V} &= \pi r^2 h && \text{Formula} \\
&= \tfrac{22}{7} \times (7)^2 \times 10 && \text{Substitute the values} \\
&= 22 \times 7 \times 10 && \text{Multiply} \\
&= 154 \times 10 \\
&= 1{,}540 \text{ cm}^3
\end{aligned}
$$

3. Find the volume of a cylinder with a radius of 14 cm and a height of 16 cm. Use $\pi = 3.14$. Give your answer to the nearest whole number.

Solution

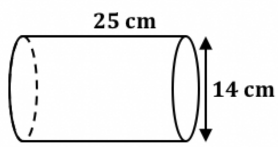
First, draw a diagram. →



Apply the volume formula:

$$
\begin{aligned}
\text{V} &= \pi r^2 h && \text{Formula} \\
&= 3.14 \times (14)^2 \times 16 && \text{Substitute the values} \\
&= 3.14 \times 196 \times 16 && \text{Multiply} \\
&= 9847.04 \\
&= 9{,}847 \text{ cm}^3
\end{aligned}
$$

4. The cylinder below is filled with water. Find its capacity in litres. Use $\pi = \frac{22}{7}$ and $1{,}000 \text{ cm}^3 = 1 \text{ litre}$.



Solution

First, find the volume of the cylinder. Then, use the fact that $1{,}000 \text{ cm}^3 = 1 \text{ litre}$ to find how many litres it can hold.

Note that the diameter is $d = 14$ cm. Find the radius: $r = \frac{d}{2} = \frac{14}{2} = 7$ cm

Apply the formula for the volume:

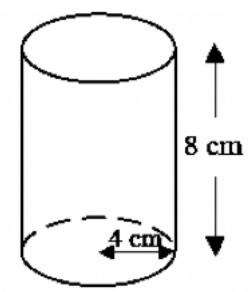
$$
\begin{aligned}
V &= \pi r^2 h && \text{Formula}\\
&= \frac{22}{7} \times (7)^2 \times 25 && \text{Substitute values}\\
&= 22 \times 7 \times 25 && \text{Multiply}\\
&= 3{,}850 \text{ cm}^3
\end{aligned}
$$

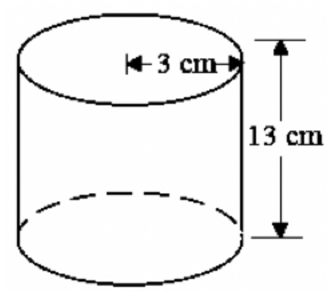
To find its capacity in litres, divide $3{,}850$ cm^3 by $1{,}000$ cm^3:

Capacity $= 3{,}850$ cm$^3 \div 1{,}000$ cm$^3 = 3.85$ litres

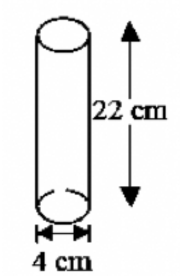
Practice

1. Find the volume of each figure to the nearest whole number. Use $\pi = 3.14$.

a. 

b. 

2. Find the volume of a cylinder with a radius of 14 mm and height of 10 mm. Take $\pi = \frac{22}{7}$.
3. Find the volume of the cylinder below. Give your answer to 1 decimal place.
   Use $\pi = 3.14$.

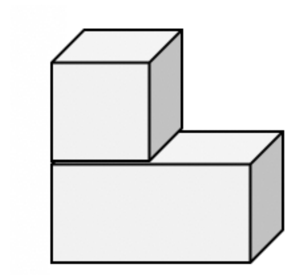


4. What is the volume of a cylinder with a radius of 3.5 cm and height of 14 cm?
   (Take $\pi = \frac{22}{7}$)
5. A cylindrical tin with a base radius of 28 cm and height of 50 cm is filled with water. What is its capacity in litres? (Take $1{,}000 \text{ cm}^3 = 1 \text{ litre}$)

Lesson Title: Volume of Composite Solids	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-073	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the volume of composite solids.

Overview



In this lesson, you will find the volume of composite solids. Composite solids are solid objects that can be divided into one or more basic solids. For example, consider the composite solid on the right. The solids within this composite solid are a cube and a rectangular prism.

To find the volume of composite solids, you use the formula for the volume of the solids that make up the composite solid. Add the volumes of the different solids together to find the total volume of the composite solid.

In the example figure, the volume would be:

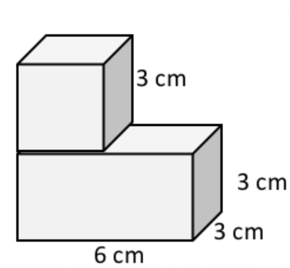
Volume of the composite solid = Volume of the cube + Volume of the rectangular prism

Recall and use the volumes of the solids you have learned:

Solid	Formula
Cube	$V = l^3$
Rectangular Prism	$V = lwh$
Triangular Prism	$A = \frac{1}{2}bh$
Cylinder	$V = \pi r^2 h$

Solved Examples

1. Find the volume of the solid shown:



Solution

Find the volume of the cube (V_1) and the volume of the rectangular prism (V_2) separately, then add them to find the total volume (V).

Volume of cube:

$$\begin{aligned} V_1 &= l^3 \\ &= (3 \text{ cm})^3 \\ &= 27 \text{ cm}^3 \end{aligned}$$

Volume of rectangular prism:

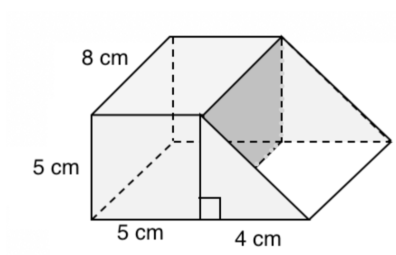
$$\begin{aligned} V_2 &= lwh \\ &= 6 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} \\ &= 54 \text{ cm}^3 \end{aligned}$$

Total volume:

$$\begin{aligned} V &= V_1 + V_2 \\ &= 27 \text{ cm}^3 + 54 \text{ cm}^3 \\ &= 81 \text{ cm}^3 \end{aligned}$$

The volume of the figure is 81 cm^3.

2. Find the volume of the figure below.



Solution

Find the volume of the rectangular prism (V_1) and the volume of the triangular prism (V_2) separately, then add them to find the total volume (V).

Volume of rectangular prism:

$$\begin{aligned} V_1 &= lwh \\ &= 8 \text{ cm} \times 5 \text{ cm} \\ &\quad \times 5 \text{ cm} \\ &= 200 \text{ cm}^3 \end{aligned}$$

Volume of triangular prism:

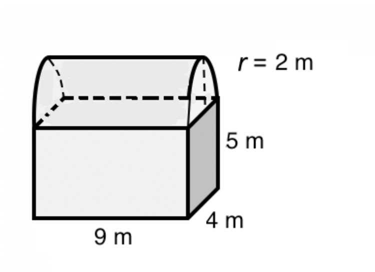
$$\begin{aligned} V_2 &= \frac{1}{2} bhl \\ &= \frac{1}{2} \times 4 \text{ cm} \times 5 \text{ cm} \times 8 \text{ cm} \\ &= 80 \text{ cm}^3 \end{aligned}$$

Total volume:

$$\begin{aligned} V &= V_1 + V_2 \\ &= 200 \text{ cm}^3 + 80 \text{ cm}^3 \\ &= 280 \text{ cm}^3 \end{aligned}$$

The volume of the figure is 280 cm^3.

3. Find the volume of the figure below. It is formed by half of a cylinder and a rectangular prism. Give your answer to 1 decimal place. Use $\pi = 3.14$.



Solution

Note that the volume of half a cylinder is found simply by multiplying $\frac{1}{2}$ by its volume formula: $\frac{1}{2}\pi r^2 h$.

Volume of rectangular prism:

$$\begin{aligned} V_1 &= lwh \\ &= 9\text{ m} \times 4\text{ m} \times 5\text{ m} \\ &= 180\text{ m}^3 \end{aligned}$$

Volume of the half cylinder:

$$\begin{aligned} V_2 &= \frac{1}{2}\pi r^2 h \\ &= \frac{1}{2}(3.14)(2^2)(9) \\ &= 56.52\text{ m}^3 \end{aligned}$$

Total volume:

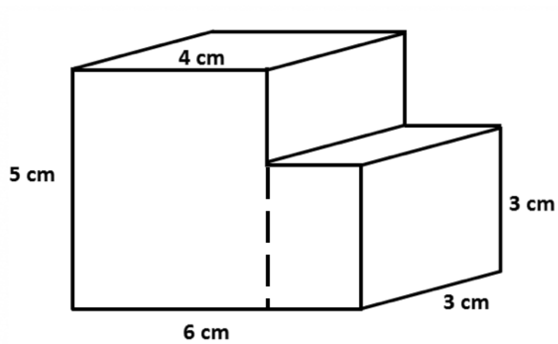
$$\begin{aligned} V &= V_1 + V_2 \\ &= 180\text{ m}^3 + 56.62\text{ m}^3 \\ &= 236.52\text{ m}^3 \end{aligned}$$

The volume of the figure is 236.5 m^3.

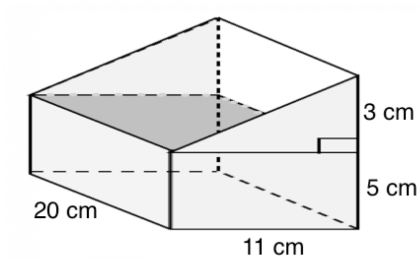
Practice

Find the volume of each solid below. Give your answers to the nearest whole number.

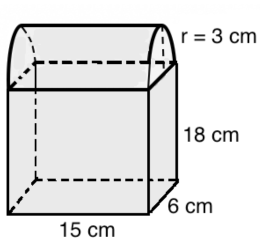
1.



2.



3.



Use $\pi = 3.14$.

Lesson Title: Volume Story Problems	Theme: Measurement and Estimation
Practice Activity: PHM-08-074	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to solve practical problems on volume.

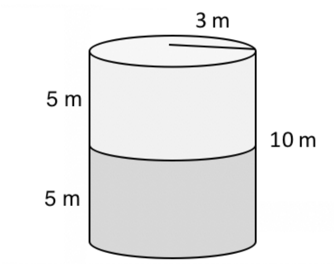
Overview

In this lesson, you will use information from previous lessons to solve story problems on volume.

Solved Examples

1. A water tank with a height of 10 m is in the shape of a cylinder with a radius of 3 m. The tank is half full of water. What is the volume of the water in the tank? Use $\pi = 3.14$.

   Solution
   First, draw a diagram:



   Find the volume of the water tank and divide it in 2, since it is only half full.

   Volume of the water tank:

V	=	$\pi r^2 h$	Formula
	=	$3.14 \times (3 \text{ m})^2 \times 10 \text{ m}$	Substitute values
	=	282.6 cm^3	

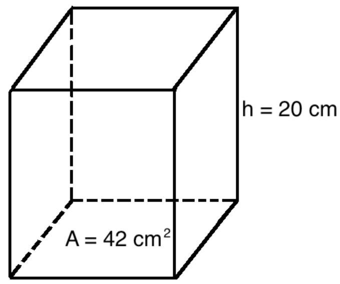
   Divide by 2 to find the volume of half of the tank: $282.6 \text{ m}^3 \div 2 = 141.3 \text{ m}^3$

   The volume of water in the tank is 141.3 m^3.

2. A carpenter built a box in the shape of a rectangular prism. The area of the bottom of the box is 42 cm^2, and the box is 20 cm tall. How many cubic centimetres of seeds will the box be able to hold?

   Solution
   Remember that the area of one side of a rectangular prism is also its cross-section. The volume formula is $V = A \times h$.

Draw a diagram. →



Apply the volume formula:

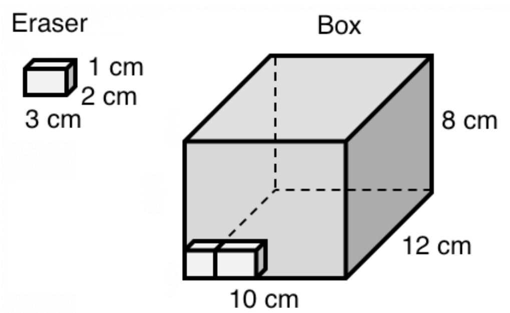
V	$=$	$A \times h$	Formula
	$=$	$42\ \text{cm}^2 \times\ 20\ \text{m}$	Substitute values
	$=$	$840\ \text{cm}^3$	

3. Fatu sells pencil erasers from her shop. She bought a large box of erasers to sell. Each eraser is 1 cm by 2 cm by 3 cm. The box of erasers is 10 cm wide, 8 cm tall, and 12 cm long. How many erasers are there in the box?

Solution

To find how many erasers are in the box, divide the total volume of the box by the volume of each eraser.

Draw a diagram. →



Volume of 1 eraser:

$$\begin{aligned} V_1 &= lwh \\ &= 3\ \text{cm} \times 2\ \text{cm} \times 1\ \text{cm} \\ &= 6\ \text{cm}^3 \end{aligned}$$

Volume of the box:

$$\begin{aligned} V_2 &= lwh \\ &= 12\ \text{cm} \times 10\ \text{cm} \times 8\ \text{cm} \\ &= 960\ \text{cm}^3 \end{aligned}$$

Divide to find the number of erasers: $960\ \text{cm}^3 \div 6\ \text{cm}^3 = 160$ erasers

There are 160 erasers in the box.

Practice

1. A tin can with height of 15 cm is in the shape of a cylinder with a radius of 7 cm. The can is 40% full of water. What is the volume of the water in the can? Use $\pi = \frac{22}{7}$.
2. A large wooden beam is 10 metres long. Its end face is a rectangle 40 cm by 60 cm.
   a. Calculate the volume of the beam in cubic metres.
   b. Find the mass of the beam if 1 cubic metre of wood has a mass of 200 kg.
3. Martin sells packs of biscuits in his shop. Each pack of biscuits is 10 cm long, 5 cm wide and 4 cm high. How many packs of biscuits can fit stacked inside a carton that is 80 cm long, 50 cm wide and 25 cm high?
4. Cooking oil costs Le 100.00 for 20 cm.^3 How much would it cost to fill a container in the shape of a cylinder with a radius of 7 cm and a height of 20 cm? Use $\pi = \frac{22}{7}$.
5. Calculate the volume of a cylindrical steel bar which is 80 cm long and 4 cm in diameter. Give your answer to the nearest whole number. Use $\pi = 3.14$.

Lesson Title: Surface Area of Solids	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-075	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify surface area as the area of the outside layer of a solid.
2. Identify and interpret measurements for surface area (units squared).

Overview

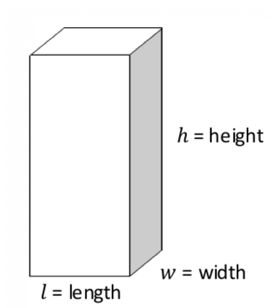
The outside layer of a solid is called the **surface area**. For example, imagine you have a box in the shape of a rectangular prism. You want to cover the outside of your box with paper. If you calculate the surface area of your box, you will know the size of paper that you need to completely cover the box. Surface area is measured in units squared. For example, cm^2.

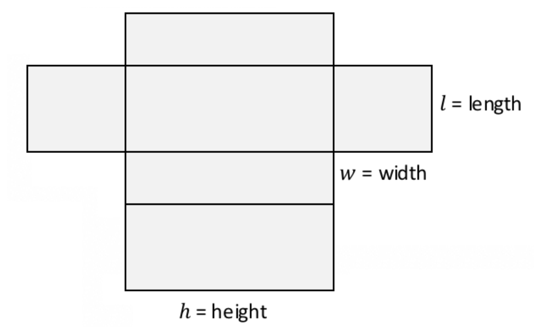
We can measure the surface area of all the solid objects we have been studying. There are formulae for calculating the surface area.

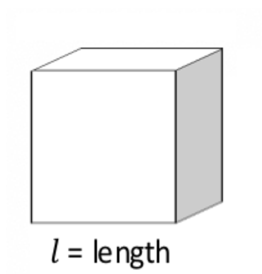
Using a **net** can help us understand this better. A net is like a paper version of a solid that can be opened up and laid flat. A net is made of plane shapes such as rectangles and circles.

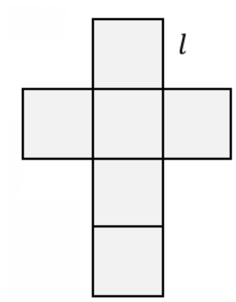
The surface area of a shape is the sum of the areas of the shapes in its net.

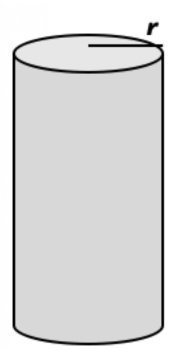
The diagrams below are some common solids and their nets.

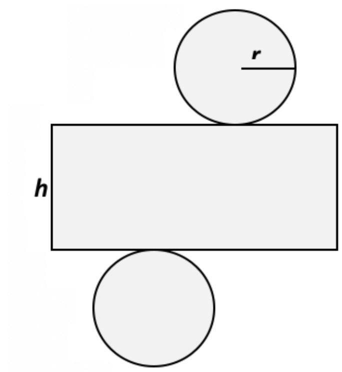


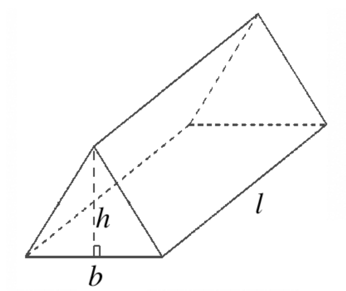


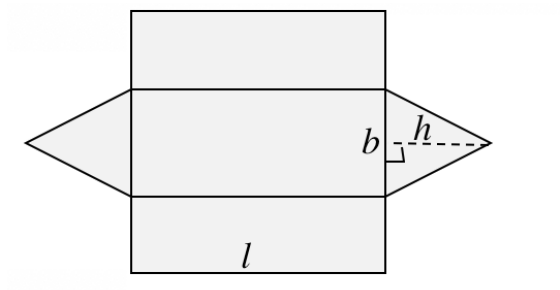








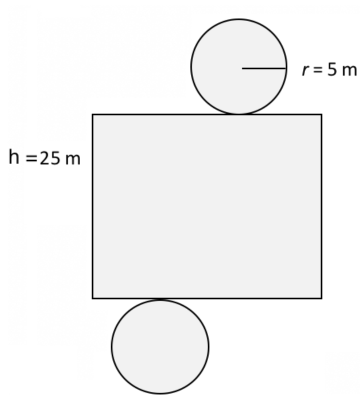




Solved Examples

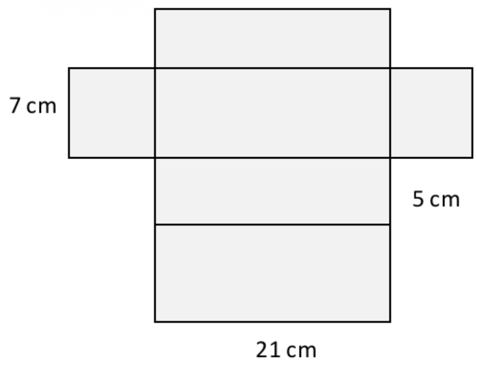
1. Sketch the net of a cylinder with a height of 25 m, and a radius of 5 m.

   Solution
   The net of a cylinder has 2 circles and 1 rectangle. Label the radius and height with the correct measurements:



2. Sketch the net of a rectangular prism with a length of 7 cm, width of 5 cm and height of 21 cm.

   Solution
   The net of a rectangular prism has 6 rectangles. Draw it to an accurate scale. For example, draw 7 cm longer than 5 cm. Draw the longest side 21 cm. Label it as shown:



3. A triangular prism has a base of 2 m, height of 3 m and length of 7 m. In what units is the surface area measured?

   Solution
   Surface area is measured in units squared. The surface area of this triangular prism is measured in m^2. This is read as “square metres” or “metres squared”.

4. A cylinder has a height of 18 mm and a radius of 4 mm. In what units is the surface area measured?

   Solution
   Surface area is measured in units squared. The surface area of this cylinder is measured in mm^2. This is read as “square millimetres” or “millimetres squared”.

Practice

1. Sketch the net of a cylinder with a height of 15 cm, and radius of 10 cm.
2. Sketch the net of a rectangular prism with a length of 17 cm, width of 18 cm and height of 38 cm.
3. Sketch the net of a **right-angled** triangular prism with a base of 8 cm, height of 5 cm, and length of 10 cm.
4. A rectangular prism has a length of 21 cm, width of 20 cm and height of 43 cm. In what units is the surface area measured?
5. A cylinder has a height of 21 m and radius of 12 m. In what units is the surface area measured?

Lesson Title: Surface Area of Cubes and Rectangular Prisms	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-076	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the surface area of a cube and rectangular prism.

Overview

In this lesson, you will find the surface area for cubes and rectangular prisms. We have formulae for finding these.

Remember that the formula for area of a square is $A = l^2$. A cube has 6 square faces. The formula for surface area of a cube is 6 times the area of each face:

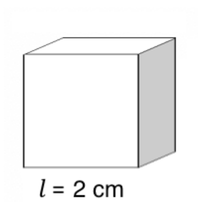
Surface area of a cube: $SA = 6l^2$

The formula for the area of a rectangle is $A = l \times w$. A rectangular prism has 6 sides, just like a cube, but they are not all the same. The opposite faces on a rectangular prism are the same. There are 3 pairs of faces which are the same. The formula for surface area is the sum of the 6 faces. Since there are pairs of faces that are the same, we get the following formula:

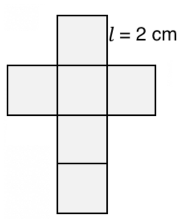
Surface area of a rectangular prism: $SA = 2lw + 2wh + 2lh$

Solved Examples

1. Draw the net of the cube shown. Then, calculate its surface area.



Solution
Draw the net of the cube each edge is 2 cm.



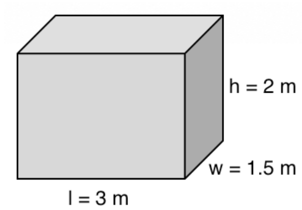
Calculate the surface area of the cube with the formula:

$$
\begin{aligned}
SA &= 6l^2 && \text{Formula} \\
&= 6(2\ \text{cm})^2 && \text{Substitute } l = 2 \text{ cm} \\
&= 6 \times 4\ \text{cm}^2 \\
&= 24\ \text{cm}^2
\end{aligned}
$$

2. Calculate the surface area of a rectangular prism with a length of 3 m, width of 1.5 m, and height of 2 m.

Solution

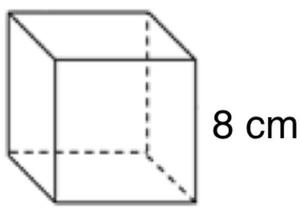
First, draw a diagram:

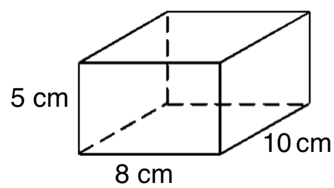


Calculate the surface area of the prism with the formula:

$$
\begin{aligned}
SA &= 2lw + 2wh + 2lh && \text{Formula} \\
&= 2(3)(1.5) + 2(1.5)(2) + 2(3)(2) && \text{Substitute the values} \\
&= 9 + 6 + 12 && \text{Multiply} \\
&= 27\ \text{m}^2
\end{aligned}
$$

3. Which has a greater surface area, the cube or the prism shown:





Solution

Calculate the area of the cube and the area of the rectangular prism. Then determine which is larger.

Surface area of the cube:

$$
\begin{aligned}
SA &= 6l^2 && \text{Formula} \\
&= 6(8\ \text{cm})^2 && \text{Substitute } l = 8 \text{ cm} \\
&= 6 \times 64\ \text{cm}^2 \\
&= 384\ \text{cm}^2
\end{aligned}
$$

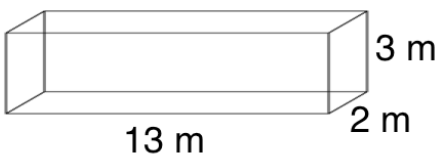
Surface area of the rectangular prism:

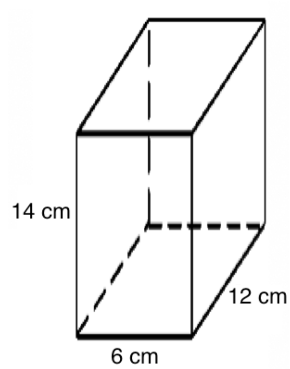
$$
\begin{aligned}
SA &= 2lw + 2wh + 2lh && \text{Formula} \\
&= 2(10)(8) + 2(8)(5) + 2(10)(5) && \text{Substitute the values} \\
&= 160 + 80 + 100 && \text{Multiply} \\
&= 340\ \text{cm}^2
\end{aligned}
$$

The surface area of the cube is larger than that of the rectangular prism:
$384\ \text{cm}^2 > 340\ \text{cm}^2$

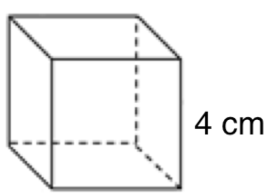
Practice

1. Calculate the surface area for each rectangular prism:





2. Calculate the surface area of the cube:



3. Calculate the surface area of a cube with sides of 12 mm.

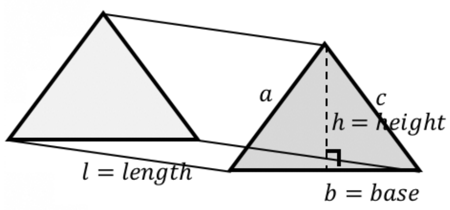
Lesson Title: Surface Area of Triangular Prisms	Theme: Measurement and Estimation
Practice Activity: PHM-08-077	**Class:** JSS 2

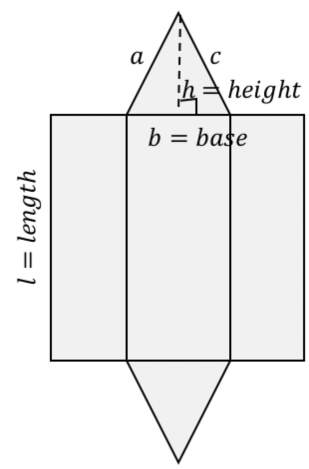
Learning Outcome
By the end of the lesson, you will be able to calculate the surface area of a triangular prism.

Overview

We calculate the surface area of the triangular prism by adding the areas of each of the shapes in the net, like a composite shape. We can also use a formula.

It is helpful to draw a net:





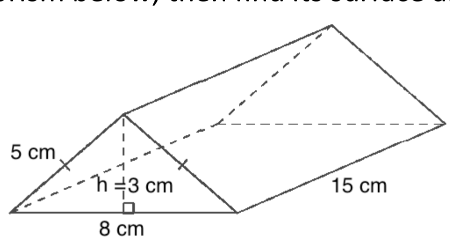
Remember that the formula for the area of a triangle is $A = \frac{1}{2}bh$. Two of the faces of a triangular prism are triangles. They are the same size. Together, their area is $2 \times \frac{1}{2}bh = bh$. This will be part of our surface area formula.

The other faces of the rectangular prism are rectangles. They all have length l. Their areas are al, bl, and cl. Together, their area is $al + bl + cl = l(a + b + c)$. This is another part of our surface area formula.

The formula for the surface area of a triangular prism is $SA = bh + (a + b + c)l$.

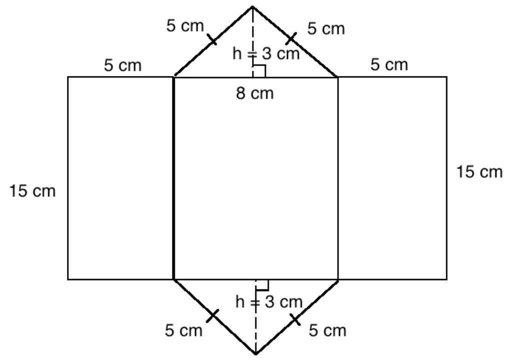
Solved Examples

1. Draw a net for the triangular prism below, then find its surface area.



Solution

First, draw the net:

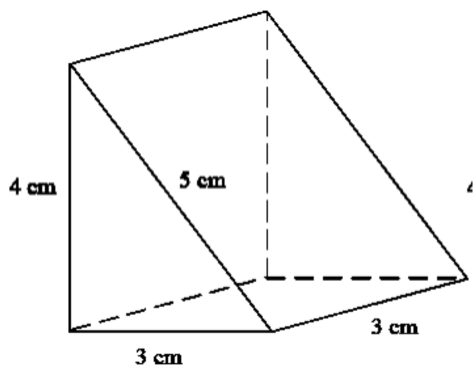


Lengths needed for the formula: $a = 5$ cm, $b = 8$ cm, $c = 5$ cm, $h = 3$ cm, $l = 15$ cm

Calculate the surface area with the formula:

SA	$=$	$bh + (a + b + c)l$	Formula
	$=$	$(8)(3) + (5 + 8 + 5)(15)$	Substitute values
	$=$	$24 + (18)(15)$	Simplify
	$=$	$24 + 270$	
	$=$	294 cm^2	

2. Find the surface area of the right-angled triangular prism:



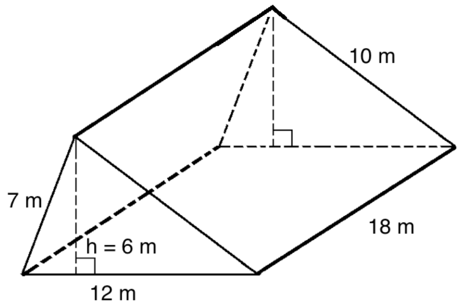
Solution

Lengths needed for the formula: $a = 4$ cm, $b = 3$ cm, $c = 5$ cm, $h = 4$ cm, $l = 3$ cm.
Note that in this case the height and 1 side of the triangle (a) are the same.

Calculate the surface area with the formula:

SA	$=$	$bh + (a + b + c)l$	Formula
	$=$	$(3)(4) + (4 + 3 + 5)(3)$	Substitute the values
	$=$	$12 + (12)(3)$	Simplify
	$=$	$12 + 36$	
	$=$	48 cm^2	

3. Find the surface area of the triangular prism:



Solution

Lengths needed for the formula: $a = 7 \text{ m}$, $b = 12 \text{ m}$, $c = 10 \text{ m}$, $h = 6 \text{ m}$, $l = 18 \text{ m}$.

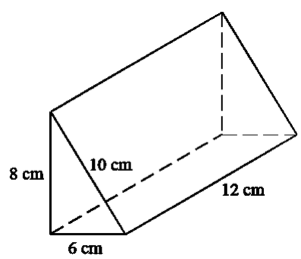
Calculate the surface area with the formula:

SA	$=$	$bh + (a + b + c)l$	Formula
	$=$	$(12)(6) + (7 + 12 + 10)(18)$	Substitute the values
	$=$	$72 + (29)(18)$	Simplify
	$=$	$72 + 522$	
	$=$	594 m^2	

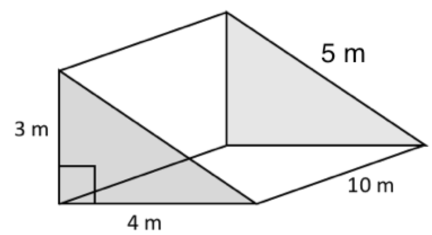
Practice

1. Find the surface area of the **right-angled** triangular prisms:

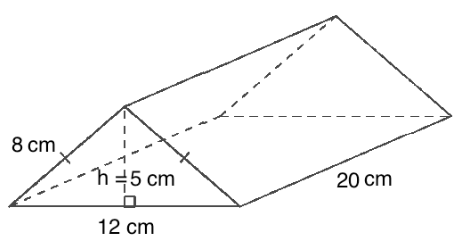
a.



b.



