Free Quality School Education

Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics*

SSS II

Term III

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

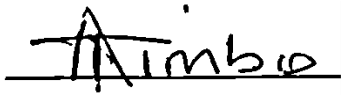
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction

to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

FACILITATION STRATEGIES

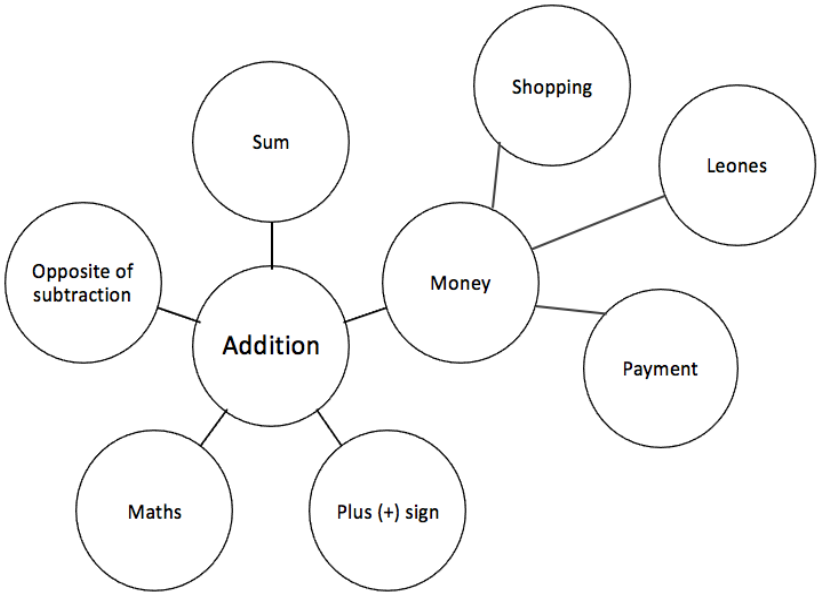
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I know' and 'What I want to know' about the topic. At the end of the lesson have pupils share 'What I learned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

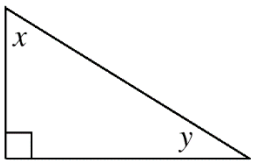
Lesson Title: Review of sine, cosine and tangent	**Theme:** Trigonometry	
Lesson Number: M2-L097	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify the trigonometric ratios (SOHCAHTOA).	**Preparation** None	

Opening *(4 minutes)*

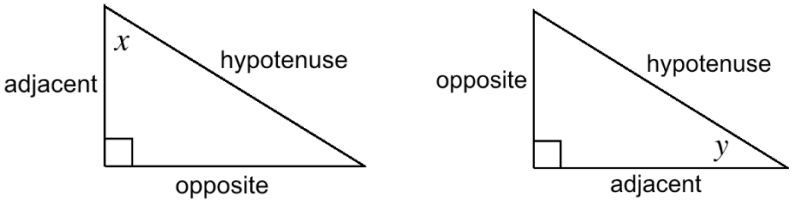
1. Discuss**:** What are some of the properties of a right-angled triangle?
2. Allow pupils to check their notes and Pupil Handbooks.
3. Allow volunteers to respond and discuss. (Example answers: Right-angled triangles have one 90^o angle; they have 2 acute angles which are also complementary angles as they add up to 90^o; they have one long side called the hypotenuse and 2 shorter sides)
4. Explain that today's lesson is on the trigonometric ratios for sine, cosine and tangent.

Teaching and Learning *(20 minutes)*

1. Draw a right-angled triangle on the board and label it as shown:



2. Ask the following questions, and invite volunteers come to the board and point out the correct side for each.
    - Which side is opposite angle x? Which side is adjacent to angle x?
    - Which side is opposite angle y? Which side is adjacent to angle y?
3. Draw the following on the board so that adjacent and opposite angles are clear:



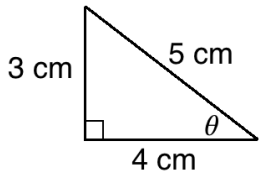
4. Explain:
    - We use the 3 types of sides (adjacent, opposite, and hypotenuse) in trigonometric ratios.
    - The 3 trigonometric ratios we are concerned with today are sine, cosine and tangent.
5. Write 3 trigonometric ratios on the board:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{O}}{\text{H}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{A}}{\text{H}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{O}}{\text{A}}$$

6. Explain:
    - Sin, cos, and tan are the abbreviations that we use for sine, cosine, and tangent in equations.
    - Trigonometric functions are functions of angles. The theta symbol (θ) is shown here, and it is often used to represent angles.
    - The trigonometric functions on the board relate an angle of a triangle to its sides.
7. Write on the board: SOHCAHTOA.
8. Explain:
    - We use the term SOHCAHTOA as a way of remembering the ratios.
    - SOH stands for “sine equals opposite over hypotenuse”.
    - CAH stands for “cosine equals adjacent over hypotenuse”.
    - TOA stands for “tangent equals opposite over adjacent”.
9. Draw the triangle on the board, labeled as shown:



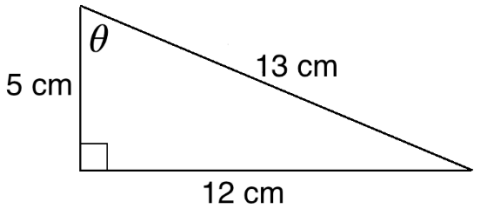
10. Apply the trigonometric ratios to angle θ the triangle. For each one, point out the relevant sides in the triangle on the board and make sure pupils understand.

$$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{3}{4}$$

11. Write the following problem on the board: Apply the trigonometric ratios to θ:



12. Ask pupils to work with seatmates to solve.
13. Invite 3 volunteers to each write a solution on the board.

    Solutions:

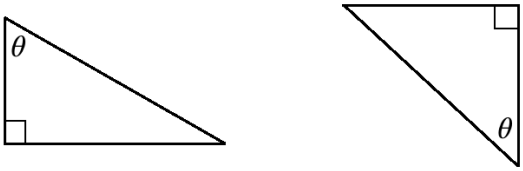
$$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{12}{13}$$

$$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{5}{12}$$

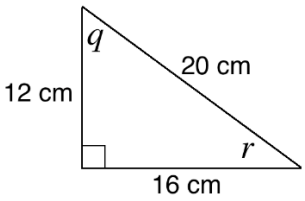
$$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{12}{13}$$

Practice *(15 minutes)*

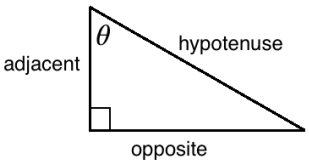
1. Write on the board:
    a. Copy the triangles below. Identify and label the adjacent and opposite angles relative to θ, and the hypotenuse.

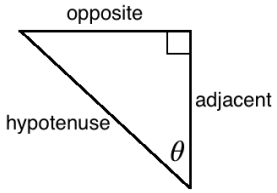


    b. For the triangle below, apply the trigonometric ratios to both angles q and r. Simplify your answers.



2. Ask pupils to work independently to solve the problems. Allow them to discuss with seatmates if needed.
3. Invite volunteers to write the solutions on the board and explain.
    Solutions:
    a.





    b.

$$\sin q = \frac{\text{O}}{\text{H}} = \frac{16}{20} = \frac{4}{5}$$

$$\cos q = \frac{\text{A}}{\text{H}} = \frac{12}{20} = \frac{3}{5}$$

$$\tan q = \frac{\text{O}}{\text{A}} = \frac{16}{12} = \frac{4}{3} = 1\frac{1}{3}$$

$$\sin r = \frac{\text{O}}{\text{H}} = \frac{12}{20} = \frac{3}{5}$$

$$\cos r = \frac{\text{A}}{\text{H}} = \frac{16}{20} = \frac{4}{5}$$

$$\tan r = \frac{\text{O}}{\text{A}} = \frac{12}{16} = \frac{3}{4}$$

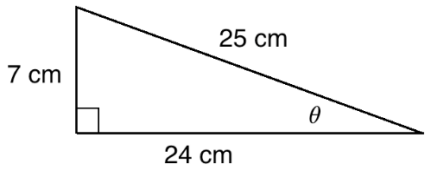
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L097 in the Pupil Handbook.

Lesson Title: Application of sine, cosine and tangent	**Theme:** Trigonometry	
Lesson Number: M2-L098	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the trigonometric ratios of tangent, sine and cosine to solve right-angled triangles, using trigonometric tables if available.	**Preparation** 1. Bring trigonometric tables if available. 2. Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Review the previous lesson. Write the following problem on the board: Write the sine, cosine, and tangent ratios for angle θ:



2. Solve as a class. Ask volunteers to give the answers, and write them on the board. Allow discussion.
   Answers:

$$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{7}{25}$$

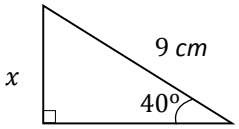
$$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{24}{25}$$

$$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{7}{24}$$

3. Explain that today's lesson is on applying the trigonometric ratios for sine, cosine and tangent to solve right-angled triangles.

Teaching and Learning *(20 minutes)*

1. Write the following problem on the board: Find the measure of missing side x:

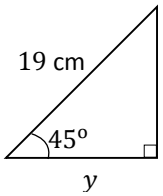


2. Discuss: Which trigonometric ratio can we use to solve this problem? Why? (Answer: Sine, because it is the ratio for opposite side and hypotenuse.)
3. Explain:
   - We use sine, because we are looking for the opposite side of the known angle. The side that we know is the hypotenuse.
   - Therefore, the ratio we can use is SOH ($\sin = \frac{\text{opposite}}{\text{hypotenuse}}$).
4. Solve on the board, explaining each step:

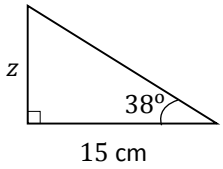
$$\sin\theta = \frac{\text{O}}{\text{H}}$$
$$\sin 40^\text{o} = \frac{x}{9}$$
$$9 \times \sin 40^\text{o} = x$$
$$9 \times 0.6428 = x$$
$$x = 5.7852$$
$$x = 5.8 \text{ cm to 1 d.p.}$$

5. Explain:
   - The trigonometric functions of angles often have many decimal places.
   