2. Find the surface area of the triangular prism:



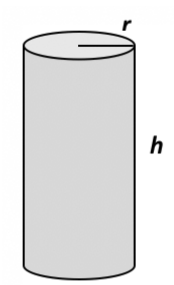
Lesson Title: Surface Area of Cylinders	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-078	**Class:** JSS 2

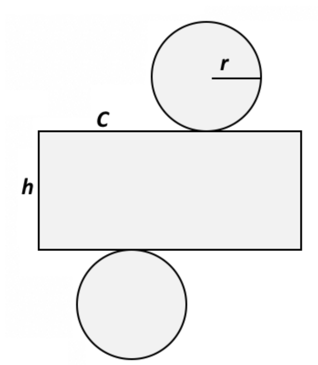
Learning Outcome
By the end of the lesson, you will be able to calculate the surface area of a cylinder.

Overview

We calculate the surface area of the cylinder by adding the areas of each of the shapes in the net, like a composite shape. We can also use a formula.

It is helpful to draw a net:





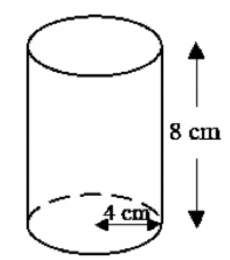
Remember that the formula for area of a circle is $A = \pi r^2$. Two of the faces of a cylinder are circles. They are the same size. Together, their area is $2\pi r^2$. This will be part of our surface area formula.

The other face of the cylinder is a rectangle. Notice that one side is the height of the cylinder h. The other side of the rectangle is equal to the circumference of the circular faces. Remember that the circumference of a circle is $C = 2\pi r$. The area of the rectangular face is the circumference times the height: $Ch = 2\pi rh$. This is another part of our surface area formula.

The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. This can also be written as $SA = 2\pi r(r + h)$.

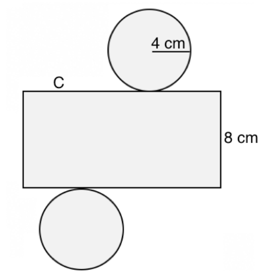
Solved Examples

1. Draw a net for the cylinder below, then find its surface area. Give your answer to the nearest whole number. Use $\pi = 3.14$.



Solution

First, draw the net:

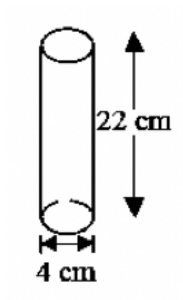


Lengths needed for the formula: $r = 4$ cm, $h = 8$ cm

Calculate the surface area with the formula:

SA	$=$	$2\pi r^2 + 2\pi rh$	Formula
	$=$	$2(3.14)(4)^2 + 2(3.14)(4)(8)$	Substitute the values
	$=$	$100.48 + 200.96$	Simplify
	$=$	301.44	
	$=$	301 cm^2	

2. Find the surface area of the cylinder below to 1 decimal place. Take $\pi = 3.14$.



Solution

Note that the diameter is given in the diagram, 4 cm. Find the radius: $r = \frac{d}{2} = \frac{4}{2} = 2$ cm. The height is 22 cm.

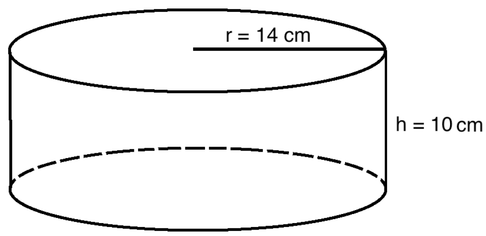
Calculate the surface area with the formula:

SA	$=$	$2\pi r^2 + 2\pi rh$	Formula
	$=$	$2(3.14)(2)^2 + 2(3.14)(2)(22)$	Substitute the values
	$=$	$25.12 + 276.32$	Simplify
	$=$	301.44	
	$=$	301.4 cm^2	

3. Find the surface area of a cylinder with a radius of 14 and height of 10. Take $\pi = \frac{22}{7}$.

Solution

First, draw a diagram:



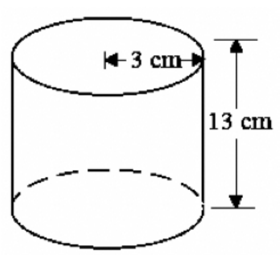
Calculate the surface area with the formula:

SA	$=$	$2\pi r^2 + 2\pi rh$	Formula
	$=$	$2(\frac{22}{7})(14)^2 + 2(\frac{22}{7})(14)(10)$	Substitute the values
	$=$	$1{,}232 + 880$	Simplify
	$=$	$2{,}112\ \text{cm}^2$	

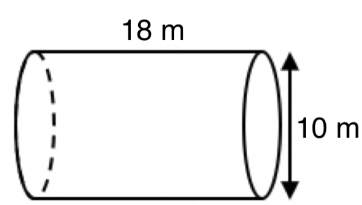
Practice

1. Find the surface area of each cylinder shown below. Use $\pi = 3.14$ and give your answers to the nearest whole number.

a.

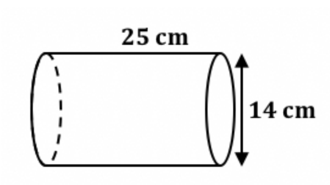


b.

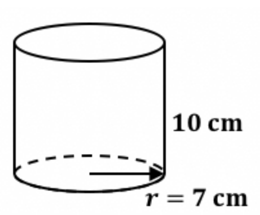


2. Find the surface area of each cylinder shown below. Use $\pi = \frac{22}{7}$ and give your answers to the nearest whole number.

a.



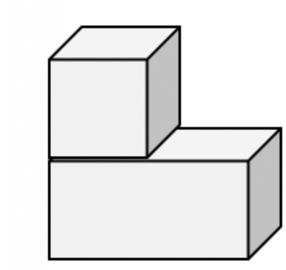
b.



Lesson Title: Surface Area of Composite Solids	**Theme:** Measurement and Estimation
Practice Activity: PHM-08-079	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the surface area of composite solids.

Overview



In this lesson, you will find the surface area of composite solids. To find the surface area of composite solids, use the formula for the area of the solids that make up the composite solid. First, add the areas of the different solids together. Then, **subtract the overlapping area.** This step is very important. The overlapping area is the part of each shape that is touching the other. For example, in the figure to the right, the overlap is one face of the cube and an equal surface area of the rectangular prism. You will subtract the area of the one face of the cube **twice**.

In the example figure, the surface area would be:

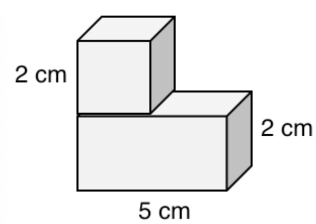
SA of the composite solid = SA of the cube + SA of the rectangular prism – 2 × area of 1 face of the cube

Recall and use the areas of the solids you have learned:

Solid	Surface Area Formula
Cube	$SA = 6l^2$
Rectangular Prism	$SA = 2lw + 2wh + 2lh$
Triangular Prism	$SA = bh + (a + b + c)l$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$

Solved Examples

1. Calculate the surface area of the composite figure:



Solution
Add the surface areas of the cube and rectangular prism, and subtract twice the area of one face of the cube.

Surface area of the cube:

$$\begin{aligned} SA_1 &= 6l^2 \\ &= 6(2\text{ cm})^2 \\ &= 6 \times 4\text{ cm}^2 \\ &= 24\text{ cm}^2 \end{aligned}$$

Surface area of the rectangular prism:

$$\begin{aligned} SA_2 &= 2lw + 2wh + 2lh \\ &= 2(2)(2) + 2(2)(5) + 2(2)(5) \\ &= 8 + 20 + 20 \\ &= 48\text{ cm}^2 \end{aligned}$$

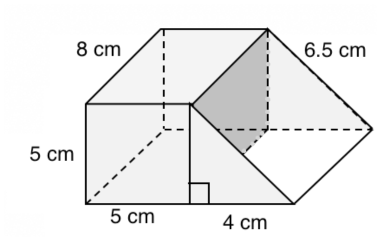
Area of 1 face of the cube: $A = l^2 = (2\text{ cm})^2 = 4\text{ cm}^2$

Surface area of the composite solid:

$$\begin{aligned} SA &= SA_1 + SA_2 - 2A \\ &= 24\text{ cm}^2 + 48\text{ cm}^2 - 2(4\text{ cm}^2) \\ &= 72\text{ cm}^2 - 8\text{ cm}^2 \\ &= 64\text{ cm}^2 \end{aligned}$$

The surface area of the composite solid is 64 cm^2.

4. Find the surface area of the figure at right.



Solution

Find the surface area of the rectangular prism (SA_1) and the surface area of the triangular prism (SA_2) separately, then subtract their shared side twice. Their shared side is a 5 cm x 8 cm rectangle.

Surface area of the rectangular prism:

$$\begin{aligned} SA_1 &= 2lw + 2wh + 2lh \\ &= 2(5)(5) + 2(5)(8) + 2(5)(8) \\ &= 50 + 80 + 80 \\ &= 210\text{ cm}^2 \end{aligned}$$

Surface area of the triangular prism:

$$\begin{aligned} SA_2 &= bh + (a + b + c)l \\ &= (4)(5) + (5 + 4 + 6.5)(8) \\ &= 20 + 124 \\ &= 144\text{ cm}^2 \end{aligned}$$

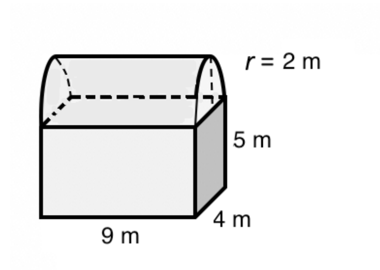
Area of 1 shared face: $A = lw = 8 \times 5 = 40\text{ cm}^2$

Surface area of the composite solid:

$$\begin{aligned} SA &= SA_1 + SA_2 - 2A \\ &= 210\text{ cm}^2 + 144\text{ cm}^2 - 2(40\text{ cm}^2) \\ &= 354\text{ cm}^2 - 80\text{ cm}^2 \\ &= 274\text{ cm}^2 \end{aligned}$$

The surface area of the figure is 274 cm^3.

5. Find the surface area of the figure at right. It is formed by half of a cylinder and a rectangular prism. Give your answer to the nearest whole number. Use $\pi = 3.14$.



Solution

Note that the surface area of half a cylinder is found simply by multiplying $\frac{1}{2}$ by its surface area formula and adding the rectangle shared face:

$\frac{1}{2}(2\pi r^2 + 2\pi rh) = \pi r^2 + \pi rh$. The shared side is a 9 m x 4 m rectangle.

Surface area of the rectangular prism:

$$\begin{aligned} SA_1 &= 2lw + 2wh + 2lh \\ &= 2(5)(4) + 2(4)(9) + 2(5)(9) \\ &= 40 + 72 + 90 \\ &= 202\ \mathrm{m}^2 \end{aligned}$$

Surface area of the $\frac{1}{2}$ cylinder:

$$\begin{aligned} SA_2 &= \pi r^2 + \pi rh \\ &= (3.14)(2^2) + (3.14)(2)(9) \\ &= 12.56 + 56.52 \\ &= 69.08\ \mathrm{m}^2 \end{aligned}$$

Area of 1 shared face: $A = lw = 9 \times 4 = 36\ \mathrm{m}^2$

Surface area of the composite solid:

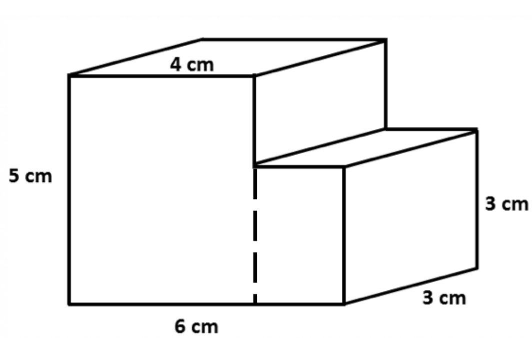
$$\begin{aligned} SA &= SA_1 + SA_2 - 2A \\ &= 202\ \mathrm{m}^2 + 69\ \mathrm{m}^2 - (36\ \mathrm{m}^2) \\ &= 271\ \mathrm{m}^2 - 36\ \mathrm{m}^2 \\ &= 235\ \mathrm{m}^2 \end{aligned}$$

The surface area of the figure is $235\ \mathrm{m}^3$.

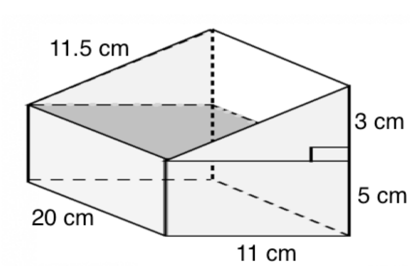
Practice

Find the surface area of each solid below. Give your answers to the nearest whole number.

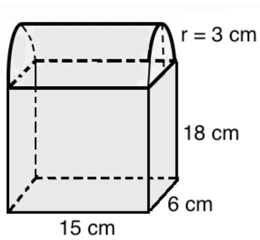
1.



2.



3.



Use $\pi = 3.14$.

Lesson Title: Surface Area Story Problems	Theme: Measurement and Estimation
Practice Activity: PHM-08-080	**Class:** JSS 2

Learning Outcome By the end of the lesson, you will be able to solve practical problems on surface area.

Overview

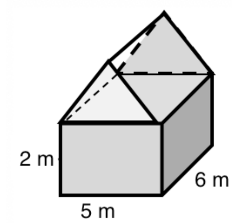
In this lesson, you will use information from previous lessons to solve story problems on surface area. It is always helpful to draw a diagram.

Solved Examples

1. Momo is painting the outside walls of his house. It is in the shape of a rectangular prism with width 5 m, length 6 m and height 2 m It takes Momo 12 minutes to paint one square metre. How many hours will it take to paint the house?

   Solution

   First, draw a diagram. →



   We need to find the surface area of the rectangular prism and then subtract the bottom and the top areas, since Momo will not paint the roof or the ground. Then we will multiply by 12 minutes per square metre.

   The formula for surface area of a rectangular prism is $SA = 2lw + 2wh + 2lh$. We can subtract the top and bottom surfaces by leaving them out to begin with: $SA = 2wh + 2lh$.

   Calculate the surface area of Momo's outside walls:

$$\begin{aligned} SA &= 2wh + 2lh \\ &= 2(5)(2) + 2(6)(2) \\ &= 20 + 24 \\ &= 44 \text{ m}^2 \end{aligned}$$

   It takes him 12 minutes to paint 1 m^2. Multiply the surface area by 12 minutes per square metre to find how long it takes him in total: $44 \text{ m}^2 \times 12 \frac{\text{min}}{\text{m}^2} = 528 \text{ minutes}$

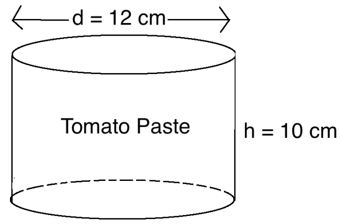
   Convert minutes to hours: $528 \div 60 = 8.8$ hours

   It will take Momo 8.8 hours to paint his house.

2. A company makes tomato paste in cans. They put labels on their cylinder-shaped cans. The cans are 10 cm tall with a diameter of 12 cm. What is the surface area of the label the company needs to print so that it will cover the side of the can from top to bottom?

Solution

Draw a picture. →



Remember that the outside curve of a cylinder is a rectangle with a length that is the same length of the circumference of the circle, and height that is the same height as the cylinder.

Use the formula $SA = 2\pi rh$ to find the size of the label.

First, find the radius: $r = \frac{d}{2} = \frac{12}{2} = 6$ cm

Substitute in the formula:

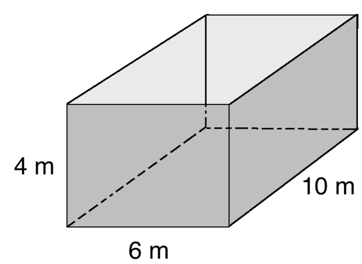
$$\begin{aligned} SA &= 2\pi rh \\ &= 2(3.14)(6)(10) \\ &= 376.8 \text{ cm}^2 \end{aligned}$$

The area of the can label is 376.8 cm^2.

3. An open rectangular tank has a length of 10 m, width of 6 m and height of 4 m. Find its surface area.

Solution

Draw a picture (seen at right). Note that the tank is a rectangular prism, but it is open on top.



Use the formula for the surface area of a rectangular prism, $SA = 2lw + 2wh + 2lh$. We can subtract the top surface by leaving it out to begin with: $SA = lw + 2wh + 2lh$. One of the faces measuring $l \times w$ has been removed.

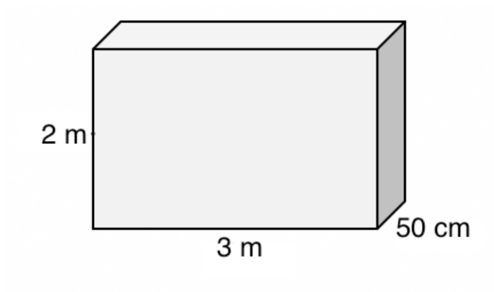
Calculate the surface area of the tank:

$$\begin{aligned} SA &= lw + 2wh + 2lh \\ &= (10)(6) + 2(6)(4) + 2(10)(4) \\ &= 60 + 48 + 80 \\ &= 188 \text{ m}^2 \end{aligned}$$

The surface area of the open tank is 188 m^2.

Practice

1. An open cylindrical tank has a radius of 3 metres and height of 2 metres. Find its surface area to the nearest whole number. (Take $\pi = 3.14$)
2. A bottle of cola has a label on it. The label is 8 cm tall, and the diameter of the bottle is 10 cm. What is the area covered by the label? (Take $\pi = 3.14$)
3. Hawa is wrapping a gift for her friend's birthday. The box she is wrapping is 15 cm tall, 20 cm long and 10 cm wide. What is the area of paper that she will need to completely cover the box?
4. A certain school built a new sign out of cement. It sits directly on the ground. The dimensions of the sign are shown below. A painter will come to paint the sign.
    a. Find the surface area of the sign. Give your answer in square metres.
    b. If one can of paint covers 3 m^2, how many cans of paint does he need to bring?



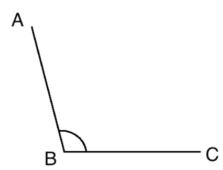
Lesson Title: Introduction to Angles	**Theme:** Geometry
Practice Activity: PHM-08-081	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify and compare types of angles (acute, obtuse, right, straight, and reflex).
2. Identify degrees as the measurement of angles.

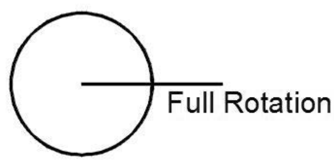
Overview

An angle is made up of 2 lines. The corner point of an angle is called **vertex** and the two straight lines are called **arms.** For example, $\angle ABC$ (‘angle ABC’) is shown below. It can be measured in degrees. Degrees are used to measure turn. There are 360 degrees in one full rotation (one complete circle). We use the little circle (°) following the number to mean degrees.

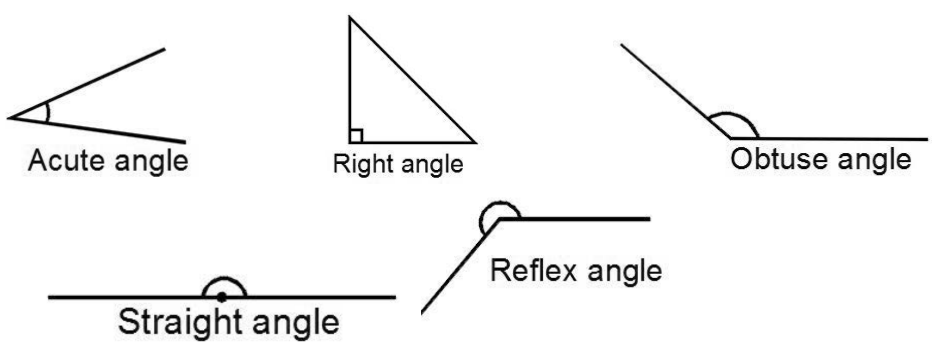
In this case, $\angle ABC = 105°$



A full rotation is $360°$.



In this lesson, you will learn about 5 types of angles, which are less than a full rotation. The different types of angles are shown and described below:

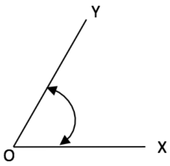


- An acute angle is an angle less than $90°$.
- A right angle is an angle that is exactly $90°$.
- An obtuse angle is an angle that is greater than $90°$ but less than $180°$.
- A straight angle is an angle that is exactly $180°$.
- A reflex angle is an angle greater than $180°$ but less than $360°$.

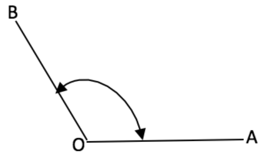
Solved Examples

1. Identify the type of each angle shown. Give your reasons.

a.



b.



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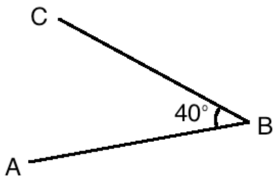
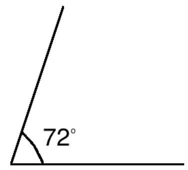
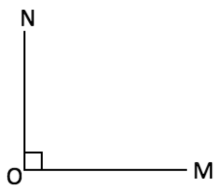


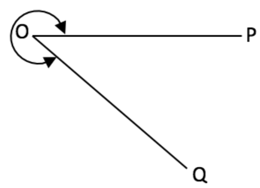
d.



e.



f.



Solutions

a. Acute angle, because it is smaller than 90°
b. Obtuse angle, because it is larger than 90° but less than 180°
c. Acute angle, because it is smaller than 90°
d. Acute angle, because it is smaller than 90°
e. Right angle, because it is exactly 90°
f. Reflex angle, because it is larger than 180°. Note that the curved arrow shows the angle.

2. Write the following angle measurements in words:

a. 104°
b. 16.3°

Solutions

a. One hundred and four degrees
b. Sixteen point three degrees

3. Write the following angular measurements in figures:

a. Three hundred twenty point three degrees
b. Ten and a half degrees

Solutions

a. 320.3°
b. 10.5°

4. Classify the type of each angle listed below.

   a. 3° b. 179° c. 180° d. 190° e. 90°

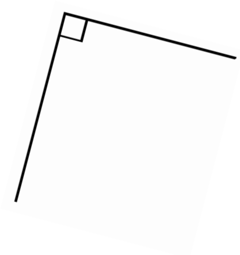
   Solutions

   a. Acute angle, b. Obtuse angle, c. Straight angle, d. Reflex angle, e. Right angle.

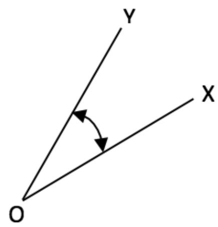
Practice

1. Identify whether each angle is obtuse, acute, right, straight, or reflex angle. Give your reasons.

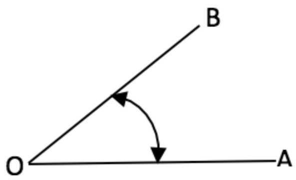
   a.



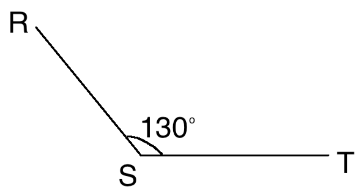
   b.



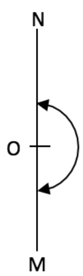
   c.



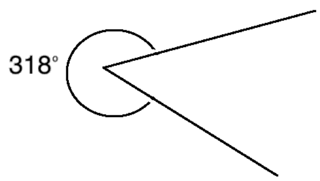
   d.



   e.



   f.



2. Write the following angular measurements in words:
   a. 280°
   b. 80.39°
   c. 16.5°
3. Write the following angular measurements in figures:
   a. Fifty-five degrees
   b. Sixty-six and a half degrees.
   c. Ninety point four degrees.
4. Classify the type of each angle listed below.

   a. 33° b. 109° c. 260° d. 9° e. 91°

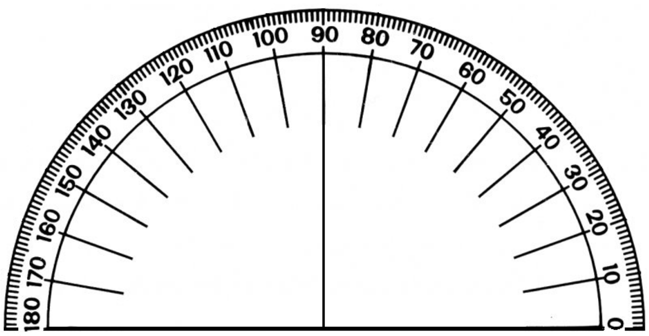
Lesson Title: Measurement of Angles	Theme: Geometry
Practice Activity: PHM-08-082	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Estimate the measure of a given angle.
2. Measure given angles (acute, obtuse, right) using a protractor.

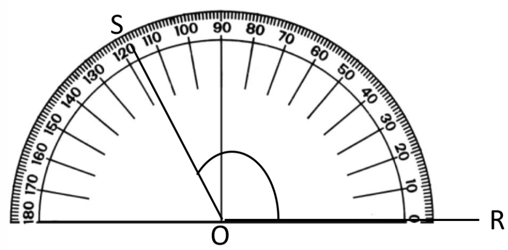
Overview

We use a tool called a protractor to measure and draw angles. Angles are measured in degrees, and this protractor can measure any angle less than 180 degrees. Look at the numbers on the protractor. They count by tens from 0° to 180°. This is like a ruler, but instead of measuring length we use it to measure how much an angle opens.

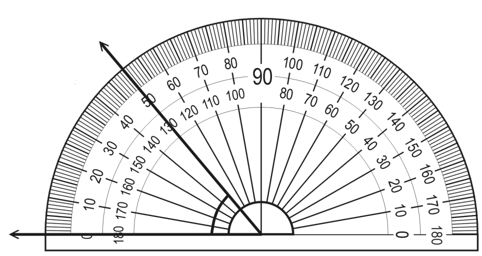
If you do not have a protractor, you can make one with paper. Trace the protractor below with a pen onto another piece of paper.



To measure an angle, place a protractor over the angle so that its centre is exactly over the vertex of the angle. The baseline of the protractor is exactly along one line of the angle. Count the degrees from the baseline to the other line of the angle.



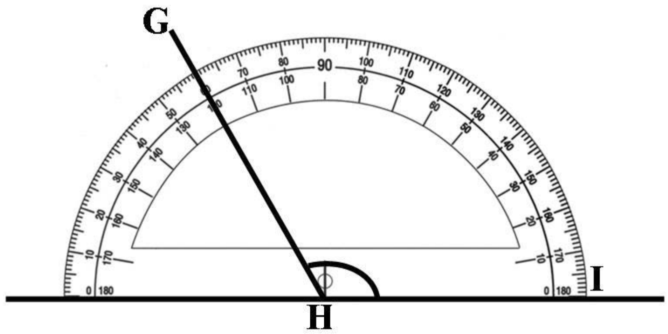
The first diagram shows angle SOR. The baseline of the protractor is along the line OR. The line SO tells us the measure of the angle. It passes through 117°. The measure of angle SOR is 117°. This can be written $\angle SOR = 117°$.



You can measure an angle using either side of the protractor. The angle shown here opens on the left. If your protractor has 2 sets of numbers, count the degrees using the outside numbers, starting on the left, from the baseline to where the other ray of the angle is pointing. The angle shown here is $50°$.

Solved Examples

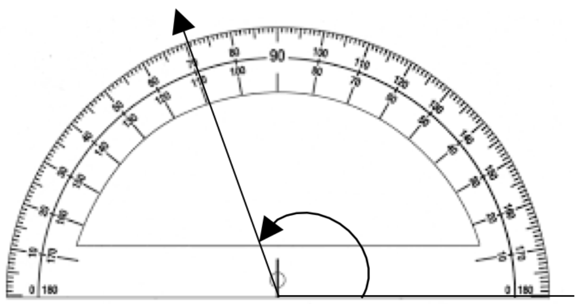
1. Identify the measure of angle GHI.



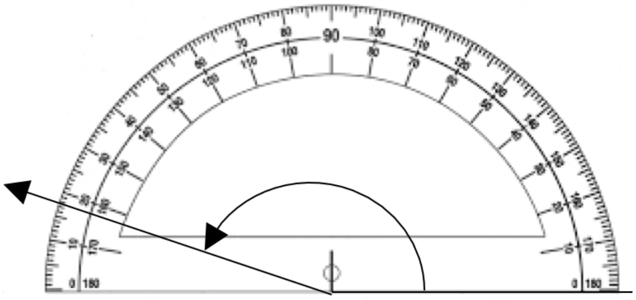
   Solution
   The measure of the angle is given by the degree that GH passes through.
   $\angle GHI = 120°$

2. Identify the values of the angles illustrated below. Determine whether each angle is acute or obtuse.

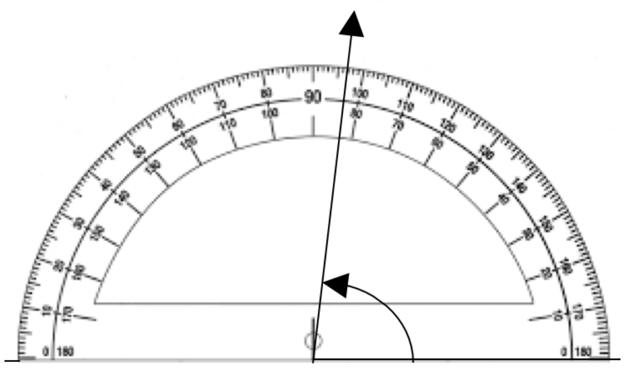
   a.



   b.



   c.

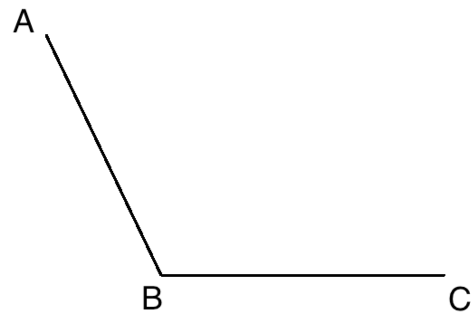


Solutions

Identify which degree the arm of each triangle passes through. The answers are:

a. 119°, obtuse
b. 163°, obtuse
c. 83°, acute

3. Use a protractor to measure of angle ABC:



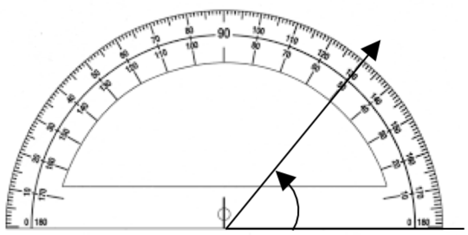
Solution

Place your protractor on the angle as shown in the diagrams above. You will find that its measure is $\angle ABC = 115°$

Practice

1. Give the values of the angles illustrated in the measurements below. Determine whether each angle is acute or obtuse.

a.



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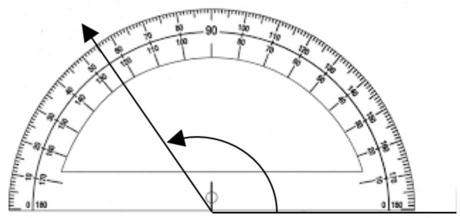

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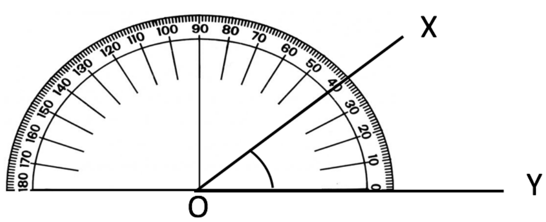
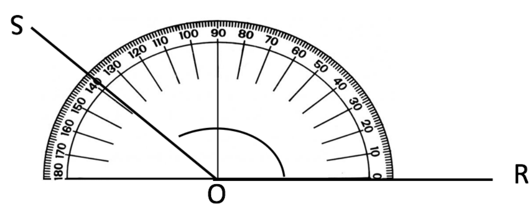
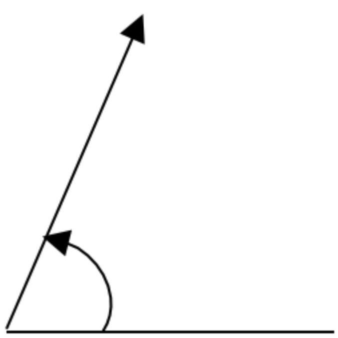




2. Use a protractor to measure the following angles:

a.



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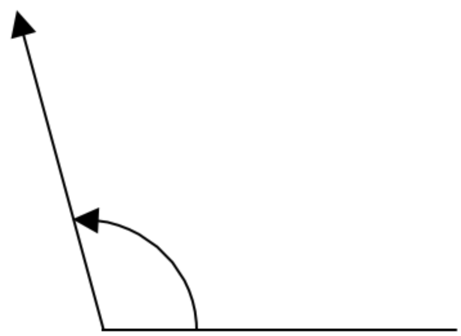

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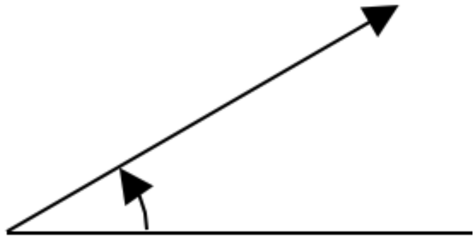
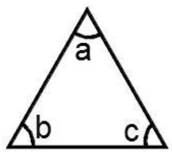


Lesson Title: Finding Unknown Angles in Triangles	**Theme:** Geometry
Practice Activity: PHM-08-083	**Class:** JSS 2

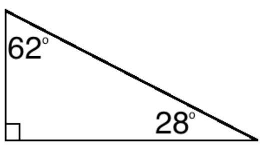
Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that the sum of the angles in a triangle is 180°.
2. Find unknown angles in a triangle.

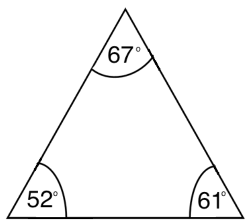
Overview

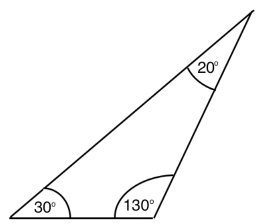
In the triangle below, the angles $a, b,$ and c are called **interior** angles. The sum of the interior angles of any triangle is equal to $180°$.



These are examples of triangles. You can try adding the angle measures – they always sum to $180°$.



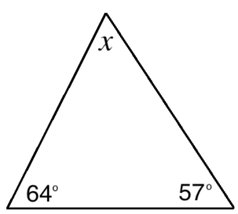




When the measure of an angle is unknown, you can find its measure by subtracting the known angles from $180°$.

Solved Examples

1. Find the measure of the angle marked x in the triangle below:



Solution

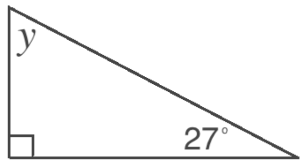
The sum of the interior angles of a triangle is $180°$.

$$\begin{aligned} x + 64° + 57° &= 180° \\ x + 121° &= 180° \\ x &= 180° - 121° \\ x &= 59° \end{aligned}$$

Check: You can always check your answer by adding the 3 angle measures together. They should sum to $180°$:

$$64° + 57° + 59° = 180°$$

2. Find the measure of angle y in the right-angled triangle below:

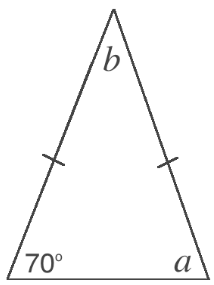


Solution

Remember that the right angle is 90°.

$$\begin{aligned} y + 90° + 27° &= 180° \\ y + 117° &= 180° \\ y &= 180° - 117° \\ y &= 63° \end{aligned}$$

3. Find the measure of angles a and b in the isosceles triangle below:



Solution

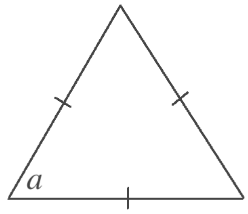
Remember that an isosceles triangle has 2 sides of equal length. These are marked in the diagram with small lines. An isosceles triangle also has 2 equal angles. The angles between each of the equal sides and the 3rd side are equal. In this triangle, $a = 70°$.

We know 2 of the angles because $a = 70°$. Use this to solve for b:

$$\begin{aligned} b + 70° + 70° &= 180° \\ b + 140° &= 180° \\ b &= 180° - 140° \\ b &= 40° \end{aligned}$$

The answer is $a = 70°$ and $b = 40°$.

4. Find the measure of angle a in the equilateral triangle below:



Solution

Remember that an equilateral triangle has all 3 sides of equal length. The 3 angle measures are also equal. Thus, we have $a + a + a = 180°$ or $3a = 180°$.

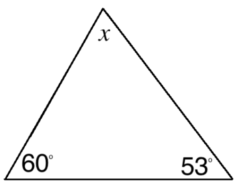
Solve for a:

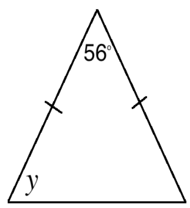
$$\begin{aligned} 3a &= 180° \\ a &= \tfrac{180°}{3} && \text{Divide both sides by 3} \\ a &= 60° \end{aligned}$$

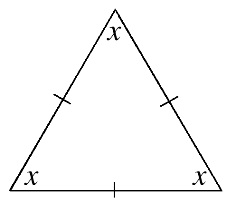
The angles of any equilateral triangle are all $60°$.

Practice

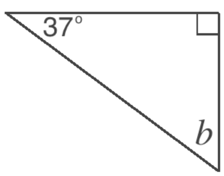
1. Find the value of the lettered angles in the diagrams below:

a. 

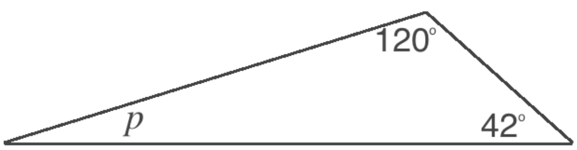
b. 

c. 

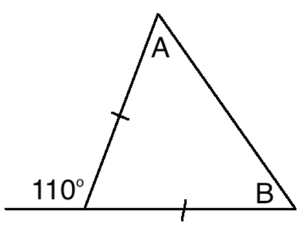
2. Find the measure of angle b in the triangle below:



3. Find the measure of angle p:



4. Find the measures of angles A and B in the diagram:



Lesson Title: Finding Unknown Angles in Quadrilaterals	**Theme:** Geometry
Practice Activity: PHM-08-084	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that the sum of the angles in any quadrilateral is 360°.
2. Find unknown angles in quadrilaterals.

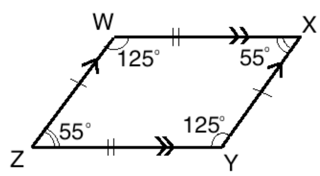
Overview

This lesson is on finding missing angles in quadrilaterals. Remember that quadrilaterals are shapes with 4 sides, including squares, rectangles, parallelograms, rhombuses, and trapeziums.

The 4 angles in **any** quadrilateral sum to 360°. If you know 3 angles of a quadrilateral, you can subtract them from 360° to find the measure of the 4th angle.

Rectangles and squares are quadrilaterals that have four 90° angles. For other shapes, you can use their characteristics to find their angle measures.

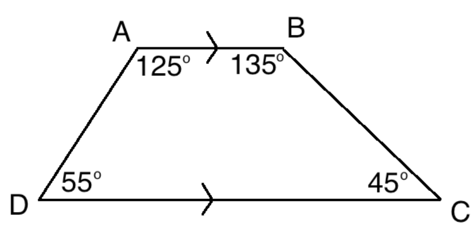
The opposite angles in a parallelogram are equal. The co-interior angles (the angles next to each other) sum to 180°. For example, consider parallelogram WXYZ:



The following angles are opposite, so are equal: W and Y; X and Z.

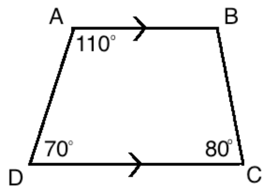
The following angles are co-interior, so sum to 180°: W and X; X and Y; Y and Z; W and Z.

In a trapezium, the angles adjacent to each other on opposite parallel sides sum to 180°. For example, in the trapezium below, $A + D = 180°$ and $B + C = 180°$:



Solved Examples

1. Find the measure of missing angle B:



Solution

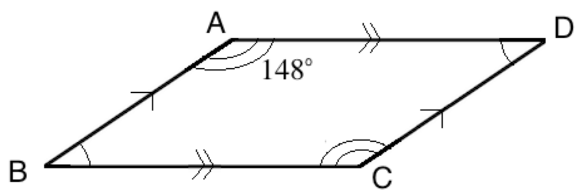
Method 1. Subtract the known angles from 360° to find B:

$$\begin{aligned} A + B + C + D &= 360° \\ 110° + B + 80° + 70° &= 360° \\ B + 260° &= 360° \\ B &= 360° - 260° \\ B &= 100° \end{aligned}$$

Method 2. Use the characteristics of a trapezium. We know that $B + C = 180°$. Use this fact to solve for B:

$$\begin{aligned} B + C &= 180° \\ B + 80° &= 180° \\ B &= 180° - 80° \\ B &= 100° \end{aligned}$$

2. Find the measures of angles B, C, and D in the parallelogram:



Solution

Note that $C = A$ because they are opposite angles in a parallelogram. Thus, $C = 148°$.

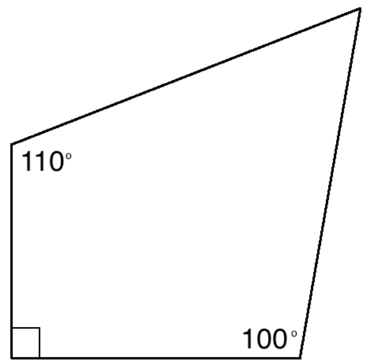
Now, B and D are unknown angles. We know that they are equal, $B = D$. Subtract the known angles from 360°, then divide by 2 to find the measure of each unknown angle.

$$\begin{aligned} A + B + C + D &= 360° \\ 148° + B + 148° + D &= 360° \\ B + D + 296° &= 360° \\ B + D &= 360° - 296° \\ B + D &= 64° \end{aligned}$$

Since $B = D$, divide 64° by 2 to find the measure of each:

$B = D = 64° \div 2 = 32°$

3. Find the measure of the unknown angle in the quadrilateral:



Solution

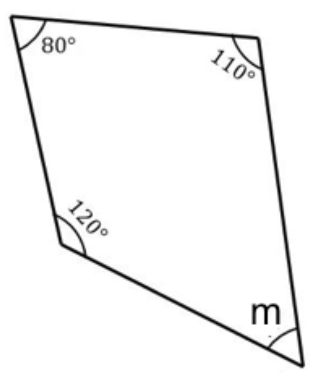
This is not a quadrilateral with any special features. Simply subtract the 3 known angles from 360° to find the angle that is not marked. It is not assigned a letter, so assign a letter to it:

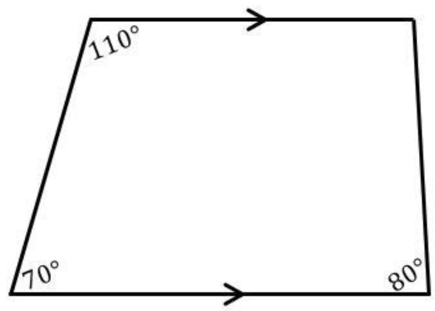
$$\begin{aligned} x + 110° + 90° + 100° &= 360° \\ x + 300° &= 360° \\ x &= 360° - 300° \\ x &= 60° \end{aligned}$$

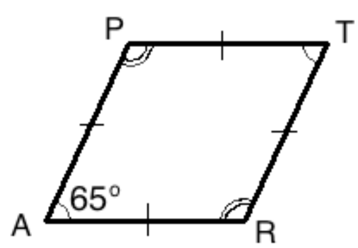
The measure of the unknown angle is 60°.

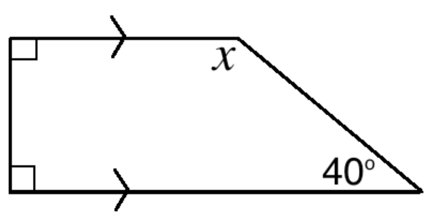
Practice

Find the measures of each of the unknown angles in the shapes below.

1. 

2. 

3. 

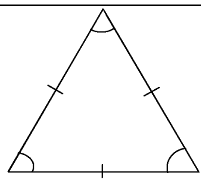
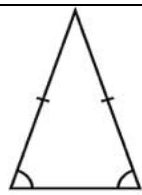
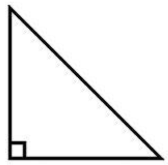
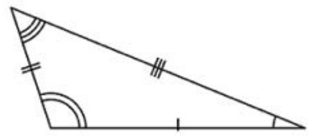
4. 

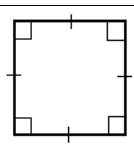
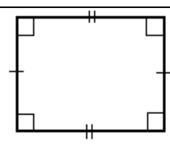
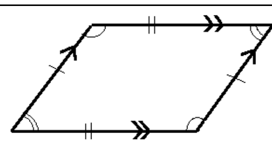
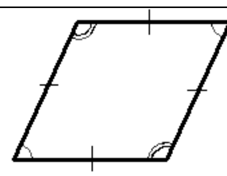
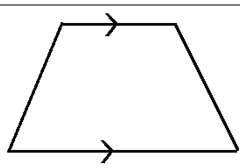
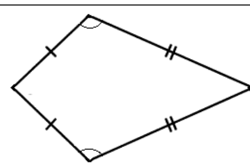
Lesson Title: Angle Practice	**Theme:** Geometry
Practice Activity: PHM-08-085	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find unknown angles in various types of triangles and quadrilaterals.

Overview

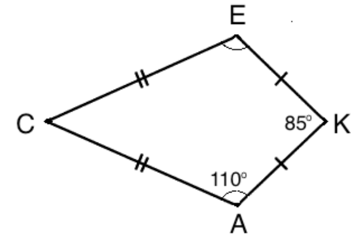
In this lesson, you will practise finding the missing angles in triangles and quadrilaterals. Use the fact that the angles in a triangle sum to 180°. The angles in a quadrilateral sum to 360°. You will also use the characteristics of different types of triangles and quadrilaterals, as shown below.

			
Equilateral Triangle All 3 angles are equal	**Isosceles triangle** 2 angles are equal	**Right-angled triangle** 1 angle is 90° Sum of the other 2 angles is 90°	**Scalene triangle** All angles are different

			
Square / Rectangle All angles 90°		**Parallelogram / Rhombus** Opposite angles equal Adjacent angles sum to 180°	
			
Trapezium Angles adjacent to each other on opposite parallel sides sum to 180°		**Kite** 2 opposite angles are equal	

Solved Examples

1. Find the measures of angles C and E in the kite CAKE:



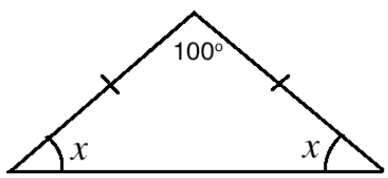
Solution

You can see that $A = E$ because the angles are marked with a curve. We have $E = A = 110°$.

Now C is the only unknown angle in the quadrilateral. Solve for angle C:

$$\begin{aligned} C + A + K + E &= 360° \\ C + 110° + 85° + 110° &= 360° \\ C + 305° &= 360° \\ C &= 360° - 305° \\ C &= 55° \end{aligned}$$

2. Find the measure of x in the triangle below:



Solution

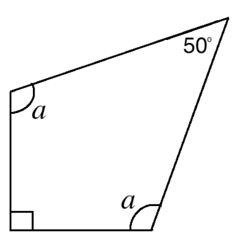
Subtract the known angle from the total degrees in a triangle, 180°:

$$180° - 100° = 80°$$

The total measurement of both of the unknown angle together is $80°$. The 2 angles are equal, so we have $x + x = 2x = 80°$. Solve for x:

$$\begin{aligned} 2x &= 80° \\ x &= \tfrac{80°}{2} \\ x &= 40° \end{aligned}$$

3. Find the measure of a in the quadrilateral below:



Solution

Subtract the known angles from the total degrees in a quadrilateral, 360°:

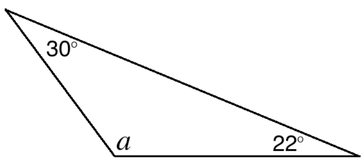
$$360° - 90° - 50° = 220°$$

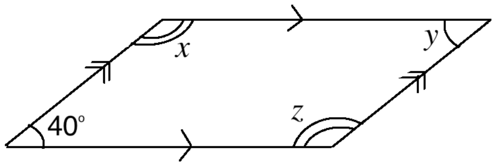
The total measurement of both of the unknown angles together is $220°$. The 2 angles are equal, so we have $a + a = 2a = 220°$. Solve for a:

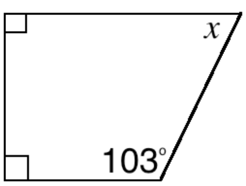
$$\begin{aligned} 2a &= 220° \\ a &= \tfrac{220°}{2} \\ a &= 110° \end{aligned}$$

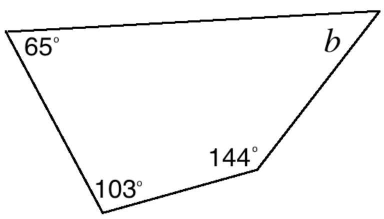
Practice

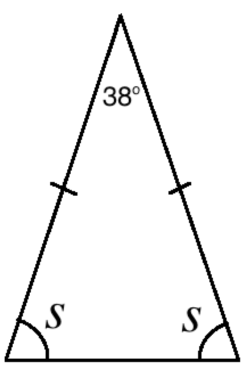
Find the measures of each of the unknown angles in the shapes below:

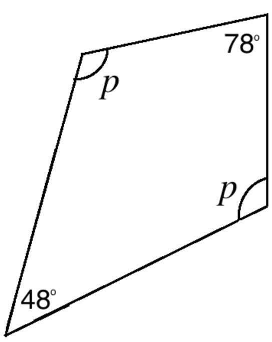
1. 

2. 

3. 

4. 

5. 

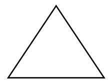
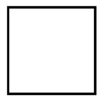
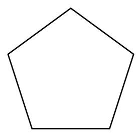
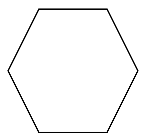
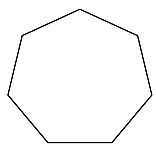
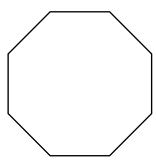
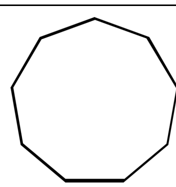
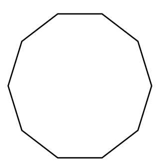
6. 

Lesson Title: Polygons	**Theme:** Geometry
Practice Activity: PHM-08-086	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify and draw polygons up to a decagon.

Overview

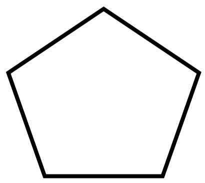
This lesson is on the properties of the polygons in the table below:

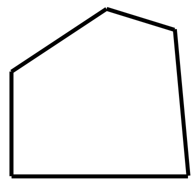
Sides	Name	Picture
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	

The name of each shape tells us how many sides and angles it has. “Tri” means 3, which is the number of sides and angles in a triangle. “Quad” means 4, which is the number of sides and angles in a quadrilateral. “Penta” means 5, “hexa” means 6, “hepta” means 7, and so on.

“Poly” means many, and the part of the word “-gon” refers to the angles. “Polygon” means a shape with many angles.

A **regular** polygon is one with equal angles and equal sides. For example, an equilateral triangle is a regular polygon. A square is also a regular polygon. A regular pentagon is shown below, alongside one that is not regular:


regular pentagon

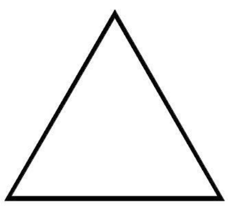
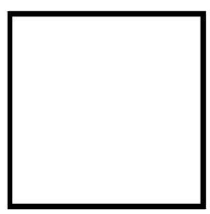
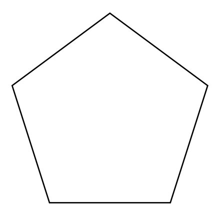
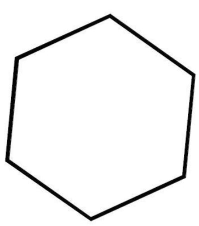
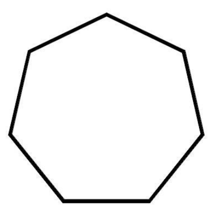
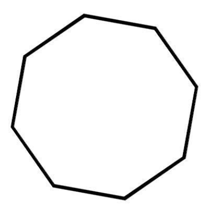

pentagon

Solved Examples

1. List and draw six types of regular polygons.

   Solution
   Regular polygons are polygons that have all sides of equal length, and all angles are equal. Six example answers are given below.

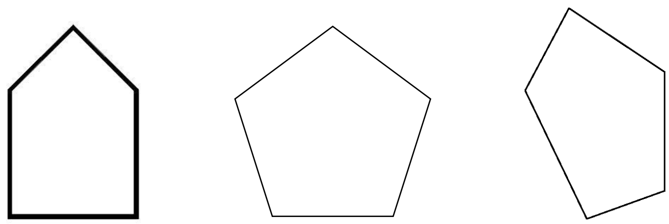
   The angles and sides do not need to be exactly equal. Draw the best you can.

3 Sides - Triangle 	4 Sides - Quadrilateral 	5 Sides - Pentagon 
6 Sides - Hexagon 	7 Sides - Heptagon 	8 Sides - Octagon 

2. Draw 3 different pentagons.

   Solution
   Pentagons are shapes with 5 sides. Your pentagons can be any size and shape, as long as they have 5 angles and 5 sides.

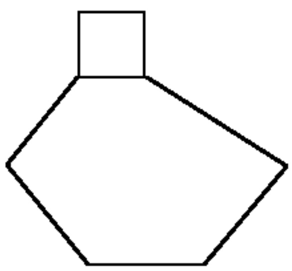
   These are some examples:



3. Draw a hexagon and square that share a side.

   Solution
   You may draw any hexagon and square. The important thing is that they share 1 side. This means that they have sides that touch and are exactly the same length.

   This is an example:



Practice

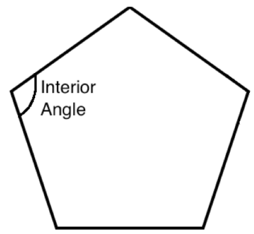
1. Draw an octagon.
2. Draw a decagon.
3. Draw 3 different hexagons.
4. Draw a pentagon and triangle that share 1 side.

Lesson Title: Sum of the Interior Angles of a Pentagon	**Theme:** Geometry
Practice Activity: PHM-08-087	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Find the sum of the interior angles of a pentagon.
2. Identify the formula for the sum of the interior angles of a polygon: $180°(n-2)$.

Overview

We have studied different types of polygons. Today, we will look at one of them: the pentagon. You will learn about the angles inside the pentagon.



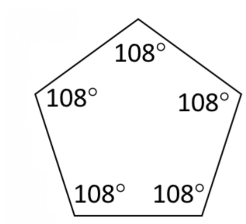
Remember that the angles in a triangle always sum to 180°, and the angles in a quadrilateral always sum to 360°. We call the angles inside a polygon **interior angles**. We can tell the sum of the interior angles of any polygon based on the number of sides it has. There is a pattern, which is that 180° is added each time:

Shape	Sum of Interior Angles
Triangle	180°
Quadrilateral	180° + 180° = 360°
Pentagon	180° + 180° + 180° = 540°

Notice that in each case, 180° is added 2 fewer times than the number of sides. From this, we have a formula that gives us the sum of the interior angles of a shape:

Sum of interior angles $= 180°(n-2)$, where n is the number of sides.

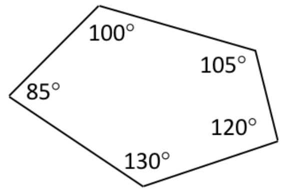
A regular polygon is one with all sides and angles equal. A regular pentagon has angles that measure 108°, as shown:



A pentagon can have any shape or size. Its interior angles will **always** sum to 540°.

Solved Examples

1. Add the angles of the pentagon below to verify that they sum to 540°.



Solution

Add the measures of the angles: $100° + 105° + 120° + 130° + 85° = 540°$

The angles of the pentagon sum to $540°$, as they are supposed to.

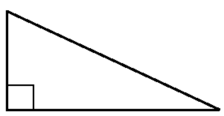
2. Apply the formula $180°(n - 2)$ to find the sum of the interior angles of a pentagon.

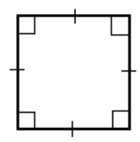
Solution

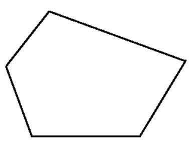
Apply the formula. Substitute $n = 5$ because a pentagon has 5 sides.

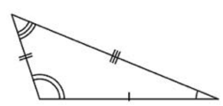
$180°(n-2)$	$=$	$180°(5-2)$	Substitute $n = 5$
	$=$	$180°(3)$	Brackets
	$=$	$540°$	Multiply

3. Next to each shape, write the sum of its interior angles:

a.  Sum: ________

b.  Sum: ________

c.  Sum: ________

d.  Sum: ________

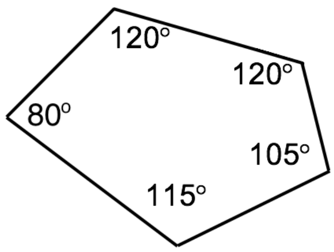
Solutions

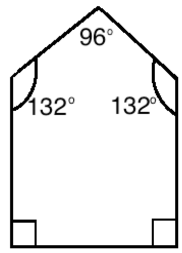
a. The shape is a triangle. The sum of the angles of a triangle is always 180°.
b. The shape is a square, which is a quadrilateral. The sum of the angles of a quadrilateral is always 360°.
c. The shape is a pentagon. The sum of the angles of a pentagon is always 540°.
d. The shape is a triangle. The sum of the angles of a triangle is always 180°.

Practice

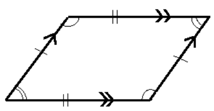
1. Write the formula for calculating the sum of the interior angles of a polygon.

2. Add the measures of the angles below to verify that the interior angles of each pentagon sum to 540°.

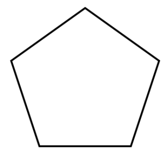
a. 

b. 

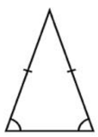
3. Next to each shape, write the sum of its interior angles:

a. 

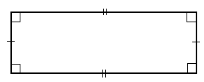
Sum: ________

b. 

Sum: ________

c. 

Sum: ________

d. 

Sum: ________

Lesson Title: Sum of the Interior Angles of a Polygon	**Theme:** Geometry
Practice Activity: PHM-08-088	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the sum of the interior angles of a polygon using the formula: $180°(n-2)$.

Overview

This lesson is on finding the sum of the interior angles of any polygon. It is helpful to memorise the sum of the angles of the polygons up to a decagon. You can also use the formula to calculate them any time.

Remember the formula for finding the **sum** of the interior angles in a polygon:

$(n-2) \times 180°$, where n is the number of sides.

The sum of the interior angles for the first 3 polygons are given in the table below. In the Solved Example and Practice problems you will calculate the others. Write them in the table.

Sides	Name	Sum of Interior Angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	

Solved Examples

1. Calculate the sum of the interior angles of a hexagon.

   Solution
   A hexagon has 6 sides, so substitute $n = 6$ in the formula and solve:

$$\begin{aligned} \text{Sum of angles} &= (n-2) \times 180° \\ &= (6-2) \times 180° \\ &= 4 \times 180° \\ &= 720° \end{aligned}$$

2. Calculate the sum of the interior angles of a polygon with 10 sides.

Solution

Substitute $n = 10$ in the formula and solve:

$$\begin{aligned}\text{Sum of angles} &= (n-2)\times 180^\circ \\ &= (10-2)\times 180^\circ \\ &= 8\times 180^\circ \\ &= 1{,}440^\circ\end{aligned}$$

3. Calculate the sum of the interior angles of a polygon with 15 sides.

Solution

Substitute $n = 15$ in the formula and solve:

$$\begin{aligned}\text{Sum of angles} &= (n-2)\times 180^\circ \\ &= (15-2)\times 180^\circ \\ &= 13\times 180^\circ \\ &= 2{,}340^\circ\end{aligned}$$

Practice

1. Calculate the sum of the interior angles of a nonagon.
2. Calculate the sum of the interior angles of a polygon with 8 sides.
3. Which polygon has interior angles that sum to 540°?
4. Calculate the sum of the interior angles of a heptagon.
5. Which polygon has interior angles that sum to 1,260°?
6. Calculate the sum of the interior angles of a polygon with 20 sides.

Lesson Title: Interior Angle Practice	Theme: Geometry
Practice Activity: PHM-08-089	Class: JSS 2

Learning Outcome
By the end of the lesson, you will be able to find unknown angles of a polygon using the sum of its interior angles.

Overview

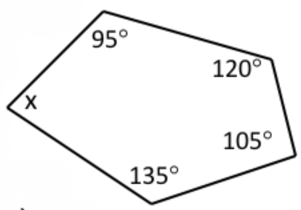
In the previous lesson, you used a formula to find the sum of the interior angles of polygons. In this lesson, you will use the information from the previous lesson to solve for missing angles.

If there is 1 unknown angle in a polygon, subtract the known angles from the sum of the angles for that polygon. See the table you completed in the Overview of the previous lesson for the sums of the angles.

Remember that a regular polygon has all of its angles equal in measure. To find the measures of the angles in a regular polygon, divide the sum of its angles by the number of angles.

Solved Examples

1. Find the measure of angle x:

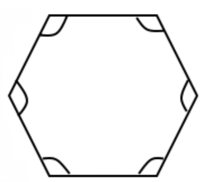


Solution
There are 5 sides and 5 angles in this polygon, which make it a pentagon. The sum of the angles of a pentagon is 540°.

Subtract the known angles from 540°:

$$\begin{aligned} x &= 540° - 95° - 120° - 105° - 135° \\ &= 85° \end{aligned}$$

2. Find the measure of each angle in the regular hexagon:



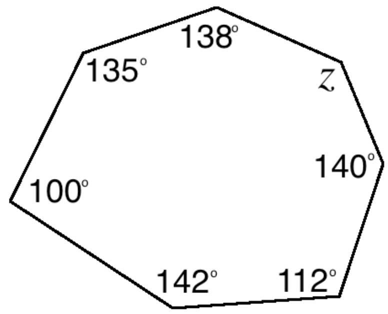
Solution

A regular hexagon has 6 equal angles. Divide the sum of the angles of a hexagon (720°) by 6:

$720° \div 6 = 120°$

All of the angles of the hexagon are $120°$.

3. Find the missing angle in the shape below:



Solution

There are 7 sides and 7 angles in this polygon, which makes it a heptagon. The sum of the angles of a heptagon is 900°.

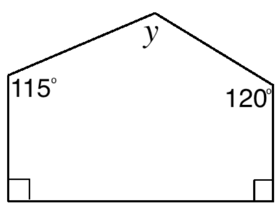
Subtract the known angles from 900°:

$$\begin{aligned} z &= 900° - 140° - 112° - 142° - 100° - 135° - 138° \\ &= 133° \end{aligned}$$

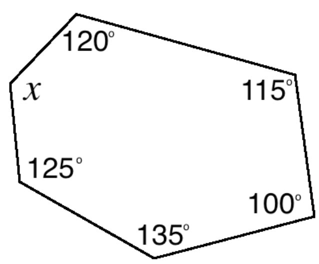
Practice

1. Find the missing angle in each of the shapes below:

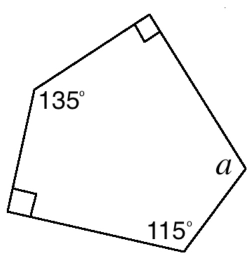
a.



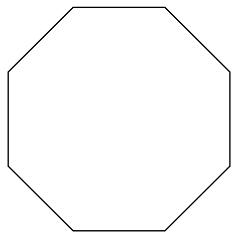
b.



c.



2. Find the measure of each angle in the regular decagon.
3. Find the measure of each angle in a regular heptagon. Give your answer to 1 decimal place.
4. Find the measure of each angle in the regular polygon below:



Lesson Title: Interior Angle Story Problems	**Theme:** Geometry
Practice Activity: PHM-08-090	**Class:** JSS 2

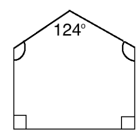
Learning Outcome
By the end of the lesson, you will be able to solve practical problems on interior angles.

Overview

In the previous lesson, we found the missing angles of polygons. In this lesson, you will use that knowledge to solve story problems.

Solved Examples

1. Issa is building a house. He wants to build a strong one, and he knows the two angles between the roof and walls must be equal. Help him by finding the missing angles in the diagram of his house.



Solution
His house is in the shape of a pentagon. Remember that the sum of the angles in a pentagon is 540°. First, subtract the 3 known angles. Because the last 2 angles are equal, divide by 2.

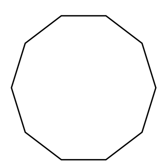
Subtract the known angles: $540° - 124° - 90° - 90° = 236°$

Divide by 2 to find the measure of each angle: $236° \div 2 = 118°$

The measure of each missing angle is $118°$.

2. Michael has 10 people in his family. He wants to build a table in the shape of a regular decagon for his family to sit around.
   a. Draw a picture of the table's top.
   b. What is the sum of the interior angles of the table top?
   c. What will the measure of each angle be?

   Solutions
   a. The table top is a regular decagon:



   b. Use the formula for sum of the angles in a polygon:

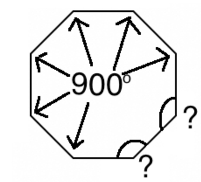
$$\begin{aligned}\text{Sum of angles} &= (n-2) \times 180° \\ &= (10-2) \times 180° \\ &= 8 \times 180° \\ &= 1{,}440°\end{aligned}$$

c. Divide the sum of the angles by 10 to find the measure of each angle:
$1{,}440° \div 10 = 144°$

3. The sum of six of the angles in an octagon is 900°. The other 2 angles are equal to each other. Calculate the sizes of the other 2 angles.

Solution
An octagon has 8 angles. We are given the total measure of 6 of them. If we subtract these 6 angles from the sum of the angles in an octagon, we can divide by 2 to find each of the other equal angles.

You may draw a picture to help imagine the problem:



If you don't have it memorised, calculate the sum of the angles in an octagon:

$$\begin{aligned} \text{Sum of angles} &= (n-2) \times 180° \\ &= (8-2) \times 180° \\ &= 6 \times 180° \\ &= 1{,}080° \end{aligned}$$

Subtract the sum of the 6 known angles: $1{,}080° - 900° = 180°$

Divide $180°$ by 2 to find the measure of each unknown angle: $\frac{180°}{2} = 90°$.

The size of each of the other 2 angles is 90°.

4. The angles of a pentagon are x, $2x$, $2x$, $3x$, and $4x$.
   a. Find the value of x.
   b. Find the measure of the largest angle.

Solutions
x is an unknown value. The smallest angle in the pentagon has a measure of x, and the largest angle has a measure of $4x$.

a. To find the measure of x, add up all 5 angles and set the sum equal to $540°$, the sum of the angles in a pentagon. Then, you will be able to find the measure of x.

$$\begin{aligned} x + 2x + 2x + 3x + 4x &= 540° \\ (1+2+2+3+4)x &= 540° \\ 12x &= 540° \\ x &= \frac{540°}{12} \\ x &= 45° \end{aligned}$$

The value of x is $45°$.

b. The largest angle is $4x$. Multiply the value of x by 4: $4x = 4(45°) = 180°$.

5. Each interior angle of a regular decagon is 144°. Find the sum of the interior angles of the polygon.

   Solution

   You may remember from a previous lesson that the sum of the angles of a decagon is always $1440°$. However, if you see this question on the essay section of the BECE exam you will be expected to show your working.

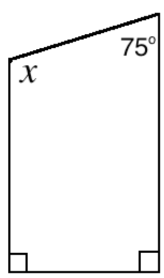
   Identify that there are 10 angles in a decagon. To calculate the sum of the interior angles, multiply 10 by the measure of each angle:

   $10 \times 144° = 1{,}440°$

Practice

1. Martin is a carpenter. He wants to build a new sign for his shop to bring in more customers. He wants to make it in the shape of a regular hexagon.
   a. Draw the shape of his sign.
   b. What is the sum of the interior angles of the sign?
   c. What will the measure of each angle be?

2. Martin wants to build a desk for his children to sit at when they study. The side of the desk is in the shape of a trapezium, as shown. Help Martin find the missing angle.



3. Each interior angle of a regular octagon is 135°. Find the sum of the interior angles of the polygon.

4. The angles of a quadrilateral are x, $2x$, $3x$, and $4x$.
   a. Find the value of x.
   b. Find the measure of the largest angle.

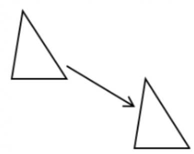
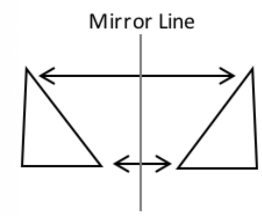
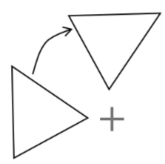
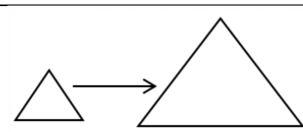
Lesson Title: Introduction to Transformation	**Theme:** Geometry
Practice Activity: PHM-08-091	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify the meaning of the words translate, rotate, reflect and enlarge.
2. Identify four simple transformations: translation, rotation, reflection and enlargement.

Overview

This is the first lesson on transformation. **Transformation** changes the position or size of an object. The objects are usually plane shapes, drawn on a Cartesian plane.

This lesson introduces 4 types of transformation: translation, reflection, rotation and enlargement. You will learn to identify each of these. The definitions and examples are given below.

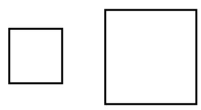
Translation	A shape moves in any direction, but keeps the same shape and size.	
Reflection	A shape reflected across a mirror line (or "line of symmetry"). The distance between the reflected shape and the mirror line is the same as between the original shape and the mirror line. A reflected shape is still the same shape and size, but it faces the opposite direction.	
Rotation	A shape moves or turns around a fixed point. It is still the same shape and size, but faces a different direction.	
Enlargement	A shape becomes larger. It has the same shape but a different size.	

Solved Examples

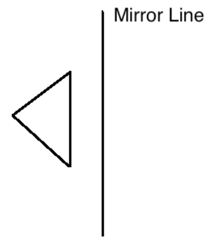
1. Draw a square. Then, draw an enlargement of your square.

   Solution

   Your first square may have any size. Make sure all of its sides are the same length. Your second square should have the same shape but a different size. For example:

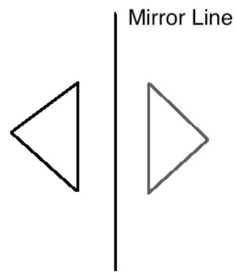


2. Draw the reflection of the triangle on the other side of the mirror line:

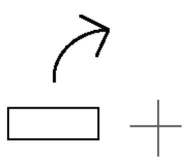


   Solution

   The new triangle is the same distance from the mirror line as the first triangle. It has the same shape and size, but faces the opposite direction. Draw it as best you can:

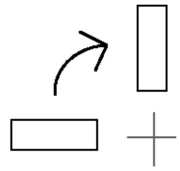


3. Rotate the rectangle in the clockwise direction around the point shown.

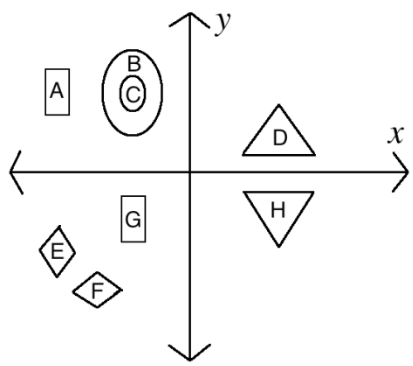


Solution

Draw another rectangle. Your rectangle should show approximately what the given rectangle would look like if it were rotated. The rectangle keeps the same size and shape, but rotates around a point. For example:



4. Use the diagram at right to find 1 example of each of the following. Give your reasons for each:
   a. Two shapes that are translations.
   b. Two shapes that are reflections.
   c. Two shapes that are rotations.
   d. Two shapes that are enlargements.



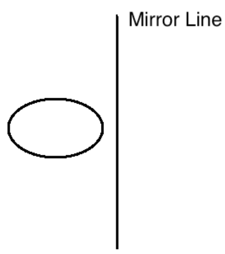
Solutions

a. A and G are translations. They are the same shape and size, but have different locations.
b. D and H are reflections. The x-axis is the mirror line.
c. E and F are rotations. They have the same shape and size, but are turned 90° about a point.
d. B is an enlargement of C. They have the same shape, but a different size.

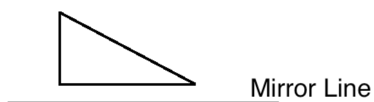
Practice

1. Draw any triangle. Then, draw an enlargement of your triangle.
2. Draw the reflection of each shape below:

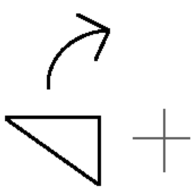
a.



b.



3. Rotate the triangle clockwise around the point shown:



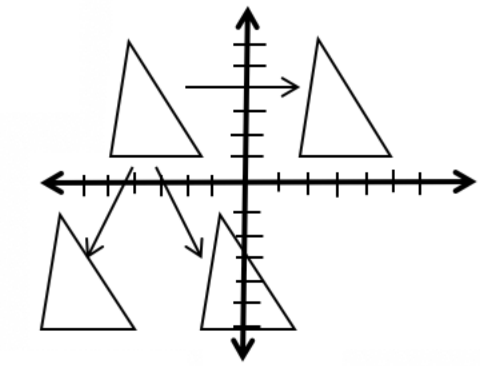
4. Draw a pentagon. Translate your pentagon in any direction.

Lesson Title: Translation	**Theme:** Geometry
Practice Activity: PHM-08-092	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that translation moves an object without changing its size or shape.
2. Recognise and perform a translation.

Overview

Translation moves an object up, down, left or right without changing its size or shape. In this lesson, you will practise translating shapes.



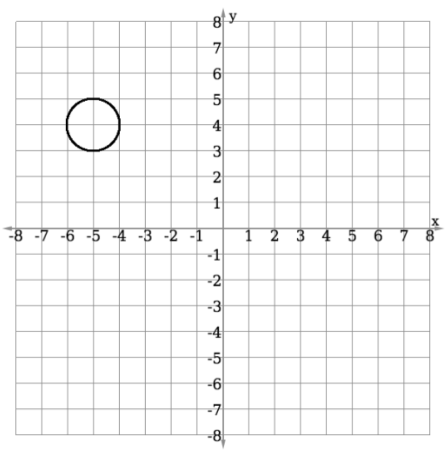
To translate a shape means to move it without changing its size or shape. It means we will have exactly the same triangle, but in a different location. You can translate a shape in any direction. The diagram at right shows 3 different translations of the triangle.

The triangles in the diagram are **congruent**. Shapes are congruent if they change but keep the same size and shape. A translated shape is always congruent to the original shape.

You may translate a shape in a given direction and distance. It is better to do this on a grid. See Solved Example 2.

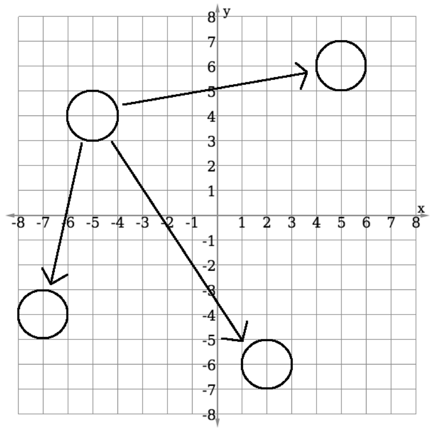
Solved Examples

1. The Cartesian plane below has a circle. Translate the circle to any 3 new locations:

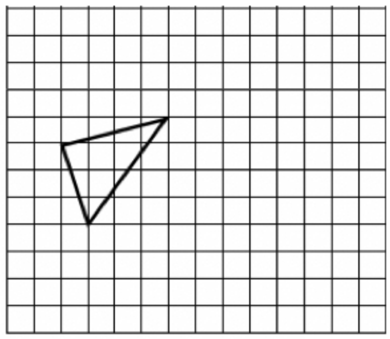


Solution
You may translate the circle to any 3 locations on the Cartesian plane that you choose. It is important that the circle keeps the same shape and size. Make sure your new circles have a diameter of 2 units.

Example answer:



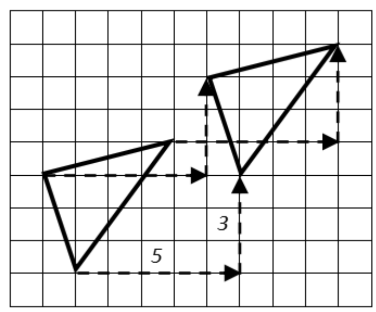
2. Translate the shape 5 units right and 3 units up.



Solution

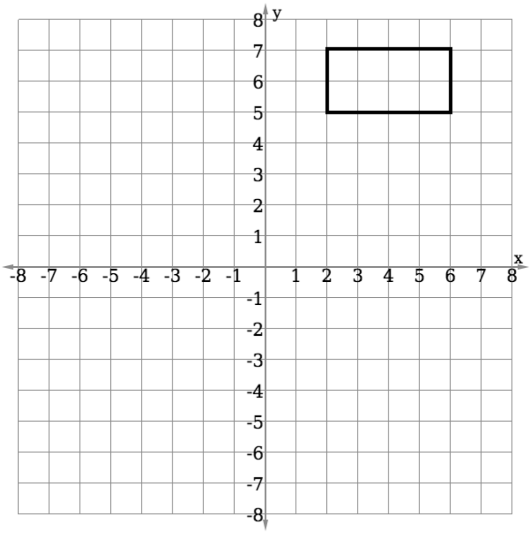
When we translate an object, every part of the object moves the same amount in the same direction.

Select points on the object. For a triangle, select the 3 angles. From each angle, count 5 units right and 3 units up. Mark each point of the new triangle. Join the 3 points to draw a triangle.

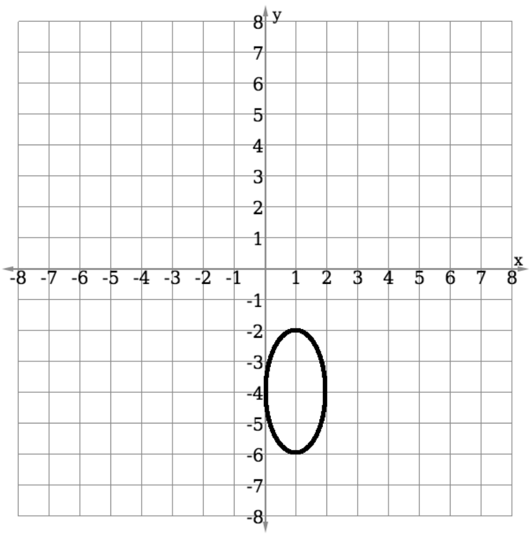


Practice

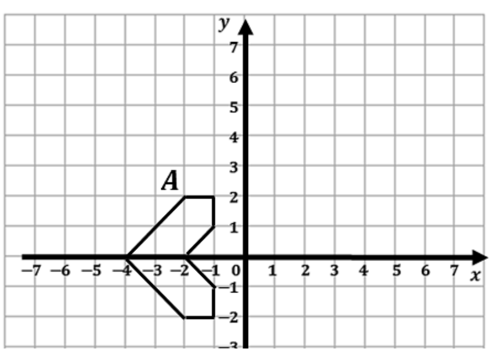
1. Translate the rectangle on the plane below to any 4 new locations:



2. Translate the oval on the Cartesian plane below to any 3 new locations:



3. Translate the shape A 5 units right and 1 unit up:

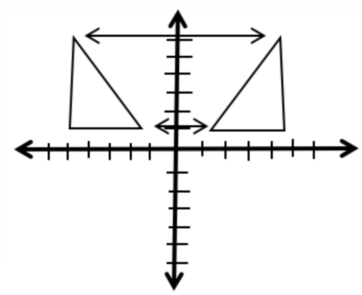


Lesson Title: Reflection	Theme: Geometry
Practice Activity: PHM-08-093	**Class:** JSS 2

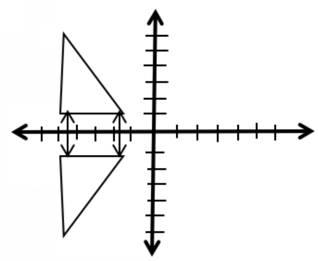
Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that reflection creates an object of the same size and shape, but facing the opposite direction.
2. Recognise and perform a reflection.

Overview

Reflection creates an image of an object of the same size and shape, across a mirror line or line of symmetry. A reflected shape will have the same shape and size as the original, but it will be facing the opposite direction and it will move to the other side of the mirror line.



Any line can be the line of symmetry. In this lesson, you will use the x-axis and y-axis. There are examples on the right. It looks like the triangles are looking in the mirror. There is a mirror image of itself on the other side of the axis.

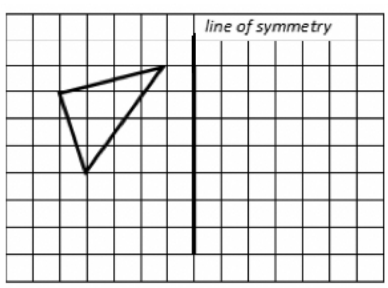


The reflected triangles are **congruent**. Shapes are congruent if they keep the same size and shape. Only the direction the triangles are facing has changed.

It is important that reflected shapes are the same distance from the mirror line.

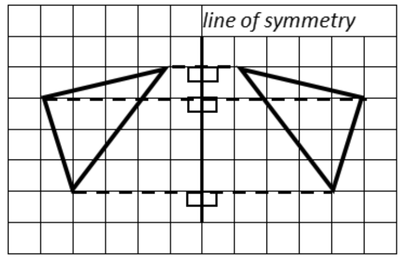
Solved Examples

1. Reflect the shape in the line of symmetry shown:



Solution

Remember that a line of symmetry is a mirror line. The triangle will have the same size, but it will face the opposite direction.

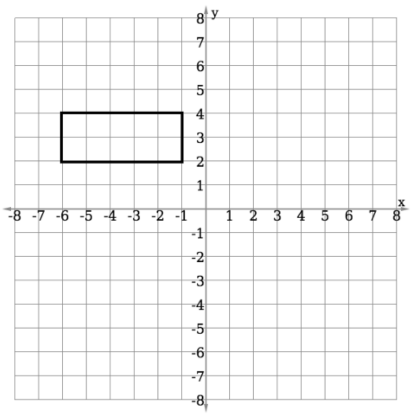


Use the fact that every point in the image is the same distance from the line of symmetry as the original object.

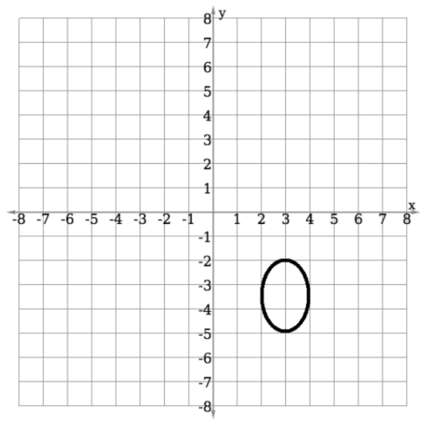
Choose points on the object. In this case, choose the 3 angles of the triangle. Draw a line at 90^o to the line of symmetry from each point on the original object to the other side of the line of symmetry. Mark on the 90^o line the **same distance** from the line of symmetry as the original object. This is the new point of the reflected object. Join the points to draw the shape at its new position after reflection.

2. Reflect each shape about the x-axis and the y-axis.

a.



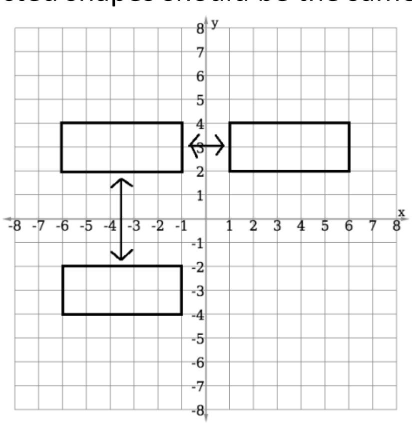
b.



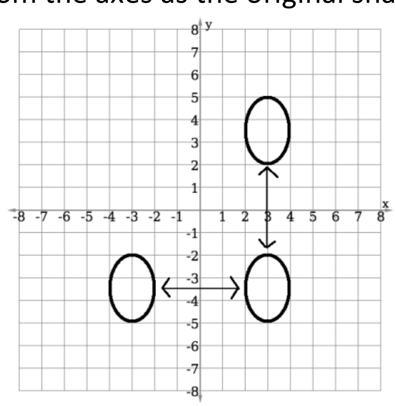
Solutions

The reflected shapes should be the same distance from the axes as the original shape:

a.

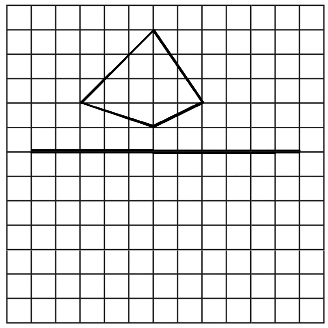


b.



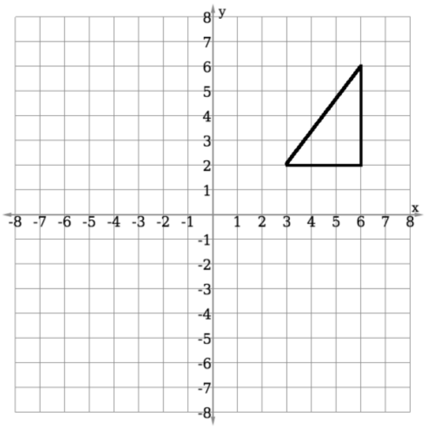
Practice

1. Reflect the shape about the line of symmetry shown:

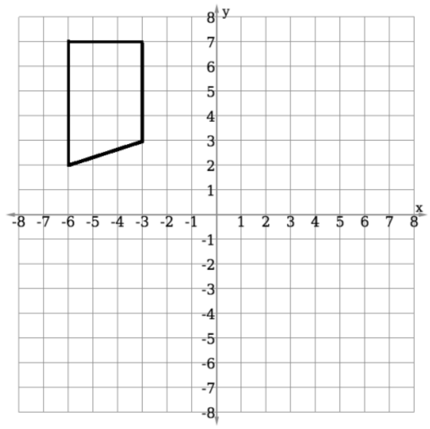


2. Reflect each shape about the x-axis and y-axis:

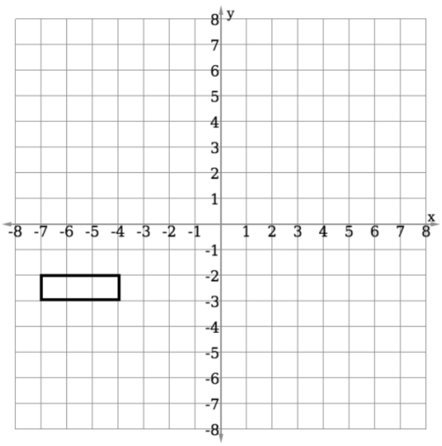
a.



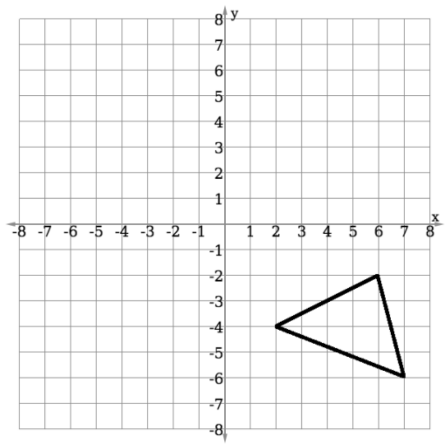
b.



c.



d.



Lesson Title: Line of Symmetry	Theme: Geometry
Practice Activity: PHM-08-094	**Class:** JSS 2

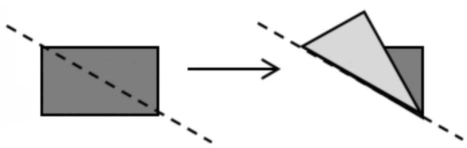
Learning Outcome
By the end of the lesson, you will be able to identify line of symmetry on two-dimensional shapes.

Overview

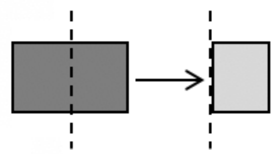
Symmetry is when the two sides of something are exactly the same on either side of a line. We call that line the **line of symmetry**. In this lesson, you will identify and draw lines of symmetry.

When you see a line through a shape, imagine folding the shape in half along the line. If one side fits perfectly over the other when folded, it is a line of symmetry.

The diagrams below show a rectangular piece of paper. The diagonal of the rectangle is **not** a line of symmetry. The vertical line through the middle of the rectangle **is** a line of symmetry:



Not a line of symmetry



Line of symmetry

Note that symmetry lines do not have to be horizontal or vertical.

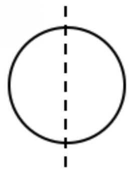
Some polygons are symmetrical. All regular polygons have lines of symmetry. The number of lines of symmetry in a **regular** polygon is the same as the number of sides. For example, an equilateral triangle, a square and a regular pentagon are all regular polygons. A square has 4 sides so it has 4 lines of symmetry.

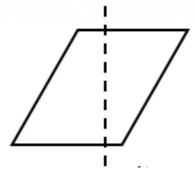
The table below shows the symmetries of some common regular and irregular polygons.

Polygons	Lines of symmetry		Polygons	Lines of symmetry
Square	4		Trapezium	0 / none
Rectangle	2		Equilateral triangle	3
Rhombus	2		Isosceles triangle	1
Parallelogram	0 / none		Scalene triangle	0 / none
Kite	1			

Solved Examples

1. Identify whether the line drawn through each shape is a line of symmetry or not.

a. 

b. 

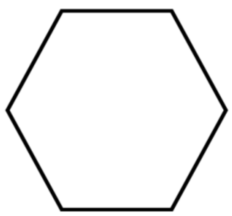
Solutions

Remember that the line is only a line of symmetry if the shape is exactly the same on both sides of the line. If you fold the shape along the line, one side will fit perfectly over the other. The answers are:

a. Yes, it is a line of symmetry.

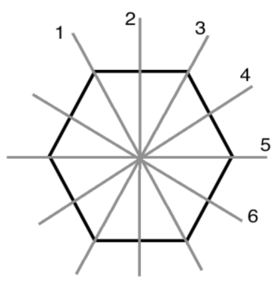
b. No, it is not a line of symmetry.

2. A regular hexagon is shown below. How many lines of symmetry does it have? Draw as many as you can find.



Solution

A regular hexagon has 6 lines of symmetry. They are shown below:



3. Draw as many lines of symmetry as you can on each of the shapes below.

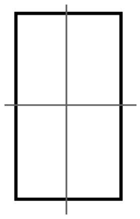
a. Rectangle:

b. Oval:

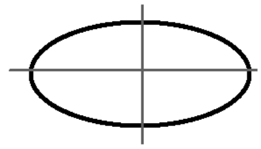
Solutions

Draw the lines of symmetry through each shape. The answers are shown below:

a. Rectangle:



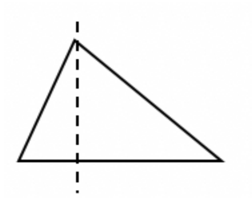
b. Oval:



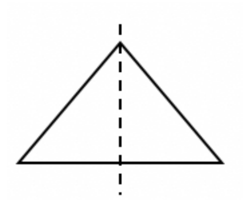
Practice

1. Identify whether the line drawn through each shape is a line of symmetry or not.

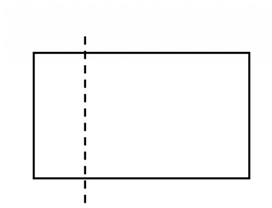
   a.



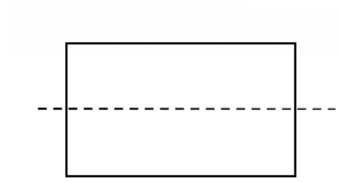
   b.



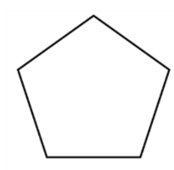
   c.



   d.



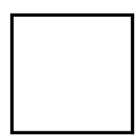
2. A regular pentagon is shown below. How many lines of symmetry does it have? Draw as many as you can find.



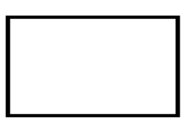
3. Draw as many lines of symmetry as you can on each of the shapes below.

   a. Star:

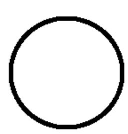
   b. Square:



   c. Rectangle:



   d. Circle:



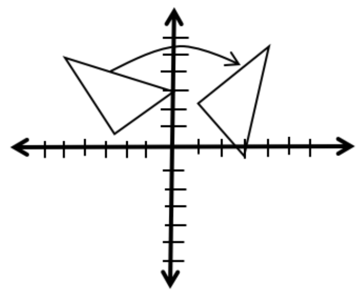
Lesson Title: Rotation	**Theme:** Geometry
Practice Activity: PHM-08-095	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that rotation moves an object circularly around a single point, without changing its size or shape.
2. Recognise and perform a rotation.

Overview

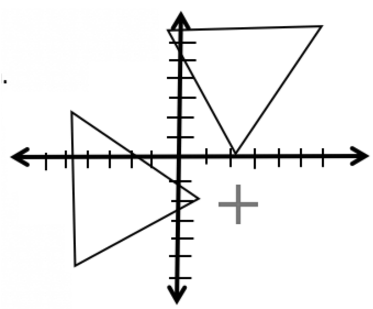
Rotation turns an object around a fixed point, called the centre of rotation, without changing its size or shape. The result will have the same shape, but it will be facing a different direction.

In the diagram at right, the triangle is rotated clockwise about the origin. The origin is the centre of rotation.



The two triangles are congruent. Shapes are congruent if they keep the same size and shape. Notice that only the direction it is facing has changed.

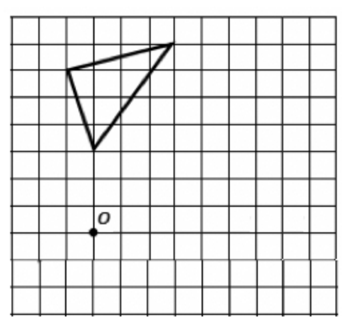
A shape can be rotated about any point on the Cartesian plane. In the second diagram at right, the triangle is rotated about the point marked with +



You may also be asked to rotate a shape a given number of degrees. In this lesson, you will rotate shapes 90°. See Solved Example 1.

Solved Examples

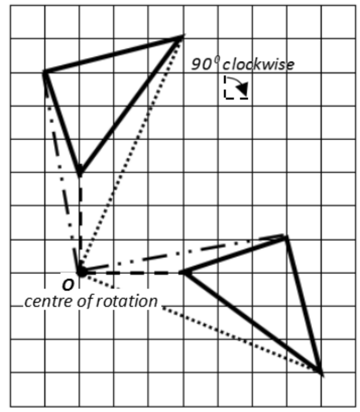
1. Rotate the shape 90° clockwise about the point shown:



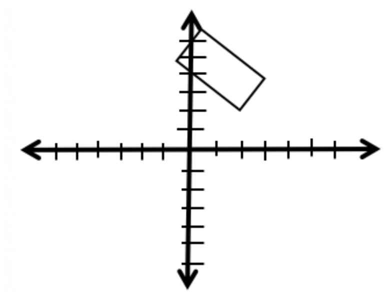
Solution

You are given 3 pieces of information in order to rotate our shape – the centre of rotation (O), the angle of rotation (90°) and the direction of rotation (clockwise).

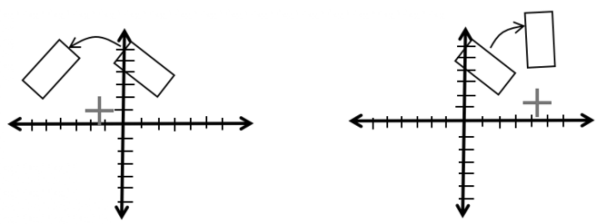
Draw a straight line from one of the angles of the triangle to the centre of rotation. Measure the line. Draw a line at an angle of 90° to the first line. Mark the measurement on the second line. That is the new point of the object after rotation. Follow the same steps for the other two angles of the triangle.



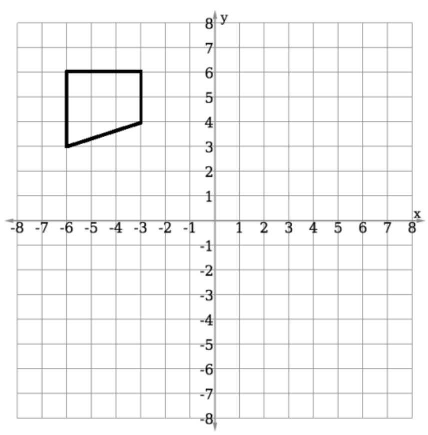
2. The diagram below shows a rectangle on the Cartesian plane. Rotate the rectangle in any direction, using any point as the centre of rotation. Draw a + symbol at your centre of rotation.



Solution
Two examples are shown below. Your own rotation could be different.

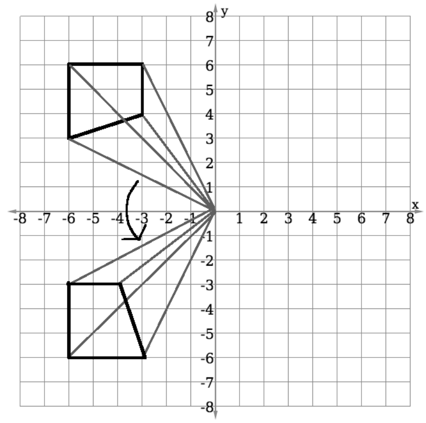


3. Rotate the shape 90° in the counter-clockwise direction about the origin:



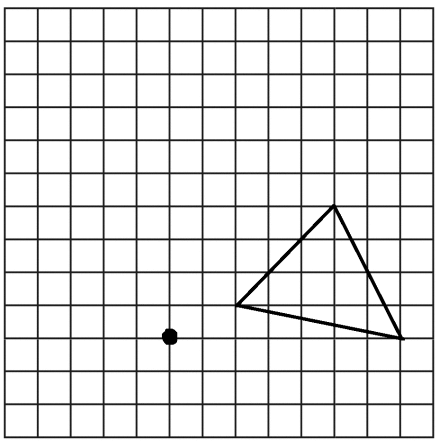
Solution

Draw a straight line from one of the angles of the shape to the origin. Measure the line. Draw a line at an angle of 90° to the first line. Mark the measurement on the second line. This is the new point of the object after rotation. Follow the same steps for the other three angles of the shape.



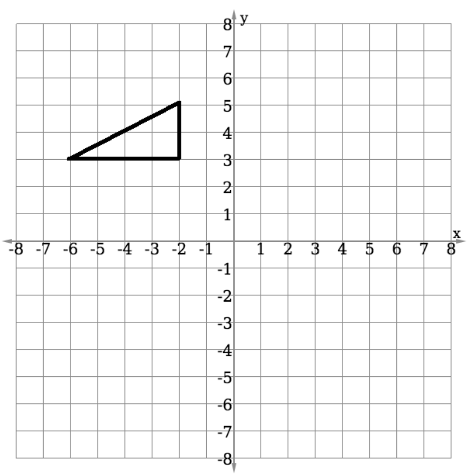
Practice

1. Rotate the triangle 90° counter-clockwise around the point shown:

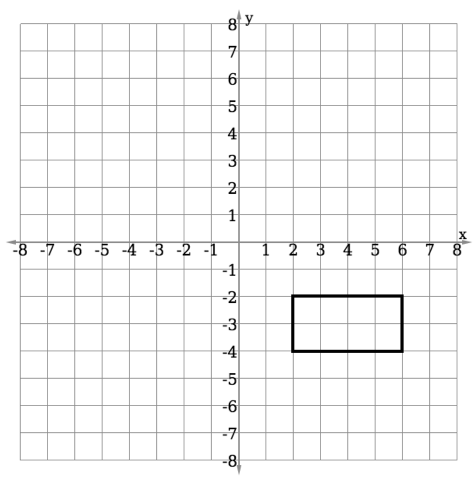


2. Rotate each shape 90° in the clockwise direction around the origin:

a.



b.



Lesson Title: Rotational Symmetry	**Theme:** Geometry
Practice Activity: PHM-08-096	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify rotational symmetry on two-dimensional shapes.

Overview

In this lesson, you will learn about rotational symmetry of shapes. Rotational symmetry is when a shape still looks the same after a rotation.

For example, consider the 'plus' symbol below. If we rotate it 90°, it becomes a plus sign again. It looks the same as it did before being rotated. We can say that the plus sign has rotational symmetry. Rotational symmetry means the shape looks the same again when we rotate it less than 360 degrees. Some shapes only look the same after being rotated the full turn ($360°$). They do not have rotational symmetry. We say they have rotational symmetry of order 1.

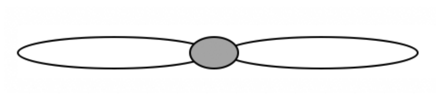
How many times the shape matches itself when we go around is called **order**. The plus symbol has order 4 because if we rotate it all the way around, we will see that we have four plus symbols that all look exactly the same.

Some polygons are symmetrical. All regular polygons have rotational symmetry. The order of rotational symmetry is the same as the number of sides. A square has 4 sides so it has a rotational symmetry of order 4. The table below shows the symmetries of some common regular and irregular polygons:

Polygons	Rotational symmetry
Square	Rotational symmetry of order 4
Rectangle	Rotational symmetry of order 2
Rhombus	Rotational symmetry of order 2
Parallelogram	Rotational symmetry of order 2
Kite	No rotational symmetry (i.e. order 1)
Trapezium	No rotational symmetry (i.e. order 1)
Equilateral triangle	Rotational symmetry of order 3
Isosceles triangle	No rotational symmetry (i.e. order 1)
Scalene triangle	No rotational symmetry (i.e. order 1)

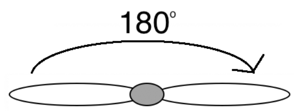
Solved Examples

1. The diagram below at right shows the propeller of an airplane. When an airplane flies, it spins around and around.
    a. Does the propeller have rotational symmetry?
    b. If it does have rotational symmetry, what is its order?



Solutions

a. Yes, the propeller does have rotational symmetry. If we rotate it 180°, it will have the same shape.



b. The order of the propeller is 2, because if we spin it all the way around we see that there are two shapes that look exactly the same.

2. Consider the star shown at right.
    a. Does the star have rotational symmetry?
    b. If it does have rotational symmetry, what is its order?

Solutions

a. Yes, the star does have rotational symmetry. If we rotate it, it will look exactly the same.
b. The star has order 5, because if we spin it all the way around we see that there are five shapes that look exactly the same.

3. Consider the heart shown at right.
    a. Does the heart have rotational symmetry?
    b. What is its order?

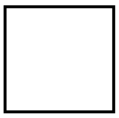
Solutions

a. No, the heart does not have rotational symmetry. If we rotate it, it will not look exactly the same until we reach 360°, a full rotation.
b. It does not have rotation symmetry, so it has an order 1.

Practice

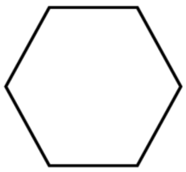
Determine whether each of the shapes below have rotational symmetry. Give the order of each shape.

1. Square:

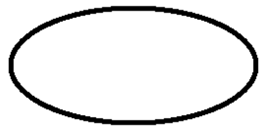


2. Rectangle:

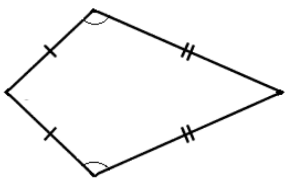
3. Hexagon:



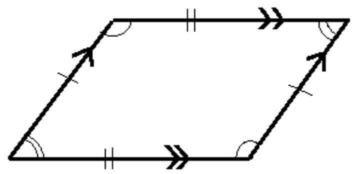
4. Oval:



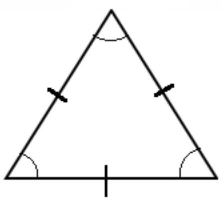
5. Kite:



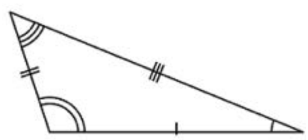
6. Parallelogram:



7. Equilateral triangle:



8. Scalene triangle:



Lesson Title: Enlargement	Theme: Geometry
Practice Activity: PHM-08-097	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that enlargement creates an object of the same shape, but a different size.
2. Recognise and perform enlargement.

Overview

In this lesson, you will perform another type of transformation called **enlargement**. To enlarge a shape means to make it larger. It keeps the same shape, but becomes a different size.

The size of the final object will depend on the **scale factor** that we use. A scale factor tells us how many times larger to make the object. For example, a scale factor of 2 will make the shape twice as large. A scale factor of 3 will make the shape 3 times as large.

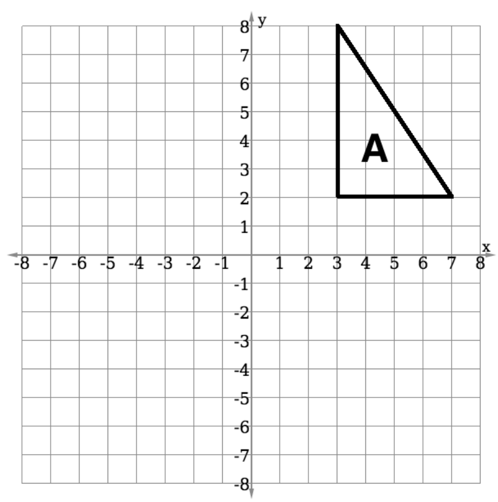
If a scale factor is a fraction, the shape becomes smaller. For example, a scale factor of $\frac{1}{2}$ will make the shape half the size of the original.

Measure the dimensions of the original shape, and multiply them by the scale factor. This will tell you the size of the enlargement. In some cases, you will be given a centre of enlargement. See Solved Example 2 for how to draw an enlargement from a centre of enlargement. If you are not given a centre of enlargement, you may draw your enlarged shape anywhere on the plane.

When you enlarge a shape, the original shape and the enlargement are **similar**. Shapes are similar if they keep the same shape and direction, and only the size changes.

Solved Examples

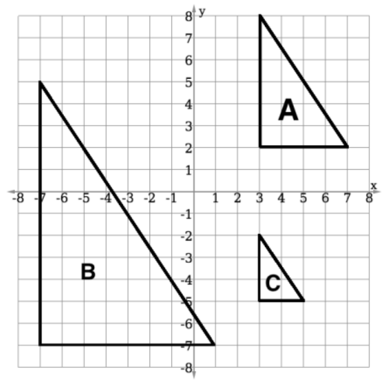
1. Consider triangle A in the diagram at right.
   a. Enlarge triangle A by a factor of 2. Label this triangle B.
   b. Enlarge triangle A by a factor of $\frac{1}{2}$. Label this triangle C.



Solutions

There is no centre of enlargement, so you may draw your enlarged triangles anywhere on the plane. For a triangle, you only need to find the lengths of two sides, the base and height. Note that the dimensions of A are base 4 units, and height 6 units.

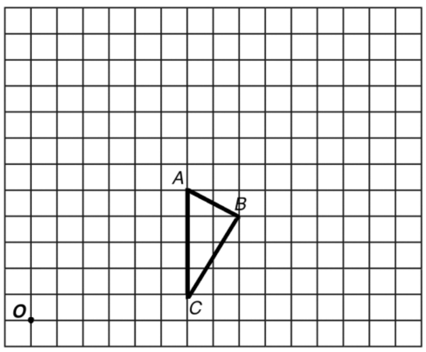
a. Find the dimensions of triangle B by multiplying the dimensions of A by 2:
Base: $2 \times 4 = 8$ units Height: $2 \times 6 = 12$



b. Find the dimensions of triangle C by multiplying the dimensions of A by $\frac{1}{2}$:
Base: $\frac{1}{2} \times 4 = 2$ units Height: $\frac{1}{2} \times 6 = 3$

Draw triangles B and C anywhere on the plane. Examples are shown at right.

2. Draw an enlargement to the triangle ABC at right with a scale factor of 2. Use the point *O* as the centre of enlargement.

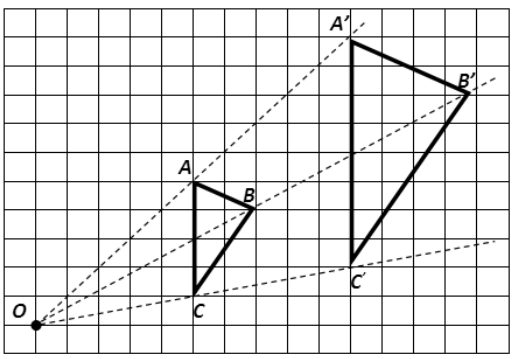


Solution

To enlarge the triangle by a scale factor of 2, we will make it twice as large.

Follow these steps to draw the enlargement:

- Draw a line from O through point A on the triangle. Extend it at least twice as far as the distance from O to A.
- Measure the distance from O to A.
- Measure the same distance on the line past A, and draw a point. This is A’.
- Follow the same process for points B and C to make B’ and C’.

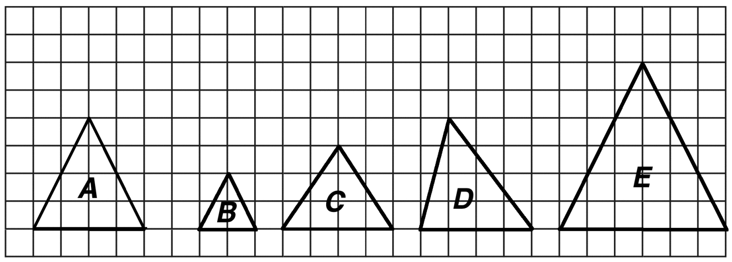


Note that the distance from O to each point on the enlarged triangle is twice as far as the distance from O to any point on the first triangle. For example, $\mathrm{OA}' = 2 \times \mathrm{OA}$.

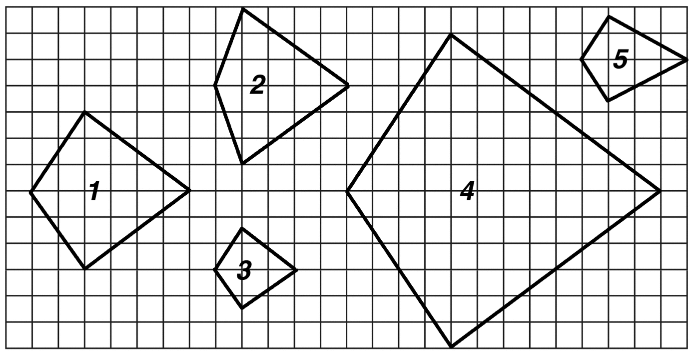
Note that the length of each side of the new triangle is twice as long as each side of the first triangle. For example, $A'B' = 2 \times AB$.

Practice

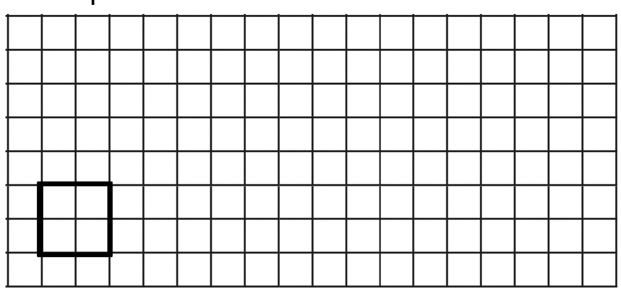
1. Which of the triangles shown below are enlargements of shape A? Give the scale factor for each enlargement.



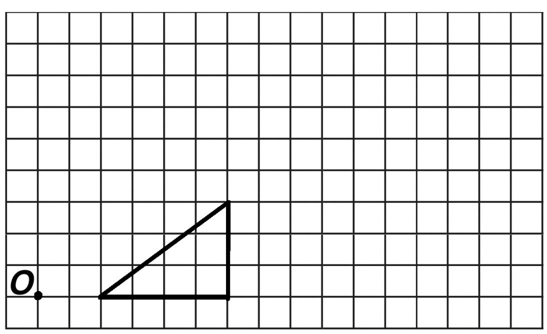
2. Which of the shapes shown below are **not** enlargements of shape 1?



3. Draw an enlargement of the square with scale factor 3.



4. Draw an enlargement of the triangle with scale factor $1\frac{1}{2}$. O is the centre of enlargement.



Lesson Title: Combining Transformations	**Theme:** Geometry
Practice Activity: PHM-08-098	**Class:** JSS 2

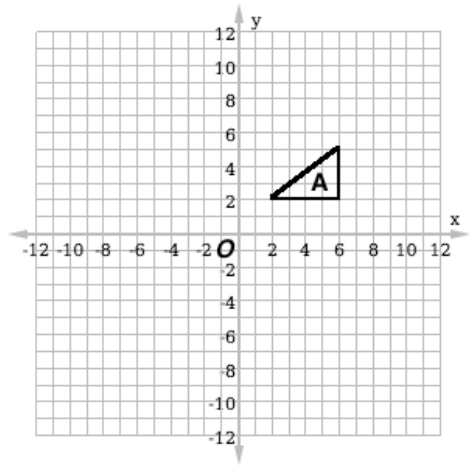
Learning Outcomes
By the end of the lesson, you will be able to:
1. Carry out combinations of all four common transformations.
2. Describe and compare the four transformations.

Overview

The 4 transformations we looked at in the previous lessons can be combined by doing one transformation and then another. For instance, we can translate an object, then reflect the translated object. Or we can rotate it, then translate the result.

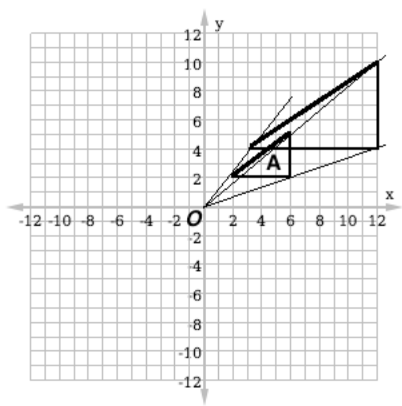
Solved Examples

1. Use the triangle in the diagram at right to complete the following:
    a. Enlarge the triangle with a scale factor of 2 using *O* as the centre of enlargement.
    b. Translate the triangle 15 units down and 10 units to the left.
    c. Is the final shape congruent or similar to triangle A?

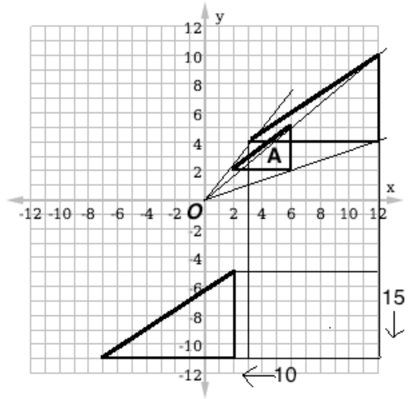


Solutions

a. Use O as the centre of enlargement and draw a triangle 2 times as large as A:

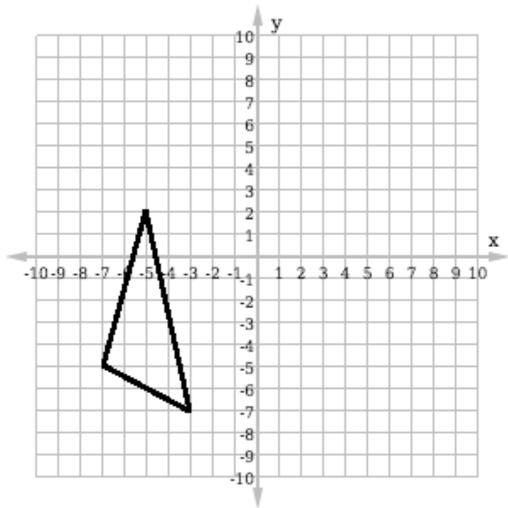


b. Translate each angle of the triangle 15 units down and 10 units to the left:



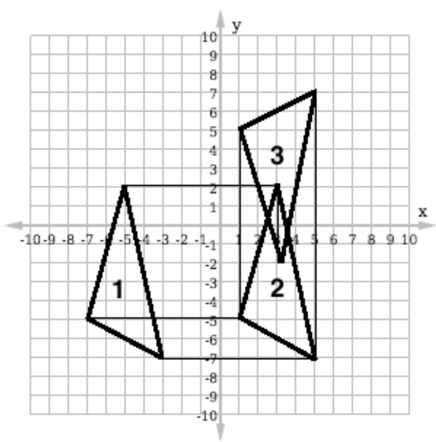
c. The final shape is **similar** to triangle A. It has the same shape but a different size.

2. Transform the shape 8 units to the right. Then reflect it in the x-axis. Is the final shape congruent or similar to the original shape?



Solution

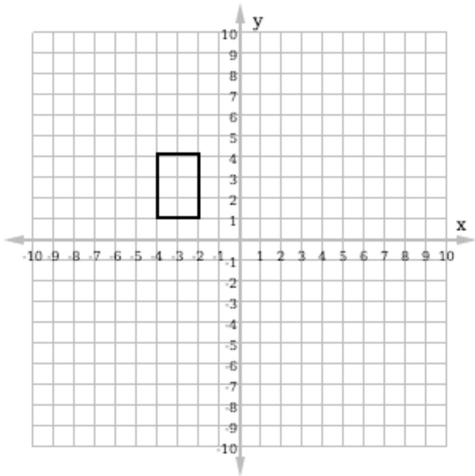
The final shape is congruent to the original shape, because it has the same shape and size. It has moved and is facing a different direction.



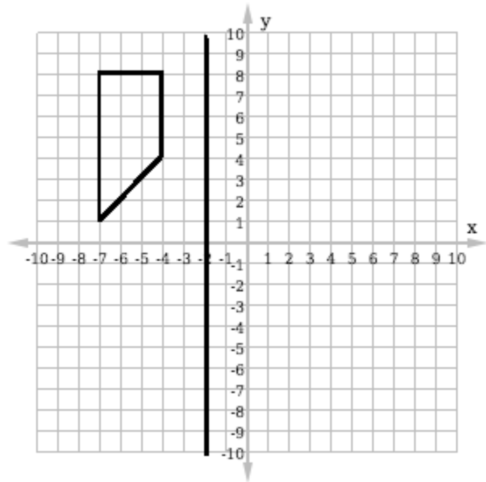
Practice

Perform the translation described in each problem. Then, determine if the final shape is congruent or similar to the original shape.

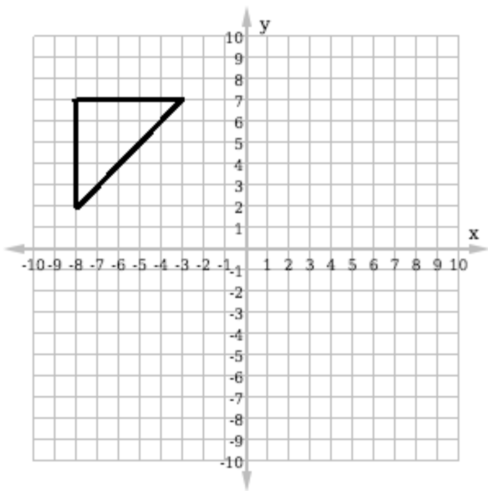
1. Rotate the shape 90° in the clockwise direction about the origin. Then enlarge it by a scale factor of 2, with the origin as the centre of enlargement.



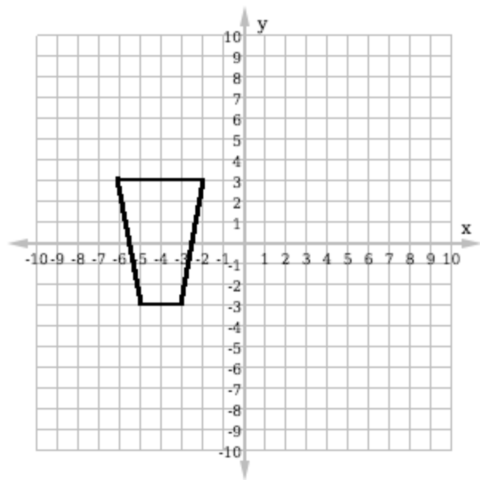
2. Reflect the shape in the line of symmetry shown. Then rotate it 90° in the clockwise direction about the origin.



3. Reflect the shape shown on the y-axis. Then transform it down 5 units and to the left 4 units.



4. Transform the shape 5 units to the right and 2 units down. Then reflect it on the x-axis.



Lesson Title: Applying Scale Factor to Drawing	Theme: Geometry
Practice Activity: PHM-08-099	**Class:** JSS 2

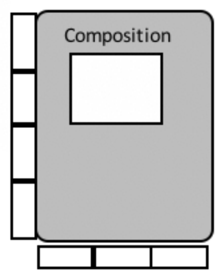
Learning Outcome
By the end of the lesson, you will be able to use a scale factor to draw an object with accurate proportions.

Overview

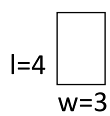
Scale factor is very useful in everyday life. For example, a scale factor is used to draw an accurate map of Sierra Leone. One centimetre might be used to represent 50 kilometres. Scale factor is useful when we want to draw pictures accurately. In this lesson, you will use scale factor to draw objects with accurate proportions.

You can create a scale factor with a piece of paper or any object. Cut or tear a piece of paper in the shape of a small rectangle, like this: 

Choose an item that you want to draw to scale. For example, an exercise book. You can use your scale factor paper to draw an exercise book with the same proportions, but a different size.



First, measure the exercise book using your scale factor. Find how many pieces of paper the length and width of the exercise book are. In the example shown at right, the exercise book is 3 units wide and 4 units tall. You can use 1 piece of paper, and hold it next to the exercise book multiple times to find how long and wide it is.

Make a sketch and record the size of your exercise book: 

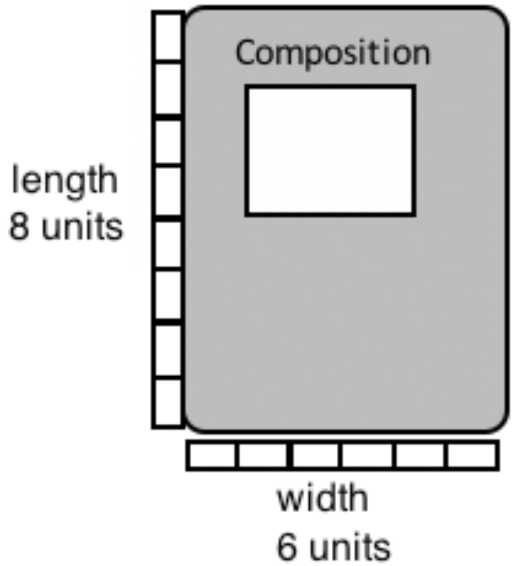
See Solved Examples 1 and 2 for how to draw the exercise book accurately with a scale factor.

Solved Examples

1. Draw the exercise book in the Overview with a scale factor of 2.

   Solution
   Step 1. Calculate the length and width of the new exercise book. Multiply the original length and width by 2: $l = 2 \times 4 = 8$ $w = 2 \times 3 = 6$

   Step 2. Draw the enlarged exercise book. The height should be 8 units, and the width should be 6 units.

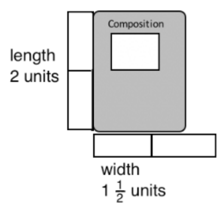


2. Draw the exercise book in the Overview with a scale factor of $\frac{1}{2}$.

Solution

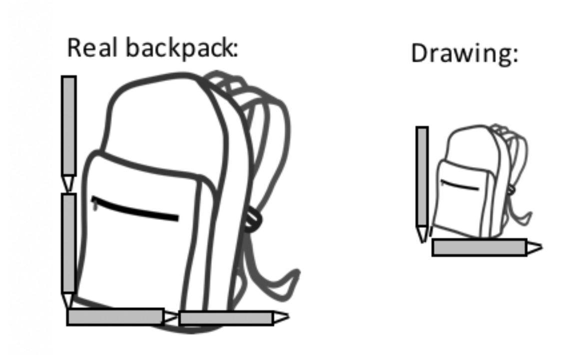
Step 1. Calculate the length and width of the new exercise book. Multiply the original length and width by $\frac{1}{2}$: $l = \frac{1}{2} \times 4 = 2$ $w = \frac{1}{2} \times 3 = 1\frac{1}{2}$

Step 2. Draw the enlarged exercise book. The height should be 2 units, and the width should be $1\frac{1}{2}$ units.



3. Choose an object in your classroom or home. Use a fraction as a scale factor to draw a smaller version of the object. You may use any object as your scale factor.

   Solution
   This is an example solution. You may use a pencil as your scale factor. The bag in this example is 2 pencils tall and 2 pencils wide. If you draw it with a scale factor of $\frac{1}{2}$, it will be 1 pencil tall and 1 pencil wide.



Practice

1. Create a scale factor using a piece of paper. Measure an object, and draw a picture of it. Make it bigger, with a scale factor greater than 1.
2. Using the same scale factor, draw an accurate picture of another object. Use a scale factor less than 1 to make your drawing smaller than the original object.

Lesson Title: Practical Applications of Scale	**Theme:** Geometry
Practice Activity: PHM-08-100	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to use a scale factor to draw an accurate map.

Overview

In the previous lesson, you used scale factor to draw objects from everyday life. In this lesson, you will use scale to draw an accurate map.

Scale helps us compare distances. For example, if a map is accurate, we can compare the distance between Bo and Freetown with the distance between Kenema and Bo. Scale also helps us with planning. We can draw an accurate map of a house before we start to build. We can draw a map of our community to show resources. This can help us to decide where to build a new school or clinic.

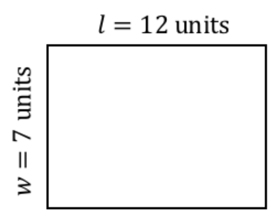
Solved Examples

1. Draw an accurate map of one room in your house. Show the walls, the door, and any windows or large objects on your map.

   Solution
   First, find an object to use for measurement. This should be large enough to easily measure the length of the room. You may use a stick, a shoe, or any other object. Find a much smaller object to do your drawing. For example, a piece of paper or pencil rubber.

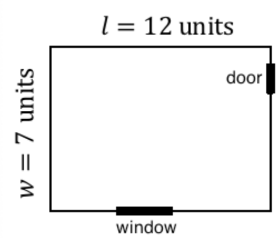
   Step 1. Measure and draw the walls.

   - Use the large item to measure the length and width of your room. For example, the room might be 12 sticks long and 7 sticks wide.
   - Use the small item to draw the walls on your paper. For example, draw the room 12 pencil rubbers long and 7 pencil rubbers wide.



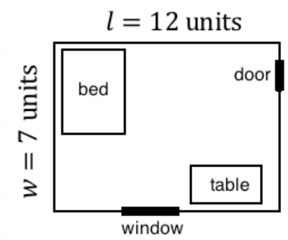
   Step 2. Measure and draw the windows and doors.

   - You only need to show the width of the windows and doors on your map. Find how many units wide the windows and doors in your room are.
   - Use the small item to draw the windows and doors on your map.



Step 3. Measure and draw any large objects in the room.

- In this example, there is a table and a bed in the room. Use the large item to measure how many units long and wide your objects are.
- Use the small item to draw the objects to scale on your map.

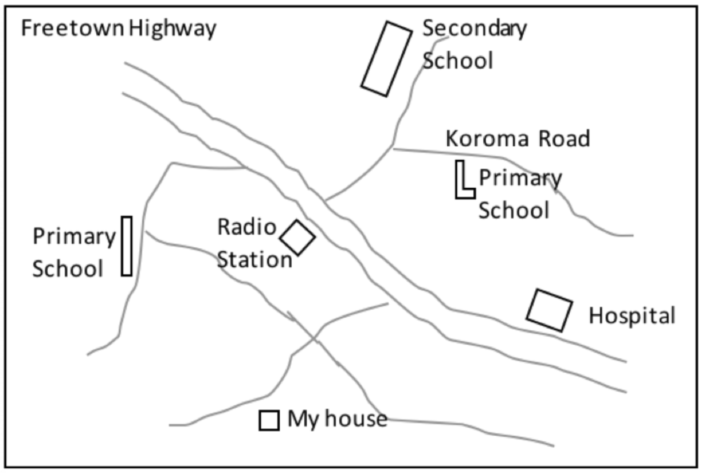


2. Draw a map of the roads and paths in your community. Write the names of the roads and places you know.

Solution

The community is too large to easily use a scale factor. It would take a long time to walk around the community to measure distances. In your drawing, you will estimate the distances between places, the lengths of roads, and the sizes of buildings.

An example community is shown below:



Practice

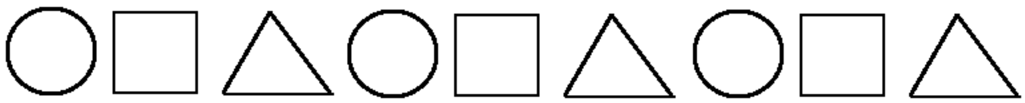
1. Draw a map of your entire house. You may use a large item to measure the length and width of the outside walls, and the length and width of each room in your house.
2. Draw a map of your yard. Include any important objects, such as a pump, tree, garden, or neighbour's house.

Lesson Title: Arithmetic Patterns	**Theme:** Algebra
Practice Activity: PHM-08-101	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify and describe arithmetic patterns.
2. Find missing terms of an arithmetic pattern.

Overview

In a pattern, objects are arranged according to rules. For example, this is a pattern involving shapes:



We also have patterns of numbers. In this lesson, you will learn about a special type of number pattern, **arithmetic patterns**. Arithmetic patterns are lists of numbers that show a pattern in which the difference between two consecutive terms is constant. Remember that the difference between two numbers is what you get when you subtract the two numbers. Two consecutive terms are two numbers next to each other in the pattern. Arithmetic patterns are often called **arithmetic sequences**.

To determine if a list of numbers is an arithmetic pattern, you must find the difference between each of the consecutive numbers. If the difference is the same, it is an arithmetic pattern.

Consider the number pattern: 5, 7, 9, 11, 13, …

Subtract the consecutive terms:

$7 - 5 = 2$ $\qquad$ $9 - 7 = 2$ $\qquad$ $11 - 9 = 2$ $\qquad$ $13 - 11 = 2$

The difference between consecutive terms is always 2. This means that the list of numbers is an arithmetic pattern. The **common difference** is 2. The common difference is a difference that is the same between each term and the next term.

If an arithmetic pattern has decreasing numbers, then the common difference is a negative number. For example, consider 12, 10, 8, 6, …

Subtract the consecutive terms. You will find that the common difference is –2:

$10 - 12 = -2$ $\qquad$ $8 - 10 = -2$ $\qquad$ $6 - 8 = -2$

Solved Examples

1. Identify whether each of the following lists of numbers is an arithmetic pattern. If it is an arithmetic pattern, give the common difference:
   a. 1, 3, 5, 7, …
   b. 1, 2, 4, 8, 16, …
   c. 4, 6, 8, 10, …
   d. 50, 40, 30, 20, 10, …

   Solutions

   a. Subtract the consecutive numbers to find the differences:
   $3 - 1 = 2$ $5 - 3 = 2$ $7 - 5 = 2$
   The difference is always 2. This is an arithmetic pattern, with a common difference of 2.

   b. Subtract the consecutive numbers to find the differences:
   $2 - 1 = 1$ $4 - 2 = 2$ $8 - 4 = 4$ …
   There is not a common difference. This is **not** an arithmetic pattern.

   c. Subtract the consecutive numbers to find the differences:
   $6 - 4 = 2$ $8 - 6 = 2$ $10 - 8 = 2$
   The difference is always 2. This is an arithmetic pattern, with a common difference of 2.

   d. Subtract the consecutive numbers to find the differences:
   $40 - 50 = -10$ $30 - 40 = -10$ $20 - 30 = -10$
   The difference is always –10. This is an arithmetic pattern with a common difference of –10. Note that the numbers decrease because the common difference is negative.

2. Find the common difference and write the missing numbers in each pattern:
   a. 5, 10, 15, ___, _____, _____, 35, 40

   b. 2, 4, 6, _____, 10, _____, ______, 16

   c. 9, 12, _____, 18, 21, 24, ______

   Solutions

   a. Subtract consecutive numbers to find the common difference:
   $10 - 5 = 5$ $15 - 10 = 5$
   The common difference is 5. Add 5 to find each missing number:
   5, 10, 15, <u>20</u>, <u>25</u>, <u>30</u>, 35, 40

b. Subtract consecutive numbers to find the common difference:
   4 – 2 = 2 6 – 4 = 2
   The common difference is 2. Add 2 to find each missing number:
   2, 4, 6, <u>8</u>, 10, <u>12</u>, <u>14</u>, 16

c. Subtract consecutive numbers to find the common difference:
   12 – 9 = 3 21 – 18 = 3
   The common difference is 3. Add 3 to find each missing number:
   9, 12, <u>15</u>, 18, 21, 24, <u>27</u>

Practice

1. Identify whether each of the following lists of numbers is an arithmetic pattern. If it is an arithmetic pattern, give the common difference:

   a. 8, 16, 24, 32, …
   b. 1, 4, 7, 10, 13, …
   c. 3, 6, 12, 24, 48, …
   d. 4, 6, 8, 10, …
   e. 3, 5, 8, 12, 17, …
   f. 11, 7, 3, -1, …
   g. 4, 1, -2, -5, …

2. Find the common difference and write the missing numbers in each pattern:

   a. 6, 12, _____, 24, 30, _____, ______, 48

   b. 3, 6, 9, _____, _____, ______

   c. 35, 40, ______, 50, ______, ______, 65

   d. 7, 12, 17, ______, ______, 32

   e. 15, 11, ______, 3, -1, ______

Lesson Title: Creating Arithmetic Patterns	Theme: Algebra
Practice Activity: PHM-08-102	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to create arithmetic patterns by using a rule to find the next terms.

Overview

In this lesson, you will create your own arithmetic pattern. To create an arithmetic pattern, all you need to do is decide what the first number will be and what the difference between two numbers will be.

Write 3 periods after a list of numbers to show that it continues. This is called ellipses.

Solved Examples

1. Write an arithmetic pattern starting with 10, with a common difference of 2. Write the first 5 terms of the pattern.

   Solution
   The first term of the pattern is 10. Find the next 4 terms using addition:

   - 10 + 2 = 12
   - 12 + 2 = 14
   - 14 + 2 = 16
   - 16 + 2 = 18

   List the first 5 terms of the pattern: 10, 12, 14, 16, 18

2. Write an arithmetic pattern starting with 0, with a common difference of 4. Write the first 6 terms of the pattern.

   Solution
   It is not necessary to write out the addition each time you write an arithmetic sequence. You can do the addition in your head and write out the list of numbers.

   Start with 0 and add 4 to get each next term: 0, 4, 8, 12, 16, 20

3. Write an arithmetic pattern that starts at 10 and has a common difference of -3.

Solution

The common difference is 3. Remember that adding a negative number is the same as subtraction, so we will subtract 3 to get the next term in the sequence.

Start with 10 and subtract 3 to get each next term: 10, 7, 4, 1, -2, -5, …

The problem does not tell us how many terms to give. In this case, write several terms and use the ellipses (…) to show that the list continues.

Practice

1. Write an arithmetic pattern starting with 3, with a common difference of 5. Write the first 7 terms of the pattern.
2. Write an arithmetic pattern starting with 10, with a common difference of 6. Write the first 5 terms of the pattern.
3. Write an arithmetic pattern that starts at 3 and has a common difference of 3.
4. Write an arithmetic pattern that starts at 5 and has a common difference of 4.
5. Write an arithmetic pattern that starts at 7 and has a common difference of -2. Write the first five terms of the pattern.

Lesson Title: Introduction to Geometric Patterns	Theme: Algebra
Practice Activity: PHM-08-103	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify and describe geometric patterns.

Overview

In this lesson, you will learn about another type of number pattern called a **geometric pattern**. A geometric pattern is a list of numbers where the ratio between two consecutive terms is constant. The ratio between two numbers is found by dividing the two numbers. Like arithmetic patterns, geometric patterns are often called **geometric sequences**.

For example, consider the pattern: $2, 4, 8, 16, 32, \ldots$

We can take the ratio of the consecutive numbers to determine the pattern. Find the ratio of each of the consecutive terms: $\frac{4}{2} = 2$ $\quad \frac{8}{4} = 2$ $\quad \frac{16}{8} = 2$ $\quad \frac{32}{16} = 2$

The ratio between each of the numbers is 2. Two is called the **common ratio** of the geometric pattern.

We can also multiply by two to get the next term in the pattern. Start from the first term and use multiplication to find the next terms: $2 \times 2 = 4$ $\quad 4 \times 2 = 8$ $\quad 8 \times 2 = 16$

When you see that numbers in a geometric pattern change between negative and positive, it means that the common ratio is a negative number. For example: $-4, 4, -4, 4, \ldots$

Remember the rules for multiplying negative numbers:

- Negative × Negative = Positive
- Negative × Positive = Negative
- Positive × Negative = Negative

See Solved Example 2 for examples of patterns with negative ratios.

Solved Examples

1. Identify whether each of the following lists of numbers is a geometric pattern. If it is a geometric pattern, give the common ratio:
   a. 1, 2, 4, 8, 16, …
   b. 3, 6, 9, 12, 15, …
   c. 1, 2, 6, 24, 96, …
   d. 1, 4, 16, 64, 256, …
   e. 10, 20, 40, 80, …

   Solutions

   a. Divide the consecutive numbers to find the ratios:
   $\frac{2}{1} = 2$ $\frac{4}{2} = 2$ $\frac{8}{4} = 2$ $\frac{16}{8} = 2$
   The ratio is always 2. This is a geometric pattern with a common ratio of 2.

   b. Divide the consecutive numbers to find the ratios:
   $\frac{6}{3} = 2$ $\frac{9}{6} = \frac{3}{2}$ $\frac{12}{9} = \frac{4}{3}$ …
   There is not a common ratio. This is not a geometric pattern.

   c. Divide the consecutive numbers to find the ratios:
   $\frac{2}{1} = 2$ $\frac{6}{2} = 3$ $\frac{24}{6} = 4$ …
   There is not a common ratio. This is not a geometric pattern.

   d. Divide the consecutive numbers to find the ratios:
   $\frac{4}{1} = 4$ $\frac{16}{4} = 4$ $\frac{64}{16} = 4$ $\frac{256}{64} = 4$
   The ratio is always 4. This is a geometric pattern with a common ratio of 4.

   e. Divide the consecutive numbers to find the ratios:
   $\frac{20}{10} = 2$ $\frac{40}{20} = 2$ $\frac{80}{40} = 2$
   The ratio is always 2. This is a geometric pattern with a common ratio of 2.

2. Identify whether each of the following lists of numbers is a geometric pattern. If it is a geometric pattern, give the common ratio:
   a. $4, -4, 4, -4, 4, -4, \ldots.$
   b. $2, -4, 8, -16, 32, \ldots$
   c. $-1, 2, -6, 24, \ldots$

Solutions

a. Divide the consecutive numbers to find the ratios:

$\frac{4}{-4} = -1$ $\frac{-4}{4} = -1$ $\frac{4}{-4} = -1$ …

The ratio is always -1. This is a geometric pattern with a common ratio of -1.

b. Divide the consecutive numbers to find the ratios:

$\frac{-4}{2} = -2$ $\frac{8}{-4} = -2$ $\frac{-16}{8} = -2$ $\frac{32}{-16} = -2$

The ratio is always -2. This is a geometric pattern with a common ratio of -2.

c. Divide the consecutive numbers to find the ratios:

$\frac{2}{-1} = -2$ $\frac{-6}{2} = -3$ $\frac{24}{-6} = -4$ …

There is not a common ratio. This is not a geometric pattern.

Practice

Identify whether each of the following lists of numbers is a geometric pattern. If it is a geometric pattern, give the common ratio:

1. 3, 6, 12, 24, …
2. 10, 20, 30, 40, …
3. $-3, 3, -3, 3, -3, \ldots$
4. 2, 4, 8, 16, 32, …
5. 1, 10, 100, 1000, …
6. 10, 30, 50, 70, …
7. $-2, 6, -18, 54, \ldots$
8. 1, 3, 12, 60, 360, …
9. $-1, -3, -5, -7, \ldots$
10. 2, 8, 32, 128, …

Lesson Title: Terms of Geometric Patterns	**Theme:** Algebra
Practice Activity: PHM-08-104	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find missing terms of geometric patterns.

Overview

In this lesson, you will learn how to find the missing terms in geometric patterns.

Remember that in a geometric pattern, the ratio between two numbers is found by dividing two consecutive numbers. The ratio is the same for all consecutive numbers, and is called a common ratio. To find the next number in a geometric pattern, multiply by the common ratio.

If you are asked to find a missing number in a geometric pattern, look at the number before the blank. Multiply it by the common ratio.

Solved Examples

1. Find the common ratio and write the missing numbers in each pattern:

   a. 5, 10, 20, _____, 80, …

   b. 2, 6, 18, ______, 162, …

   c. 3, 9, ______, _______, _______, …

   Solutions

   a. Divide consecutive numbers to find the common ratio:
   $\frac{10}{5} = 2 \qquad \frac{20}{10} = 2$ …
   The common ratio is 2. Multiply 20 by 2 to find the next number in the pattern:
   $20 \times 2 = 40$
   You can check your answer by multiplying it by the common ratio. If it gives the next number in the pattern (80), your answer is correct: $40 \times 2 = 80$.

   The answer is 5, 10, 20, <u>40</u>, 80

   b. Divide consecutive numbers to find the common ratio:
   $\frac{6}{2} = 3 \qquad \frac{18}{6} = 3$ …
   The common ratio is 3. Multiply 18 by 3 to find the next number in the pattern:
   $18 \times 3 = 54$

   The answer is 2, 6, 18, <u>54</u>, 162

c. Divide consecutive numbers to find the common ratio:

$\frac{9}{3} = 3$

The common ratio is 3. Multiply by 3 to find the next numbers in the pattern:

$9 \times 3 = 27 \qquad 27 \times 3 = 81 \qquad 81 \times 3 = 243$

The answer is 3, 9, <u>27</u>, <u>81</u>, <u>243</u>

2. Find the common ratio and the missing numbers in the geometric patterns:

   a. $2, -2, 2, -2,$ ______, $-2,$ ______, ...

   b. $1, -2, 4,$ ______, $16,$ ______, ...

   c. $3, -6, 12, -24,$ ______, $-96,$ ...

   d. $-2, -4, -8,$ ____, ______, ...

Solutions

Remember that when the consecutive terms in a geometric sequence change from positive to negative numbers, the ratio must be a negative number.

a. Divide consecutive numbers to find the common ratio:

$\frac{-2}{2} = -1 \qquad \frac{2}{-2} = -1 \qquad \ldots$

The common ratio is -1. Multiply -2 by -1 to find the next number in the pattern:

$-2 \times -1 = 2$.

The answer is $2, -2, 2, -2, \underline{2}, -2, \underline{2}, \ldots$

b. Divide consecutive numbers to find the common ratio:

$\frac{-2}{1} = -2 \qquad \frac{4}{-2} = -2 \qquad \ldots$

The common ratio is -2. Multiply by -2 to find the next number in the pattern:

$4 \times -2 = -8$ and $16 \times -2 = -32$

The answer is $1, -2, 4, \underline{-8}, 16, \underline{-32}, \ldots$

c. Divide consecutive numbers to find the common ratio:

$\frac{-6}{3} = -2 \qquad \frac{12}{-6} = -2 \qquad \ldots$

The common ratio is -2. Multiply by -2 to find the next number in the pattern:

$-24 \times -2 = 48$

The answer is $3, -6, 12, -24, \underline{48}, -96, \ldots$

d. Divide consecutive numbers to find the common ratio:
$\frac{-4}{-2} = 2$ $\frac{-8}{-4} = 2 \ldots$
The common ratio is 2. Multiply by 2 to find the next number in the pattern:
$-8 \times 2 = -16$ and $-16 \times 2 = -32$

The answer is $-2, -4, -8, \underline{-16}, \underline{-32}, \ldots$

Practice

Find the common ratio and write the missing numbers in each pattern:

1. 2, 4, _____, ______, 32, …
2. 10, 30, ______, 270, _____, …
3. $-1, -3, -9,$ _____, ______, …
4. $-1, 2,$ ____, _____, -16, …
5. $-10, 10,$ ____, $10, -10,$ _____, …
6. 7, 14, _____, 56, 112, _____, …
7. $-2, -6,$ ____, _____, -162, ….
8. 1, 10, 100, _______, _______, 100,000, ….
9. $2, -6, 18,$ ____, _____,…..
10. 1, 3, 9, _____, ______, ______, ….

Lesson Title: Creating Geometric Patterns	Theme: Algebra
Practice Activity: PHM-08-105	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to create geometric patterns by using a rule to find the next terms.

Overview

In this lesson, you will create your own geometric pattern. To create a geometric pattern, all you need to do is decide what the first number will be and what the ratio between two numbers will be.

Solved Examples

1. Write a geometric pattern starting with 4, with a common ratio of 3. Write the first 5 terms of the pattern.

   Solution
   The first term of the pattern is 4. Find the next 5 terms using multiplication:

   - $4 \times 3 = 12$
   - $12 \times 3 = 36$
   - $36 \times 3 = 108$
   - $108 \times 3 = 324$

   List the first 5 terms of the pattern: 4, 12, 36, 108, 324

2. Write a geometric pattern starting with 1, with a common ratio of -3. Write the first 4 terms of the pattern.

   Solution
   It is not necessary to write out the multiplication each time you write a geometric pattern. You can do the multiplication in your head or on paper, and write out the list of numbers.

   Start with 1 and multiply -3 to get each next term: $1, -3, 9, -27, \ldots$

3. Write a geometric pattern that starts at -2 and has a common ratio of 3.

   Solution
   Start with -2 and multiply by 3 to get each next term: $-2, -6, -18, -54, \ldots$.

   The problem does not tell us how many terms to give. In this case, write several terms and use the ellipses (…) to show that the list continues.

Practice

1. Write a geometric pattern starting with 3, with a common ratio of 10. Write the first 5 terms of the pattern.
2. Write a geometric pattern starting with -1, with a common ratio of -2. Write the first 4 terms of the pattern.
3. Write a geometric pattern that starts at 3 and has a common ratio of 3.
4. Write a geometric pattern that starts at 2 and has a common ratio of -5.
5. Write a geometric pattern that starts at -2 and has a common ratio of 4.

Lesson Title: Simplifying Algebraic Expressions	**Theme:** Algebra
Practice Activity: PHM-08-106	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify and combine like terms where variables have power 0 or 1.

Overview

In algebra, **like terms** are terms that have the same variable, and the variables have the same power. For example, $2a, 3a, 5a$ are all like terms, with the variable a to the power 1. As another example, $5p^2$ and $8p^2$ are like terms. The coefficients (number parts) can be different, but the rest of the term must be exactly the same.

Integers are also like terms. If you see two or more integers in an expression, they can be combined.

Like terms can be combined by adding or subtracting to give a single term. The result will have the same variable with the same power. The coefficients of the terms are added or subtracted. For example, consider these combinations:

$$3a + 2a = (3 + 2)a = 5a$$

$$4a - a = (4 - 1)a = 3a$$

$$4a + a - 2a = (4 + 1 - 2)a = 3a$$

Unlike terms are two or more terms that are not like terms. That is, they do not have the same variables or powers. Examples of unlike terms are:

$$2a + 3b + 4c$$
$$3x + 2x^2$$

Unlike terms cannot be combined.

Solved Examples

1. Simplify: $8x - 2y - 4x + 5y$

   Solution

$$\begin{aligned} 8x - 2y - 4x + 5y &= 8x - 4x + 5y - 2y && \text{Collect like terms} \\ &= (8 - 4)x + (5 - 2)y && \text{Combine like terms} \\ &= 4x + 3y \end{aligned}$$

2. Simplify: $7p - 3q + 3p + 2q$

Solution

$$\begin{aligned} 7p - 3q + 3p + 2q &= 7p + 3p + 2q - 3q && \text{Collect like terms} \\ &= (7+3)p + (2-3)q && \text{Combine like terms} \\ &= 10p - q \end{aligned}$$

3. Simplify: $4x + 9 - 3x + 4$

Solution

$$\begin{aligned} 4x + 9 - 3x + 4 &= 4x - 3x + 9 + 4 && \text{Collect like terms} \\ &= (4-3)x + (9+4) && \text{Combine like terms} \\ &= x + 13 \end{aligned}$$

4. Simplify the following algebraic expressions:
   a. $12e + 5f - 4e - 2f$
   b. $2m + 5 - 3m - 4$
   c. $11x - 10y - 10x + 12y$
   d. $3u - 3 + 4v - 2u + 7 - 2v$

   Solutions

   a.

$$\begin{aligned} 12e + 5f - 4e - 2f &= 12e - 4e + 5f - 2f && \text{Collect like terms} \\ &= (12-4)e + (5-2)f && \text{Combine like terms} \\ &= 8e + 3f \end{aligned}$$

   b.

$$\begin{aligned} 2m + 5 - 3m - 4 &= 2m - 3m + 5 - 4 && \text{Collect like terms} \\ &= (2-3)m + (5-4) && \text{Combine like terms} \\ &= -m + 1 \end{aligned}$$

   c.

$$\begin{aligned} 11x - 10y - 10x + 12y &= 11x - 10x + 12y - 10y \\ &= (11-10)x + (12-10)y \\ &= x + 2y \end{aligned}$$

   d.

$$\begin{aligned} 3u - 3 + 4v - 2u + 7 - 2v &= 3u - 2u + 4v - 2v + 7 - 3 \\ &= (3-2)u + (4-2)v + (7-3) \\ &= u + 2v + 4 \end{aligned}$$

Practice

Simplify the following algebraic expressions:

1. $4y - 3x + 5x - 3y$
2. $9a + 4b - 11a + 3b$
3. $12x - 5 + 2y - 7x + 8$
4. $2a + 5b - 7 + 3a - b$
5. $6m + 11n - 4m + 2n - m + n$
6. $8n + 9m + 4m - 10m - 2n$
7. $4 + 7b - 3a - 3 - 2b + 7a$
8. $m + 2n + p - m + 5n - 2$
9. $7xy + 5x - 4x + 2xy - 3$
10. $a + 3ab + 67a - 2ab - 1$

Lesson Title: Simplifying Expressions with Higher Powers	**Theme:** Algebra
Practice Activity: PHM-08-107	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify and combine like terms where variables have power 2 or greater.

Overview

In this lesson, you will simplify expressions where the variables are raised to powers of 2 or greater. Remember that like terms have the same variable, and the variables have the same power. This means that $3x^3$ and $4x^3$ are like terms. Similarly, x^5 and $7x^5$ are like terms.

Follow the same steps that you used in the previous lesson to combine like terms. Add their coefficients. For example: $3x^3 + 4x^3 = (3+4)x^3 = 7x^3$.

When you group like terms, remember to always write the terms in order of highest power to lowest power.

Solved Examples

1. Simplify: $4x^2 + x + 2x^2 - 3x$

   Solution

$$\begin{aligned} 4x^2 + x + 2x^2 - 3x &= 4x^2 + 2x^2 + x - 3x && \text{Collect like terms} \\ &= (4+2)x^2 + (1-3)x && \text{Combine like terms} \\ &= 6x^2 - 2x \end{aligned}$$

2. Simplify: $x^3 + x^2 - 5 + 5x^3 + 8x^2$

   Solution

$$\begin{aligned} x^3 + x^2 - 5 + 5x^3 + 8x^2 &= x^3 + 5x^3 + x^2 + 8x^2 - 5 && \text{Collect like terms} \\ &= (1+5)x^3 + (1+8)x^2 - 5 && \text{Combine like terms} \\ &= 6x^3 + 9x^2 - 5 \end{aligned}$$

3. Simplify the expression $2x^2 + 1 - x + 3x^3 + 2 + 3x - 4x^2 + 4x^3$

   Solution

$$\begin{aligned} 2x^2 + 1 - x + 3x^3 + 2 + 3x - 4x^2 + 4x^3 &= 3x^3 + 4x^3 + 2x^2 - 4x^2 - x + 3x + 2 + 1 \\ &= (3+4)x^3 + (2-4)x^2 + (-1+3)x + 3 \\ &= 7x^3 - 2x^2 + 2x + 3 \end{aligned}$$

4. Simplify the expression: $x^2 + y^2 - 5y + 2x^2 + 5y^2 + 2x - y$

Solution

Remember that like terms must have the same variable **and** power. For example, x^2 and y^2 cannot be combined because the variables are different.

$$\begin{aligned} x^2 + y^2 - 5y + 2x^2 + 5y^2 + 2x - y &= x^2 + 2x^2 + y^2 + 5y^2 + 2x - 5y - y \\ &= (1+2)x^2 + (1+5)y^2 + 2x + (-5-1)y \\ &= 3x^2 + 6y^2 + 2x - 6y \end{aligned}$$

Practice

Simplify each expression:

1. $3x^2 + x + x^2 - 4$
2. $x^2 + 3x + 5x^2 - x + 6$
3. $4x^2 - 8 - 12x^2 + x + 21x$
4. $3x - 4x^2 + 6x + 10x^2$
5. $x^3 + 7x - x^2 + 3x + 8x^3 + 4x^2$
6. $10x^2 + 8y^2 + 7y - y^2 + 3x^2 - 8 + x - 2y$
7. $7x^2 + 10 + 9y - x + 3x^2 - 3 + 5x - 12$
8. $2x^3 - x - 12 - 11x + 6 - 5x^3 + x^2$
9. $4x^2y + 5xy^2 + 3x^2y - 2xy^2$
10. $11v^2w - 2v^2 + 3v^2 - 8v^2w - 2v^2$

Lesson Title: Simplifying Expressions with Fractions	**Theme:** Algebra
Practice Activity: PHM-08-108	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify and combine like terms that involve fractions.

Overview

In this lesson, you will combine like terms that involve fractions. Follow the same steps that you used in the previous lesson to combine like terms. Add their coefficients. If you need to review addition and subtraction of fractions, see the box below.

Adding and subtracting fractions
You will use your skills in adding and subtracting fractions. Review the following steps:

1. Find the LCM of the denominators.
2. Write each fraction as an equivalent fraction with the LCM denominator.
3. Add or subtract the fractions.

For example, consider $\frac{3}{4}+\frac{1}{8}$.

1. The LCM of 4 and 8 is 8.
2. Rewrite the fractions: $\frac{6}{8}+\frac{1}{8}$
3. Add the fractions: $\frac{6}{8}+\frac{1}{8}=\frac{6+1}{8}=\frac{7}{8}$

Remember that if fractions have the same denominator, the numerators are simply added or subtracted.

Solved Examples

1. Simplify: $5+\frac{1}{2}x+x^2+\frac{1}{4}x+1\frac{1}{2}$

   Solution

$$
\begin{aligned}
5+\tfrac{1}{2}x+x^2+\tfrac{1}{4}x+1\tfrac{1}{2} &= x^2+\tfrac{1}{2}x+\tfrac{1}{4}x+5+1\tfrac{1}{2} && \text{Collect like terms}\\
&= x^2+\left(\tfrac{1}{2}+\tfrac{1}{4}\right)x+\left(5+1\tfrac{1}{2}\right) && \text{Combine like terms}\\
&= x^2+\left(\tfrac{2}{4}+\tfrac{1}{4}\right)x+\left(5+1\tfrac{1}{2}\right) && \text{Write fractions with LCM}\\
&= x^2+\left(\tfrac{2+1}{4}\right)x+\left(6\tfrac{1}{2}\right) && \text{Add fractions}\\
&= x^2+\tfrac{3}{4}x+6\tfrac{1}{2}
\end{aligned}
$$

2. Simplify: $\left(\frac{1}{2}\right)x^2 + \left(\frac{3}{8}\right)x + \left(\frac{1}{2}\right)x^2 + \left(\frac{1}{8}\right)x$

Solution

Note that $\left(\frac{1}{2}\right)x^2$ is the same as $\frac{1}{2}x^2$. Terms with fractions can be written either way.

$\left(\frac{1}{2}\right)x^2 + \left(\frac{3}{8}\right)x + \left(\frac{1}{2}\right)x^2 + \left(\frac{1}{8}\right)x$	$= \left(\frac{1}{2}\right)x^2 + \left(\frac{1}{2}\right)x^2 + \left(\frac{3}{8}\right)x + \left(\frac{1}{8}\right)x$	Collect like terms
	$= \left(\frac{1}{2} + \frac{1}{2}\right)x^2 + \left(\frac{3}{8} + \frac{1}{8}\right)x$	Combine like terms
	$= \left(\frac{1+1}{2}\right)x^2 + \left(\frac{3+1}{8}\right)x$	Add fractions
	$= \left(\frac{2}{2}\right)x^2 + \left(\frac{4}{8}\right)x$	
	$= 1x^2 + \left(\frac{1}{2}\right)x$	Simplify
	$= x^2 + \frac{1}{2}x$	

3. Simplify: $-\frac{1}{6}x^3 - 5x + \frac{1}{3}x^3 - 3 - x + \frac{1}{2}$

Solution

$-\frac{1}{6}x^3 - 5x + \frac{1}{3}x^3 - 3 - x + \frac{1}{2}$	$= -\frac{1}{6}x^3 + \frac{1}{3}x^3 - 5x - x - 3 + \frac{1}{2}$	Collect like terms
	$= \left(-\frac{1}{6} + \frac{1}{3}\right)x^3 + (-5-1)x + \left(-3 + \frac{1}{2}\right)$	Combine like terms
	$= \left(-\frac{1}{6} + \frac{2}{6}\right)x^3 + (-5-1)x + \left(-\frac{6}{2} + \frac{1}{2}\right)$	Write fractions with LCM
	$= \left(\frac{-1+2}{6}\right)x^3 + (-6)x + \left(\frac{-6+1}{2}\right)$	Add fractions
	$= \left(\frac{1}{6}\right)x^3 + (-6)x + \left(\frac{-5}{2}\right)$	
	$= \frac{1}{6}x^3 - 6x - 2\frac{1}{2}$	

Practice

Simplify the following expressions:

1. $5x^2 - x + \frac{1}{2}x^2$
2. $\frac{2}{3}x + 5 - \frac{1}{2}x - 2\frac{1}{2}$
3. $\frac{1}{5}x + x^2 + \frac{2}{3}x + \frac{1}{3} - \frac{1}{2}x^2$
4. $\frac{5}{6}x^2 + 8 + \frac{1}{3}x + \frac{1}{3}x^2 + \frac{1}{5}$
5. $-\frac{1}{6} + \frac{1}{3}x + \frac{1}{4}x^3 + \frac{1}{3} + \frac{1}{4}x^2 - \frac{1}{8}x^3$
6. $\frac{1}{2}x^2 + x - \frac{1}{3}x - \frac{1}{10}x^2 - \frac{1}{6}$

Lesson Title: Multiplying an Algebraic Expression by an Integer	Theme: Algebra
Practice Activity: PHM-08-109	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to expand an algebraic expression by multiplying by an integer.

Overview

In today's lesson, you will handle algebraic expressions that contain brackets. For example, $2(5x-4)$. In order to simplify such expressions, we must first remove the brackets. In removing brackets, **multiply** the term outside the bracket by each of the terms inside the bracket. Multiplying a number by the terms in brackets is also called **expanding** the expression.

We must be very careful with signs when removing brackets. When there is a positive (+) number before the bracket, the sign inside the brackets does not change when the brackets are removed. When there is a negative number (–) in front of the brackets, the signs inside the bracket change when the brackets are removed. This is because of the rules of multiplication.

Remember the rules for multiplying negative numbers:

- Negative x Positive = Negative
- Positive x Negative = Negative
- Negative x Negative = Positive

Remember that when you multiply a number by a term with a coefficient, the 2 numbers are multiplied. For example: $2 \times 5x = 10x$

Solved Examples

1. Simplify: $2(5x-4)$

   Solution
   Multiply 2 by each term inside brackets. The terms inside brackets are $5x$ and -4.

   $$\begin{aligned} 2(5x-4) &= (2\times 5x)+(2\times -4) && \text{Multiply each term by 2} \\ &= 10x-8 \end{aligned}$$

2. Simplify: $-4(2y-3)$

Solution

This is an example of a problem with a negative number in front of the brackets.

$$\begin{aligned} -4(2y-3) &= (-4 \times 2y) + (-4 \times -3) && \text{Multiply each term by –4} \\ &= -8y + 12 \end{aligned}$$

3. Simplify: $2(x^2 - x + 5)$

Solution

$$\begin{aligned} 2(x^2 - x + 5) &= (2 \times x^2) + (2 \times -x) + (2 \times 5) && \text{Multiply each term by 2} \\ &= 2x^2 - 2x + 10 \end{aligned}$$

4. Simplify: $-3(2m + 3n + 4)$

Solution

$$\begin{aligned} -3(2m + 3n) &= (-3 \times 2m) + (-3 \times 3n) + (-3 \times 4) && \text{Multiply each term by } -3 \\ &= -6m - 9n - 12 \end{aligned}$$

5. Simplify: $-(a + 4b)$

Solution

If there is a negative sign in front of the brackets, it is the same as having -1 in front of the bracket. Multiply each term inside the brackets by -1. This changes the sign on each term. In other words, the negative sign is distributed to each term inside the brackets.

$$\begin{aligned} -(a + 4b) &= (-1 \times a) + (-1 \times 4b) && \text{Multiply each term by } -3 \\ &= -a - 4b \end{aligned}$$

Practice

Remove brackets and simplify the following algebraic expressions:

1. $5(x-4)$
2. $-7(3y-4)$
3. $-2(m+n)$
4. $3(2v+3)$
5. $-(2x^2 - x + 7)$
6. $8(-3m + 2n)$
7. $-2(-2a - 3)$
8. $10(x^2 - 3x)$

Lesson Title: Multiplying Variables	**Theme:** Algebra
Practice Activity: PHM-08-110	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to multiply two monomials with variables, applying the rules of indices.

Overview

A monomial is an expression that consists of only one term. 'Mono' means one. These are examples of monomials:

$$x \qquad y \qquad 3 \qquad x^2 \qquad 3xy \quad 2x^2y \quad 7xyz$$

Monomials can be numbers, variables, or a mixture of both.

A number in a monomial that also has a variable is called a **coefficient**. In $3xy$, 3 is the coefficient of the monomial. Monomials cannot have negative or fractional indices or powers. They have no operations like addition or subtraction in them.

In this lesson, you will multiply two monomials with variables using the laws of indices. Recall the first law of indices. When two indices with the same base are multiplied, their powers are added: $a^m \times a^n = a^{m+n}$

When two monomials are multiplied together, the coefficients are multiplied and the variable parts are multiplied. Only combine variables if they have the same base.

Solved Examples

1. Multiply: $x^4 \times x^7$

   Solution
   Apply the law of indices. Add the powers: $x^4 \times x^7 = x^{4+7} = x^{11}$

2. Multiply: $2y \times x^7$

   Solution
   The variables are different, so they cannot be combined. Remember that when variables are written next to each other, it means they are multiplied. The answer is $2y \times x^7 = 2yx^7$.

3. Multiply: $2y \times 3y^2$

   Solution

   Multiply the coefficient part and the variable part separately.

$$\begin{aligned} 2y \times 3y^2 &= (2 \times 3)(y \times y^2) \\ &= 6y^{1+2} \\ &= 6y^3 \end{aligned}$$

4. Multiply: $4x^2 \times 3xy$

   Solution

   Multiply the coefficient part and the variable part separately. Remember to combine variables only if they are the same.

$$\begin{aligned} 4x^2 \times 3xy &= (4 \times 3)(x^2 \times x)y \\ &= 12x^{2+1}y \\ &= 12x^3y \end{aligned}$$

5. Multiply: $-7x \times 4x^2$

   Solution

$$\begin{aligned} -7x \times 4x^2 &= (-7 \times 4)(x \times x^2) \\ &= -28x^{1+2} \\ &= -28x^3 \end{aligned}$$

Practice

Simplify each expression by multiplying:

1. $y^5 \times y^2$
2. $y^3 \times x^2$
3. $2y^5 \times 3xy$
4. $-3x^3 \times 2x^2$
5. $-x^2 \times x^3$
6. $2x^2 \times 5x^3y$
7. $xy \times xy$
8. $xy^2 \times 5x^2y$
9. $7x^2y^4 \times 2x^3y^2$
10. $15p^3q^2 \times 12p^5q^3$

Lesson Title: Multiplying an Algebraic Expression by a Variable	Theme: Algebra
Practice Activity: PHM-08-111	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to expand an algebraic expression by multiplying by a variable.

Overview

In this lesson, you will multiply algebraic expressions by variables. For example, consider $x(x+3)$. The variable x is multiplied by the expression $x+3$. Multiplying a variable by an expression is also called **expansion**.

Use the same method that you used in Lesson 109, when you multiplied an expression by a number. Multiply each term in brackets by the variable. Use the techniques you learned in the previous lesson to multiply variables.

Solved Examples

1. Multiply: $x(x+3)$

   Solution
   Multiply x by each term inside brackets. The terms inside the brackets are x and 3.

$x(x+3)$	$= (x \times x) + (x \times 3)$	Multiply each term by x
	$= x^2 + 3x$	

2. Expand: $x(x^2 - 2)$

   Solution
   In this case, 'expand' means to multiply. Remember to apply the laws of indices to multiply the variables.

$x(x^2-2)$	$= (x \times x^2) + (x \times -2)$	Multiply each term by x
	$= x^{1+2} - 2x$	Apply the law of indices
	$= x^3 - 2x$	

3. Multiply: $-x(4-5x)$

Solution
Each term in brackets is multiplied by $-x$. Remember the rules for multiplying negative values.

$$\begin{aligned} -x(4-5x) &= (-x \times 4) + (-x \times -5x) \quad \text{Multiply each term by } -x \\ &= -4x + 5x^{1+1} \\ &= -4x + 5x^2 \end{aligned}$$

4. Multiply: $x(x+y-4)$

Solution
There are 3 terms in the brackets. The variable x is multiplied by each of them.

$$\begin{aligned} x(x+y-4) &= (x \times x) + (x \times y) + (x \times -4) \quad \text{Multiply each term by } x \\ &= x^2 + xy - 4x \end{aligned}$$

5. Expand: $3x(x-4)$

Solution
In this case, the expression is multiplied by a monomial with both a number (3) and a variable (x). $3x$ is multiplied by both terms in the brackets. Use the techniques you learned in the previous lesson on multiplying monomials.

$$\begin{aligned} 3x(x-4) &= (3x \times x) + (3x \times -4) \quad \text{Multiply each term by } 3x \\ &= 3x^2 - 12x \end{aligned}$$

Practice

Expand the following expressions:

1. $x(x-2)$
2. $x(4-x)$
3. $x(2x-1)$
4. $x(x^2-3x)$
5. $-x(x+12)$
6. $-x(9-x+y)$
7. $3x(x-4)$
8. $-2x(4x+1)$
9. $4x(x^2+x)$
10. $3a(a+4ab-b)$

Lesson Title: Simplifying and Expanding Algebraic Expressions	**Theme:** Algebra
Practice Activity: PHM-08-112	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to apply operations to simplify algebraic expressions involving integers and variables.

Overview

In this lesson, you will use the skills that you learned from previous lessons to simplify algebraic expressions. Follow the correct order of operations, BODMAS. Remember that this stands for Bracket, Of, Division, Multiplication, Addition, Subtraction. This means that you will always remove brackets before combining like terms (addition/subtraction).

Solved Examples

1. Simplify: $x(2x+1)+2x^2$

   Solution
   Remove the brackets first. Then, you will combine any like terms (addition/subtraction).

$$\begin{aligned} x(2x+1)+2x^2 &= (x\times 2x)+(x\times 1)+2x^2 && \text{Remove the brackets} \\ &= 2x^2+x+2x^2 \\ &= 2x^2+2x^2+x && \text{Collect like terms} \\ &= (2+2)x^2+x && \text{Combine like terms} \\ &= 4x^2+x \end{aligned}$$

2. Simplify: $5a+a(a-2)+3a^2$

   Solution

$$\begin{aligned} 5a+a(a-2)+3a^2 &= 5a+(a\times a)+(a\times -2)+3a^2 && \text{Remove the brackets} \\ &= 5a+a^2-2a+3a^2 \\ &= a^2+3a^2+5a-2a && \text{Collect like terms} \\ &= (1+3)a^2+(5-2)a && \text{Combine like terms} \\ &= 4a^2+3a \end{aligned}$$

3. Expand and simplify: $x(x+1) - 2x(3-x)$

Solution

There are 2 sets of brackets in this problem. Remove the brackets first, treating the 2 sets of brackets separately. Then, you will combine any like terms (addition/subtraction).

$$\begin{aligned} x(x+1) - 2x(3-x) &= (x \times x) + (x \times 1) + (-2x \times 3) + (-2x \times -x) \\ &= x^2 + x - 6x + 2x^2 \\ &= x^2 + 2x^2 + x - 6x \\ &= (1+2)x^2 + (1-6)x \\ &= 3x^2 - 5x \end{aligned}$$

4. Expand and simplify $2a[(a+3b) + 4(2a-b)]$

Solution

Note that when there are brackets inside a bracket as in this case, always remove the inside bracket first.

$$\begin{aligned} 2a[(a+3b) + 4(2a-b)] &= 2a[a + 3b + 8a - 4b] \\ &= 2a[a + 8a + 3b - 4b] \\ &= 2a[9a - b] \\ &= 18a^2 - 2ab \end{aligned}$$

Practice

1. $y(y-4) + 7y$
2. $5(x-y) + 12x - 5$
3. $x(3-x) + x(2x-4)$
4. $3x + x^2 + x(x-5)$
5. $y + 3y(y-1)$
6. $5z^3 + z(z^2+5) - z$
7. $x(y-x) + 2x^2 + 5$
8. $3x + x(x-5) + 7(x+2)$
9. $(e+f-g) - (e-f+g)$
10. $2(6-5x) - 3(2+2x) - 4(3x-1)$

Lesson Title: Algebraic Expression Story Problems	Theme: Algebra
Practice Activity: PHM-08-113	Class: JSS 2

Learning Outcome
By the end of the lesson, you will be able to write algebraic expressions for situations in story problems.

Overview

This lesson is on writing algebraic expressions from story problems. There are many types of word problems which involve relations among known and unknown numbers. These can be written in the form of expressions. The unknown values in story problems are assigned variables, such as x.

Variables can represent different things. For example:

- Variable t = time
- Variable h = hours
- Variable s = seconds

When you read a story problem, you will choose variables to represent anything you need it to. For example, in a story about apples and peanuts, you may have:

- Variable a = apples
- Variable p = peanuts

Always simplify your expressions if possible by expanding or combining like terms.

Solved Examples

1. Bendu bought some rice, and her sister gave her 3 more cups of rice. Write an expression for the amount of rice she has now.

   Solution
   We do not know the amount of rice Bendu bought, so we will use a variable. Let r be the amount of rice she bought in cups.

   The amount she has now is $r + 3$ cups. The algebraic expression is $r + 3$, where r is the number of cups Bendu bought.

2. Foday reads books every day. It takes him 3 minutes to read each page. Write an expression for the amount of time Foday spends reading each day.

Solution

The unknown value in this problem is the number of pages that Foday reads each day. Let p be the number of pages he reads.

Since he spends 3 minutes on each page, you want to multiply the 3 minutes times the number of pages he reads. This will give you the total amount of time spent on reading.

The amount of time Foday spends reading is $3p$, where p is the number of pages.

3. You want to buy rice and chicken. Rice costs Le 1,200.00 per cup, and chicken costs Le 4,000.00 for each piece. Write an algebraic expression for how much money you spend in the market depending on how much rice and chicken you buy.

Solution

There are 2 unknown values in this problem. The unknown values in this problem are the amount of rice you buy, and the amount of chicken you buy. Let the amount of rice be r cups and the amount of chicken be c pieces.

Multiply the cost of each item by the number of cups or pieces you will buy. The cost of rice will be $1{,}200r$ and the cost of chicken will be $4{,}000c$.

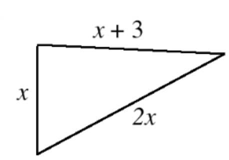
Add to find the total amount you will spend: $1{,}200r + 4{,}000c$, where r is cups of rice and c is pieces of chicken.

4. Write an expression for the perimeter of a triangle with sides x, $2x$, and $x + 3$.

Solution

In this case, the variable is given in the problem. You will use your knowledge of perimeter to solve this problem. Remember that a triangle is a 3-sided shape. We find its perimeter by adding the lengths of its 3 sides.

It may help to draw a picture. For example:



Add the lengths of the 3 sides, and simplify:

$$\begin{aligned} P &= x + 2x + (x + 3) && \\ &= x + 2x + x + 3 && \text{Add the 3 sides} \\ &= (1 + 2 + 1)x + 3 && \text{Group like terms} \\ &= 4x + 3 && \text{Combine like terms} \end{aligned}$$

The expression for the perimeter of this triangle is $P = 4x + 3$.

5. Write an expression for the combined ages of four friends aged x, $x+3$, $x-1$ and $x+2$.

Solution

The words 'combined age' tell us to add their ages. Add all 4 of the given expressions together:

$$\begin{aligned} \text{Combined age} &= x+(x+3)+(x-1)+(x+2) && \text{Add ages} \\ &= x+x+3+x-1+x+2 \\ &= x+x+x+x+3-1+2 && \text{Group like terms} \\ &= 4x+4 \end{aligned}$$

The expression for their combined age is $4x+4$.

Practice

1. You want to buy rice and fish. Rice costs Le 1,500.00 per cup, and fish costs Le 5,000.00 for each piece. Write an algebraic expression for how much money you spend in the market depending on how much rice and fish you buy.
2. Sia had some rice, but she sold 10 cups of it. Write an expression for the amount of rice Sia has left.
3. Hawa is painting a wall. It takes her 4 minutes to paint each square metre. Write an expression for the amount of time Hawa will spend painting the wall.
4. Write an expression for the perimeter of a quadrilateral with sides of length x, $2x$, $x+8$ and $x+2$.
5. Bockarie is 11 years older than Kallon. If Kallon is x years old, write an expression for Bockarie's age.
6. Hawa is twice as old as Musa. If Musa is $x+3$ years old, write an expression for Hawa's age.
7. Three friends measured their weights on a scale. If their weights are x, $x+5$ and $x-7$, write an expression for their combined weight.
8. A man has $6x$ sheep and $5y$ goats. He sells $3x$ sheep and $2y$ goats. How many animals are left after the sales?

Lesson Title: Factoring Integers from Algebraic Expressions	**Theme:** Algebra
Practice Activity: PHM-08-114	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify integers that are common factors in an algebraic expression.
2. Divide common factors from an algebraic expression.

Overview

In this lesson, you will identify that **factorisation** involves using division to break an expression into parts. You will identify and factorise integers that are common factors in an algebraic expression.

Factorisation is the **opposite** of expanding a bracket. Remember that we multiply to expand a bracket: $2(x+3) = 2 \times x + 2 \times 3 = 2x + 6$

In this lesson, you will take an expression like $2x + 6$ and use division to find all its factors. This process is called factorisation. You will use factorisation to get from $2x + 6$ to $2(x + 3)$.

Use these steps to factor an expression:

1. Find the highest common factor (HCF) of the terms in the expression.
2. Write the HCF outside of empty brackets.
3. Divide each term in the expression by the HCF, and write the result in the brackets.

Always check your result to make sure the expression in brackets cannot be factorised further. If you choose a factor that is not the GCF, you may need to factorise more than once to complete the factorisation.

Solved Examples

1. Factorise the expression $2x + 6$.

Solution
First, look for the HCF of the expression. It is 2. This is the largest number which can divide $2x$ and 6.

$$\begin{aligned} 2x + 6 &= 2(\quad\quad) && \text{Factor the HCF, 2} \\ &= 2(x+3) && \text{Divide each term in } 2x+6 \text{ by 2} \end{aligned}$$

Answer: $2(x + 3)$

Check your answer by expanding the brackets:

$$\begin{aligned} 2(x+3) &= 2 \times x + 2 \times 3 && \text{Multiply each term in the brackets by 2} \\ &= 2x + 6 && \text{Check that this is the original expression} \end{aligned}$$

2. Factorise the expression $12x - 24$.

Solution
Note that the HCF of the expression is 12.

$$\begin{aligned} 12x - 24 &= 12(\qquad) && \text{Factor the HCF, 12} \\ &= 12(x-2) && \text{Divide each term in } 12x - 24 \text{ by 12} \end{aligned}$$

Answer: $12(x-2)$

Check your answer by expanding the brackets:

$$\begin{aligned} 12(x-2) &= 12 \times x + (12 \times -2) && \text{Multiply each term in brackets by 12} \\ &= 12x - 24 && \text{Original expression} \end{aligned}$$

3. Factorise $10 + 3y - 2 + y$.

Solution
Note that there is no HCF for the 4 terms of this expression. However, there are like terms in the expression. Combine the like terms first, then try to factorise the expression.

$$\begin{aligned} 10 + 3y - 2 + y &= 10 - 2 + 3y + y && \text{Collect like terms} \\ &= 8 + 4y && \text{Combine like terms} \\ &= 4(\qquad) && \text{Factor the HCF, 4} \\ &= 4(2+y) && \text{Divide each term in } 8 + 4y \text{ by 4} \end{aligned}$$

Check your answer by expanding the brackets:

$$\begin{aligned} 4(2+y) &= 4 \times 2 + (4 \times y) && \text{Multiply each term in brackets by 4} \\ &= 8 + 4y && \text{Original expression} \end{aligned}$$

4. Factorise $5x^3 + 15x^2 + 35x + 20$

Solution
Note that the HCF of the expression is 5. This expression has higher powers of x. However, the factorisation process is the same. Divide each term by the HCF (5). The result will have the same powers of x.

$$\begin{aligned} 5x^3 + 15x^2 + 35x + 20 &= 5(\qquad) && \text{Factor the HCF, 5} \\ &= 5(x^3 + 3x^2 + 7x + 4) && \text{Divide each term by 5} \end{aligned}$$

Check your answer by expanding the brackets:

$$5(x^3 + 3x^2 + 7x + 4) = 5x^3 + 15x^2 + 35x + 20$$

Practice

Factorise the expressions below. Please check all answers.

1. $4x + 12$
2. $7x - 21y$
3. $14 - 2x$
4. $20x + 30$
5. $4y - 6$
6. $10s + 12t - 4t$
7. $9 - 18p + 3$
8. $3x^2 + 12x + 30$
9. $9x^2 - 12$
10. $2x^3 + 40x^2 + 12x + 24$

Lesson Title: Factoring Variables from Algebraic Expressions	**Theme:** Algebra
Practice Activity: PHM-08-115	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify variables that are common factors in an algebraic expression.
2. Divide common factors from an algebraic expression.

Overview

In the previous lesson, you factorised numbers from algebraic expressions. In this lesson, you will factorise variables.

When you factorise a variable, divide each term in the expression by that variable. You will apply the second law of indices. This says that when dividing indices, subtract their powers: $a^m \div a^n = a^{m-n}$.

If 2 terms contain the same variable, then that variable is a common factor. Any common factor can be factorised out of the expression.

For example, consider $x^2 + 2x$. The variable x is a common factor because it is in both terms. Write x outside of the brackets, and divide each term by x: $x^2 + 2x = x(x + 2)$.

It is not necessary to show the division each time you factorise an expression, but the division of each term of this example is shown below:

$\frac{x^2}{x} = x^{2-1} = x^1 = x$

$\frac{2x}{x} = 2$, x cancels

In some cases, you can factor both an integer and a variable. Divide the expression by both of them at the same time. For example, consider $10x^2 + 20x$. The common factors are 10 and x. The HCF of this expression is found by multiplying the common factors. The HCF is $10x$. When you factorise the expression you will divide by the HCF: $10x^2 + 20x = 10x(x + 2)$

Solved Examples

1. Factorise: $x^2 + 5x$

   Solution
   Note that the HCF of the expression is x.

   $$\begin{aligned} x^2 + 5x &= x(\quad) && \text{Factor the HCF, } x \\ &= x(x+5) && \text{Divide each term in } x^2+5x \text{ by } x \end{aligned}$$

   Answer: $x(x+5)$

   Check your answer by expanding the brackets:

   $$\begin{aligned} x(x+5) &= x \times x + (x \times 5) && \text{Multiply each term by } x \\ &= x^2 + 5x && \text{Original expression} \end{aligned}$$

2. Factorise: $3x^3 + 2x^2 + x$

   Solution
   Note that the HCF of the expression is x.

   $$\begin{aligned} 3x^3 + 2x^2 + x &= x(\quad) && \text{Factor the HCF, } x \\ &= x(3x^2 + 2x + 1) && \text{Divide each term by } x \end{aligned}$$

   Answer: $x(3x^2 + 2x + 1)$

   Check your answer by expanding the brackets:

   $$\begin{aligned} x(3x^2 + 2x + 1) &= (x \times 3x^2) + (x \times 2x) + (x \times 1) \\ &= 3x^3 + 2x^2 + x \end{aligned}$$

3. Factorise: $12x^2 - 6x$

   Solution
   Note that there are 2 common factors in the expression, 6 and x. This means the HCF is $6x$. Bring $6x$ outside of brackets.

   $$\begin{aligned} 12x^2 - 6x &= 6x(\quad) && \text{Factor the HCF, } 6x \\ &= 6x(2x-1) && \text{Divide each term by } 6x \end{aligned}$$

   Check your answer by expanding the brackets:

   $$\begin{aligned} 6x(2x-1) &= (6x \times 2x) + (6x \times -1) \\ &= 12x^2 - 6x \end{aligned}$$

4. Factorise $a^2 + a + 7a + 3a^2$

Solution

Note that there are like terms in this expression. Remember to combine like terms before factorising the expression.

$a^2 + a + 7a + 3a^2$	$=$	$a^2 + 3a^2 + a + 7a$	Collect like terms
	$=$	$4a^2 + 8a$	Combine like terms
	$=$	$4a(\quad)$	Factor the HCF, $4a$
	$=$	$4a(a+2)$	Divide each term by $4a$

Check your answer by expanding the brackets:

$$\begin{aligned} 4a(a+2) &= (4a \times a) + (4a \times 2) \\ &= 4a^2 + 8a \end{aligned}$$

Practice

Factorise the expressions below. Please check all answers.

1. $xy + y$
2. $xy + yz$
3. $2a^2 - a$
4. $3x^2 + 8x$
5. $y^3 + y^2$
6. $x^3 + 7x^2 - 3x$
7. $3x^3 + 9x^2 - 18x$
8. $5x^3 - 15x^2$
9. $9a^2 + 13a - 3a - 4a^2$
10. $5x^3 + 12x^2 + 9x^3 - 5x^2$

Answer Key – JSS 2 Term 2

Lesson Title:	Personal Expenditure
Practice Activity:	PHM-08-056

1. 10%
2. 5%
3. Completed table:

Expense	Amount (Le)	% of income
New clothes	200,000.00	10%
Food	400,000.00	20%
Transportation	250,000.00	12.5%
Mobile phone use	100,000.00	5%
Health care	500,000.00	25%

Lesson Title:	Income Tax
Practice Activity:	PHM-08-057

1. Le 1,297,500.00
2. Le 16,200,000.00
3. a. Le 1,586,000.00; b. Le 14,274,000.00

Lesson Title:	Sales Tax
Practice Activity:	PHM-08-058

1. a. Le 22,500.00; b. Le 472,500.00
2. Le 530,000.00
3. Le 3,120,000.00

Lesson Title:	Time and Duration
Practice Activity:	PHM-08-059

1. a. 01:00; b. 00:45; c. 20:00; d. 12:30
2. a. 2 am or 2:00 am; b. 10:15 am; c. 7 pm or 7:00 pm; d. 11:30 pm
3. 9:45 am
4. 19:30
5. 4 hours and 15 minutes
6. 1 hour

Lesson Title:	Problem Solving with Time
Practice Activity:	PHM-08-060

1. 3 hours and 15 minutes
2. 12:55
3. 5:15 pm
4. 2 hours and 30 minutes
5. 19:35
6. 8:30am

Lesson Title:	Perimeter and Area of Rectangles and Squares
Practice Activity:	PHM-08-061

1. a. $P = 78\text{ m}, A = 350\text{ m}^2$; b. $P = 24.8\text{ m}, A = 38.44\text{ m}^2$
2. $P = 56\text{ cm}, A = 196\text{ cm}^2$
3. $P = 16\text{ m}, A = 15\text{ m}^2$
4. The square
5. 8 cm
6. 264 metres

Lesson Title:	Perimeter and Area of Parallelograms
Practice Activity:	PHM-08-062

1. a. $A = 150\text{ cm}^2$; $P = 32\text{ cm}$
   b. $A = 24\text{ cm}^2$; $P = 24\text{ cm}$
2. 4.4 m^2
3. 110 m
4. $A = 63\text{ m}^2$, $P = 33.2\text{ m}$
5. $A = 175\text{ m}^2$

Lesson Title:	Perimeter and Area of Trapeziums
Practice Activity:	PHM-08-063

1. $P = 36.5\text{ m}, A = 66\text{ m}^2$
2. 52.8 cm^2
3. $P = 53\text{ cm}, A = 105\text{ cm}^2$
4. $P = 207\text{ mm}, A = 2210\text{ mm}^2$
5. 88 cm^2

Lesson Title:	Perimeter and Area of Triangles
Practice Activity:	PHM-08-064

1. a. $110\ \text{cm}^2$; b. $45\ \text{cm}^2$
2. a. $P = 48\ \text{cm}$, $A = 85.2\ \text{cm}^2$; b. $P = 125\ \text{mm}$, $A = 500\ \text{mm}^2$
3. 62 m
4. 36 m

Lesson Title:	Perimeter and Area of Circles
Practice Activity:	PHM-08-065

1. $C =\ 44\ \text{cm}$, $A = 154\ \text{cm}^2$
2. $C =\ 628\ \text{cm}$
3. $C =\ 220\ \text{cm}$
4. $A = 28.3\ \text{cm}^2$
5. $A = 50.29\ \text{m}^2$
6. $C =\ 440\ \text{cm}$
7. $C =\ 176\ \text{m}$; $A = 2{,}464\ \text{m}^2$
8. $C =\ 188.4\ \text{cm}$; $A = 2{,}826\ \text{cm}^2$
9. $115.5\ \text{cm}^2$

Lesson Title:	Perimeter and Area of Composite Shapes
Practice Activity:	PHM-08-066

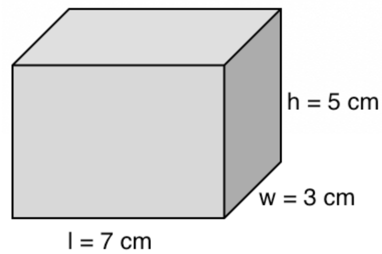
1. $A = 77\ \text{cm}^2$; $P = 44\ \text{cm}$
2. $A = 98.24\ \text{cm}^2$; $P = 37.12\ \text{cm}$
3. $A = 100\ \text{m}^2$; $P = 54\ \text{m}$
4. $A = 67\ \text{cm}^2$; $P = 36\ \text{cm}$

Lesson Title:	Perimeter and Area Story Problems
Practice Activity:	PHM-08-067

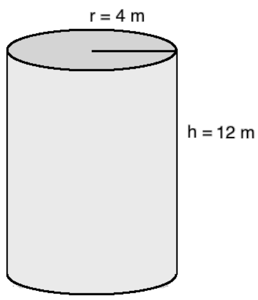
1. $P = 62\ \text{m}$
2. a. $P = 110\ \text{m}$; b. Le 440,000.00
3. a. $A = 707.1\ \text{m}^2$; b. $P = 94.3\ \text{m}$
4. $P = 24\ \text{m}$
5. a. $A = 120\ \text{m}^2$; b. 3,000 tiles
6. 5 : 3
7. 2,500 slabs

Lesson Title:	Volume of Solids
Practice Activity:	PHM-08-068

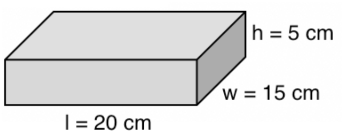
1. $V = l \times w \times h$ or $V = A \times h$
2. $V = \pi r^2 h$ or $V = A \times h$
3. Rectangular solid:



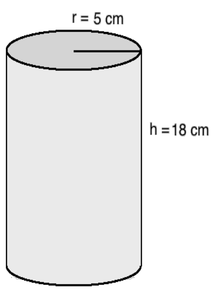
4. Cylinder:



5. Dictionary:



6. a. see below; b. cm^3



7. a. cm^3; b. cm^3; c. m^3; d. cm^3; d. ft^3; e. ft^3
8. a. area; b. volume; c. volume; d. area; e. area

Lesson Title:	Volume of Cubes
Practice Activity:	PHM-08-069

1. 343 cm^3
2. 15.6 m^3
3. 729 cm^3

4. 512 m^3
5. 2 ft

Lesson Title:	Volume of Rectangular Prisms
Practice Activity:	PHM-08-070

1. a. 160 cm^3; b. 280 cm^3
2. 78 m^3
3. 120,000 cm^3
4. 6 m^3
5. 45 packets

Lesson Title:	Volume of Triangular Prisms
Practice Activity:	PHM-08-071

1. a. 288 cm^3
   b. 18 cm^3

2. 42 m^3

3. 300 cm^3

4. a. 18 cm^2 b. 270 cm^3

Lesson Title:	Volume of Cylinders
Practice Activity:	PHM-08-072

1. a. 402 cm^3
   b. 367 cm^3

2. 6,160 mm^3

3. 276.3 cm^3

4. 539 cm^3

5. 123.2 litres

Lesson Title:	Volume of Composite Solids
Practice Activity:	PHM-08-073

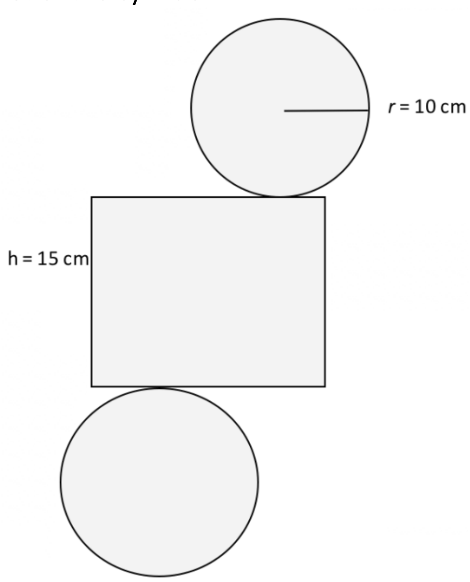
1. 78 cm^3
2. 1,430 cm^3
3. 1,832 cm^3

Lesson Title:	Volume Story Problems
Practice Activity:	PHM-08-074

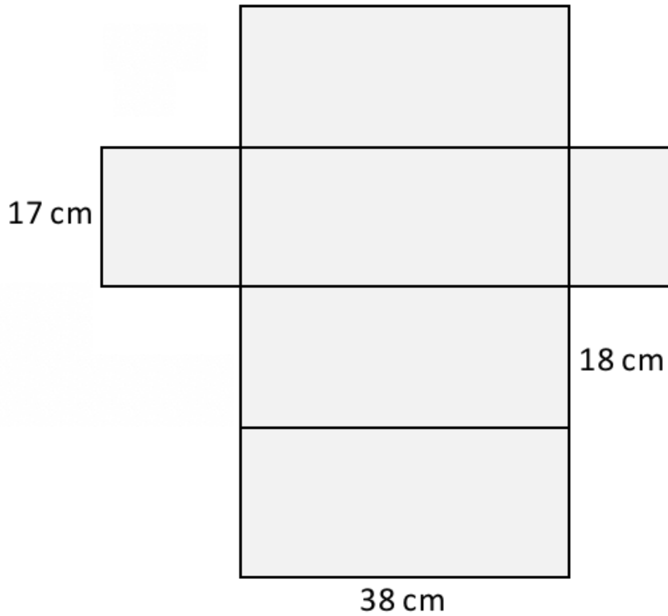
1. 924 cm^3
2. a. 2.4 m^3; b. 480 kg
3. 500 packs of biscuits
4. Le 15,400.00
5. 1,005 cm^3

Lesson Title:	Surface Area of Solids
Practice Activity:	PHM-08-075

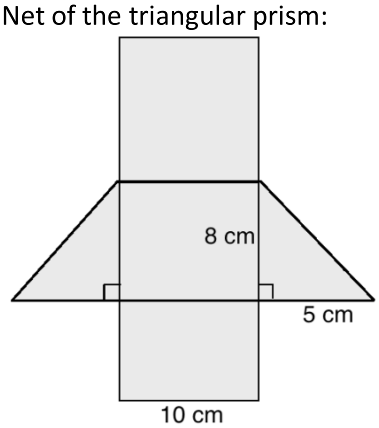
1. Net of the cylinder:



2. Net of the rectangular prism:



3. Net of the triangular prism:



4. cm^2
5. m^2

Lesson Title: Surface Area of Cubes and Rectangular Prisms
Practice Activity: PHM-08-076

1. 142 m^2
2. 648 cm^2
3. 96 cm^2
4. 864 mm^2

Lesson Title:	Surface Area of Triangular Prisms
Practice Activity:	PHM-08-077

1. (a) 336 cm^2
   (b) 132 m^2

2. 620 cm^2

Lesson Title:	Surface Area of Cylinders
Practice Activity:	PHM-08-078

1. a. 301 cm^2; b. 722 m^2
2. a. 1,408 cm^2; b. 748 cm^2

Lesson Title:	Surface Area of Composite Solids
Practice Activity:	PHM-08-079

1. 118 cm^2
2. 853 cm^2
3. 926 cm^2

Lesson Title:	Surface Area Story Problems
Practice Activity:	PHM-08-080

1. 66 m^2
2. 251.2 cm^2
3. 1,300 cm^2
4. a. 15.5 m^2; b. It will take 5.167 cans; the painter probably has full cans, so he should bring 6 cans with him to cover the sign.

Lesson Title:	Introduction to Angles
Practice Activity:	PHM-07-081

1. a. right; b. acute; c. acute; d. obtuse; e. straight; f. reflex
2. a. Two hundred and eighty degrees
   b. Eighty point three nine degrees
   c. Sixteen and a half degrees **or** Sixteen point five degrees
3. a. 55°; b. 66.5°; c. 90.4°
4. a. acute; b. obtuse; c. reflex; d. acute; e. obtuse

Lesson Title:	Measurement of Angles
Practice Activity:	PHM-07-082

1. a. $50°$, acute; b. $125°$, obtuse; c. $\angle XOY = 40°$, acute; d. $\angle SOR = 139°$, obtuse
2. a. $67°$; b. $105°$; c. $30°$

Lesson Title:	Finding Unknown Angles in Triangles
Practice Activity:	PHM-07-083

1. a. $x = 67°$; b. $x = 62°$; c. $x = 60°$
2. $b = 53°$
3. $p = 18°$
4. $A = B = 55°$

Lesson Title:	Finding Unknown Angles in Quadrilaterals
Practice Activity:	PHM-07-084

1. $m = 50°$
2. $100°$
3. $P = 115°, T = 65°, R = 115°$
4. $140°$

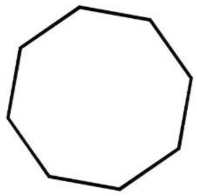
Lesson Title:	Angle Practice
Practice Activity:	PHM-07-085

1. $a = 128°$
2. $x = 140°, y = 40°, z = 140°$
3. $x = 77°$
4. $b = 48°$
5. $s = 71°$
6. $p = 117°$

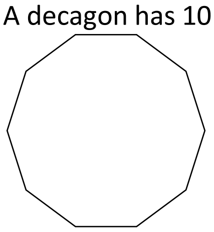
Lesson Title:	Polygons
Practice Activity:	PHM-07-086

Your drawings may look different than these. Make sure your shapes have the correct number of sides and angles.

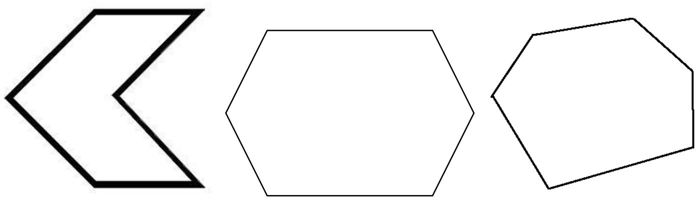
1. An octagon has 8 sides:



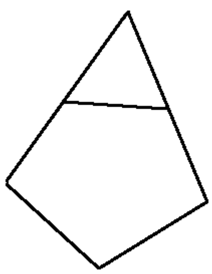
2. A decagon has 10 sides:



3. A hexagon has 6 sides. These are 3 examples:



4. Example:



Lesson Title:	Sum of the Interior Angles of a Pentagon
Practice Activity:	PHM-07-087

1. Sum of interior angles $= 180°(n-2)$, where n is the number of sides
2. a. $120° + 120° + 105° + 115° + 80° = 540°$; b. $90° + 90° + 132° + 132° + 96° = 540°$
3. a. $360°$; b. $540°$; c. $180°$; d. $360°$

Lesson Title:	Sum of the Interior Angles of a Polygon
Practice Activity:	PHM-07-088

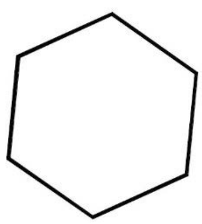
1. 1,260°
2. 1,080°
3. Pentagon
4. 900°
5. Nonagon
6. 3,240°

Lesson Title:	Interior Angle Practice
Practice Activity:	PHM-07-089

1. a. $y = 125°$; b. $x = 125°$; c. $a = 110°$
2. $144°$
3. $128.6°$
4. $135°$

Lesson Title:	Interior Angle Story Problems
Practice Activity:	PHM-07-090

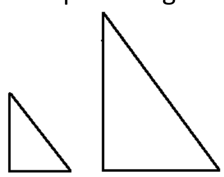
1. a. See shape below; b. 720°; c. 120°



2. 105°
3. 1080°
4. a. 36°; b. 144°

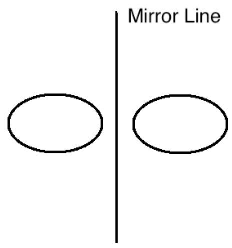
Lesson Title: Introduction to Transformation	
Practice Activity: PHM-08-091	

1. Example triangles:

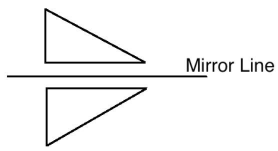


2.

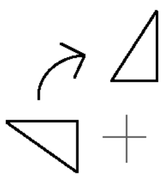
a.



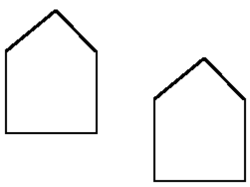
b.



3. Rotation:

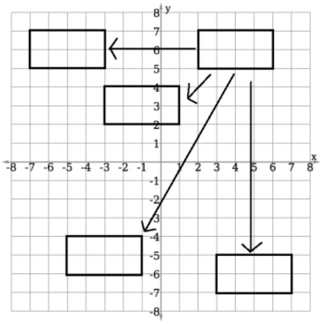


4. Example pentagons:

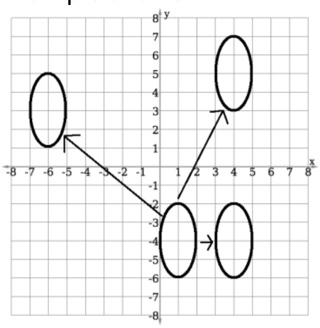


Lesson Title: Translation	
Practice Activity: PHM-08-092	

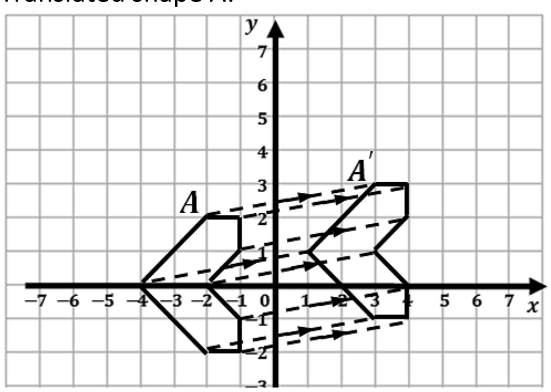
1. Example answer:



2. Example answer:

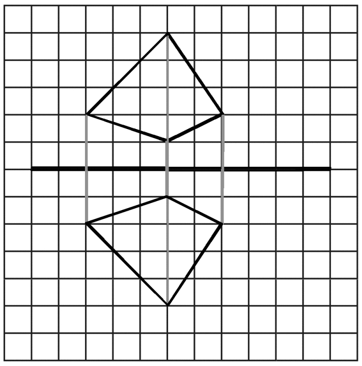


3. Translated shape A:



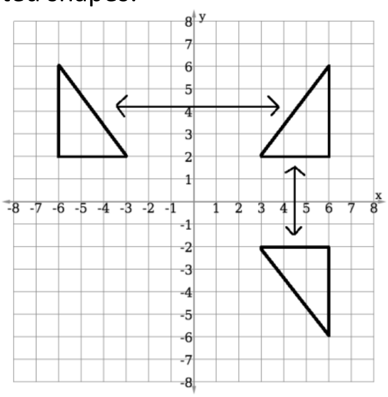
Lesson Title:	Reflection
Practice Activity:	PHM-08-093

1. Reflected shape:

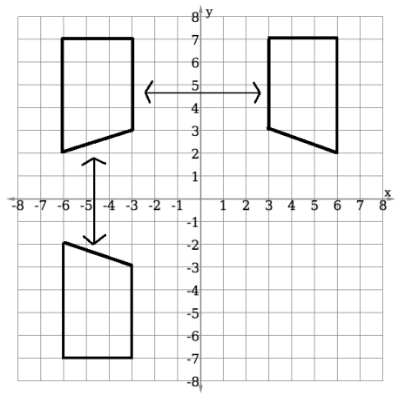


2. Reflected shapes:

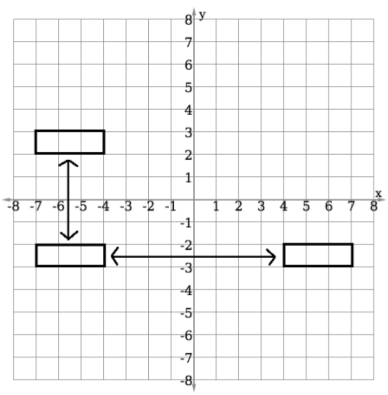
a.



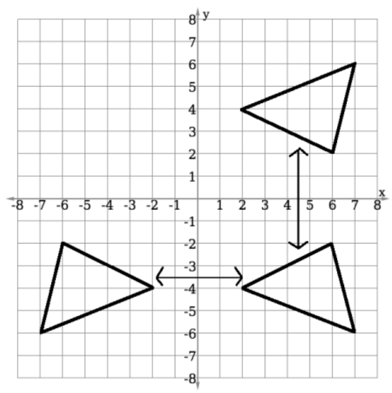
b.



c.

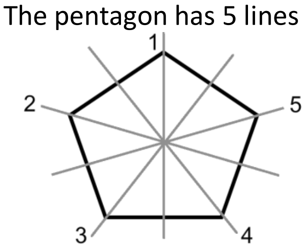


d.

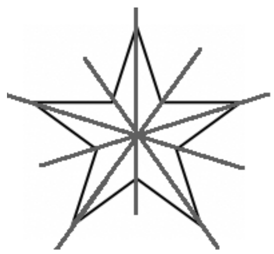


Lesson Title:	Line Symmetry
Practice Activity:	PHM-08-094

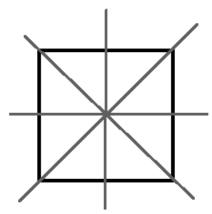
1. a. No; b. Yes; c. No; d. Yes
2. The pentagon has 5 lines of symmetry:



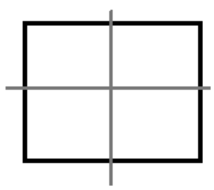
3. The lines of symmetry are shown below:
   a. Star:



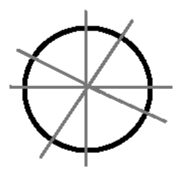
   b. Square:



   c.

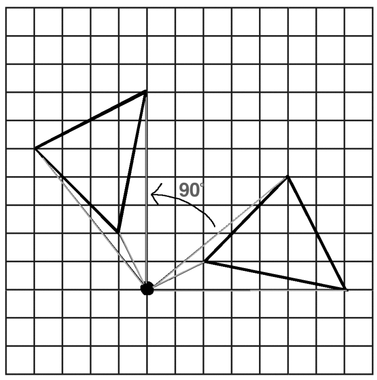


   d. A circle has infinitely many lines of symmetry. Any line passing through its centre is a line of symmetry. For example:



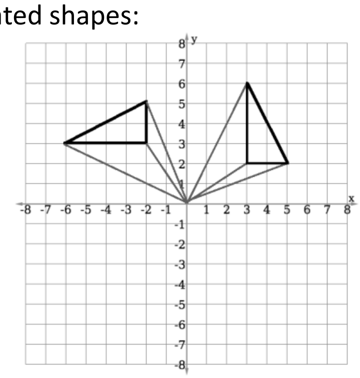
Lesson Title: Rotation	
Practice Activity: PHM-08-095	

1. Rotated triangle:

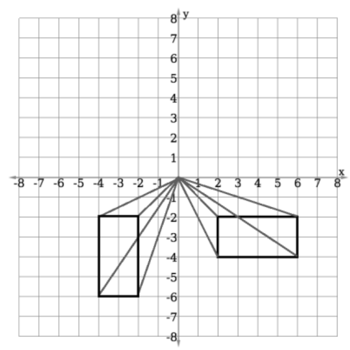


2. Rotated shapes:

a.



b.

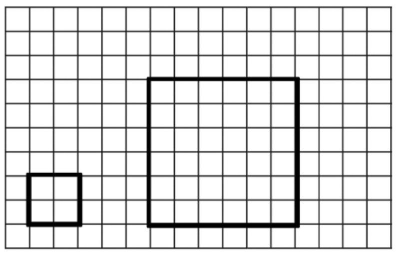


Lesson Title: Rotational Symmetry	
Practice Activity: PHM-08-096	

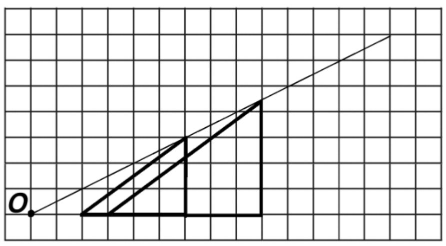
1. Yes; order 4
2. Yes; order 2
3. Yes; order 6
4. Yes; order 2
5. No; order 1
6. Yes; order 2
7. Yes; order 3
8. No; order 1

Lesson Title: Enlargement
Practice Activity: PHM-08-097

1. B is an enlargement of A with scale factor $\frac{1}{2}$. E is an enlargement of A with a scale factor of $1\frac{1}{2}$.
2. Shapes 2 and 5 are not enlargements of 1.
3. Enlarged square:

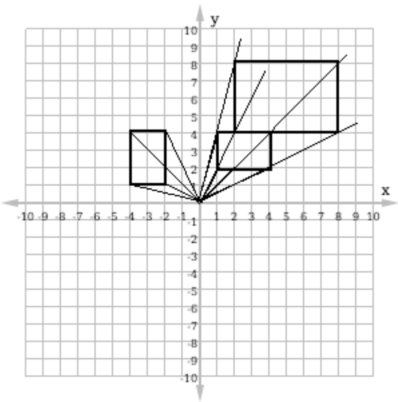


4. Enlarged triangle:

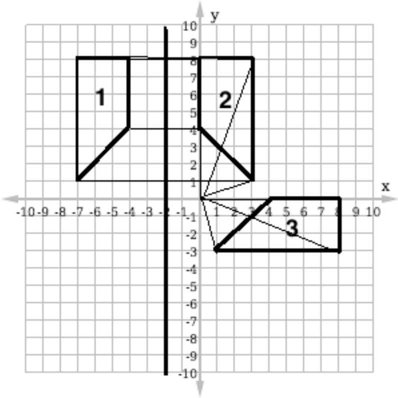


Lesson Title: Combining Transformations
Practice Activity: PHM-08-098

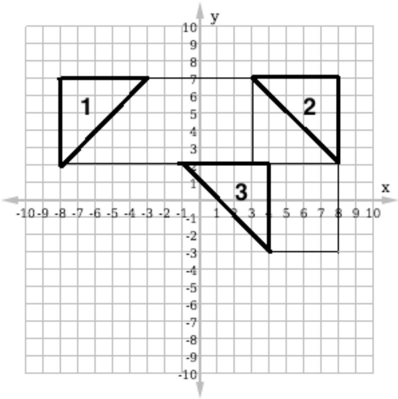
1. The final shape is **similar** to the original. Transformations are shown below:



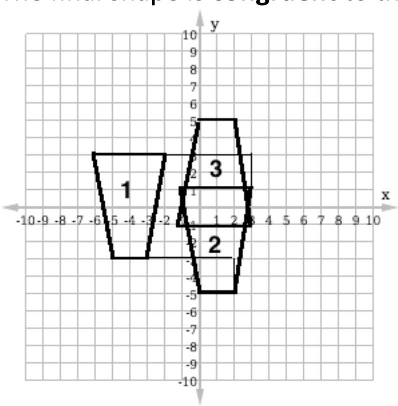
2. The final shape is **congruent** to the original. Transformations are shown below:



3. The final shape is **congruent** to the original. Transformations are shown below:



4. The final shape is **congruent** to the original. Transformations are shown below:



Lesson Title:	Applying Scale Factor to Drawing
Practice Activity:	PHM-08-099

The answers to this lesson should be accurate drawings of real-life objects that you find. The drawing for problem 1 is larger than the original object. The drawing for problem 2 is smaller than the original object.

Lesson Title:	Practical Applications of Scale
Practice Activity:	PHM-08-100

The answers to this lesson should be accurate maps of your house and yard. Check the size of the objects and the distance between objects to make sure they are drawn accurately.

Lesson Title:	Arithmetic Patterns
Practice Activity:	PHM-08-101

1. a. Arithmetic pattern with common difference 8
   b. Arithmetic pattern with common difference 3
   c. Not an arithmetic pattern
   d. Arithmetic pattern with common difference 2
   e. Not an arithmetic pattern
   f. Arithmetic pattern with common difference -4
   g. Arithmetic pattern with common difference -3

2. a. Common difference: 6; Pattern: 6, 12, <u>18</u>, 24, 30, <u>36</u>, <u>42</u>, 48
   b. Common difference: 3; Pattern: 3, 6, 9, <u>12</u>, <u>15</u>, <u>18</u>
   c. Common difference: 5; Pattern: 35, 40, <u>45</u>, 50, <u>55</u>, <u>60</u>, 65
   d. common difference: 5; pattern: 7, 12, 17, <u>22</u>, <u>27</u>, 32
   e. common difference: -4; pattern: 15, 11, <u>7</u>, 3, -1, <u>-5</u>

Lesson Title:	Creating Arithmetic Patterns
Practice Activity:	PHM-08-102

1. 3, 8, 13, 18, 23, 28, 33
2. 10, 16, 22, 28, 34
3. 3, 6, 9, 12, 15, …
4. 5, 9, 13, 17, 21, …
5. 7, 5, 3, 1, -1

Lesson Title:	Introduction to Geometric Patterns
Practice Activity:	PHM-08-103

1. Geometric pattern with common ratio 2
2. Not a geometric pattern
3. Geometric pattern with common ratio -1
4. Geometric pattern with common ratio 2
5. Geometric pattern with common ratio 10
6. Not a geometric pattern
7. Geometric pattern with common ratio -3

8. Not a geometric pattern
9. Not a geometric pattern
10. Geometric pattern with common ratio 4

Lesson Title:	Terms of Geometric Patterns
Practice Activity:	PHM-08-104

1. Common ratio: 2; Pattern: 2, 4, 8, 16, 32, …
2. Common ratio: 3; Pattern: 10, 30, 90, 270, 810, …
3. Common ratio: 3; Pattern: -1, -3, -9, -27, -81, …
4. Common ratio: -2; Pattern: -1, 2, -4, 8, -16, …
5. Common ratio: -1; Pattern: -10, 10, -10, 10, -10, 10, …
6. Common ratio: 2; Pattern: 7, 14, 28, 56, 112, 224, …
7. Common ratio: 3; Pattern: -2, -6, -18, -54, -162, …
8. Common ratio: 10; Pattern: 1, 10, 100, 1,000, 10,000, 100,000, …
9. Common ratio: -3; Pattern: 2, -6, 18, -54, 162, …
10. Common ratio: 3; Pattern: 1, 3, 9, 27, 81, 243, …

Lesson Title:	Creating Geometric Patterns
Practice Activity:	PHM-08-105

1. 3, 30, 300, 3,000, 30,000, …
2. $-1, 2, -4, 8, \ldots$
3. 3, 9, 27, 81, …
4. $2, -10, 50, -250, \ldots$
5. $-2, -8, -32, -128$

Lesson Title:	Simplifying Algebraic Expressions
Practice Activity:	PHM-08-106

1. $2x + y$
2. $-2a + 7b$
3. $5x + 2y + 3$
4. $5a + 4b - 7$
5. $m + 14n$
6. $3m + 6n$
7. $4a + 5b + 1$
8. $7n + p - 2$
9. $x + 9xy - 3$
10. $68a + ab - 1$

Lesson Title:	Simplifying Expressions with Higher Powers
Practice Activity:	PHM-08-107

1. $4x^2 + x - 4$
2. $6x^2 + 2x + 6$
3. $-8x^2 + 22x - 8$
4. $6x^2 + 9x$
5. $9x^3 + 3x^2 + 10x$
6. $13x^2 + 7y^2 + x + 5y - 8$
7. $10x^2 + 4x + 9y - 5$
8. $-3x^3 + x^2 - 12x - 6$
9. $7x^2y + 3xy^2$
10. $3v^2w - v^2$

Lesson Title:	Simplifying Expressions with Fractions
Practice Activity:	PHM-08-108

1. $5\frac{1}{2}x^2 - x$
2. $\frac{1}{6}x + 2\frac{1}{2}$
3. $\frac{1}{2}x^2 + \frac{13}{15}x + \frac{1}{3}$
4. $1\frac{1}{6}x^2 + \frac{1}{3}x + 8\frac{1}{5}$
5. $\frac{1}{8}x^3 + \frac{1}{4}x^2 + \frac{1}{3}x + \frac{1}{6}$
6. $\frac{2}{5}x^2 + \frac{2}{3}x - \frac{1}{6}$

Lesson Title:	Multiplying an Algebraic Expression by an Integer
Practice Activity:	PHM-08-109

1. $5x - 20$
2. $-21y + 28$
3. $-2m - 2n$
4. $6v + 9$
5. $-2x^2 + x - 7$
6. $-24m + 16n$
7. $4a + 6$
8. $10x^2 - 30x$

Lesson Title: Multiplying Variables
Practice Activity: PHM-08-110

1. y^7
2. x^2y^3
3. $6xy^6$
4. $-6x^5$
5. $-x^5$
6. $10x^5y$
7. x^2y^2
8. $5x^3y^3$
9. $14x^5y^6$
10. $180p^8q^5$

Lesson Title: Multiplying an Algebraic Expression by a Variable
Practice Activity: PHM-08-111

1. $x^2 - 2x$
2. $4x - x^2$
3. $2x^2 - x$
4. $x^3 - 3x^2$
5. $-x^2 - 12x$
6. $-9x + x^2 - xy$
7. $3x^2 - 12x$
8. $-8x^2 - 2x$
9. $4x^3 + 4x^2$
10. $3a^2 + 12a^2b - 3ab$

Lesson Title: Simplifying and Expanding Algebraic Expressions
Practice Activity: PHM-08-112

1. $y^2 + 3y$
2. $17x - 5y - 5$
3. $x^2 - x$
4. $2x^2 - 2x$
5. $3y^2 - 2y$
6. $6z^3 + 4z$
7. $x^2 + xy + 5$
8. $x^2 + 5x + 14$
9. $2f - 2g$
10. $-28x + 10$

Lesson Title:	Algebraic Expression Story Problems
Practice Activity:	PHM-08-113

1. $1{,}500r + 5{,}000f$, where r is cups of rice and f is pieces of fish.
2. $r - 10$, where r is cups of rice.
3. $4m$, where m is the number of square metres.
4. $5x + 10$
5. $x + 11$
6. $2x + 6$
7. $3x - 2$
8. $3x + 3y$

Lesson Title:	Factoring Integers from Algebraic Expressions
Practice Activity:	PHM-08-114

1. $4(x + 3)$
2. $7(x - 3y)$
3. $2(7 - x)$
4. $10(2x + 3)$
5. $2(2y - 3)$
6. $2(5s + 4t)$
7. $6(2 - 3p)$
8. $3(x^2 + 4x + 10)$
9. $3(3x^2 - 4)$
10. $2(x^3 + 20x^2 + 6x + 12)$

Lesson Title:	Factoring Variables from Algebraic Expressions
Practice Activity:	PHM-08-115

1. $y(x + 1)$
2. $y(x + z)$
3. $a(2a - 1)$
4. $x(3x + 8)$
5. $y^2(y + 1)$
6. $x(x^2 + 7x - 3)$
7. $3x(x^2 + 3x - 6)$
8. $5x^2(x - 3)$
9. $5a(a + 2)$
10. $7x^2(2x + 1)$

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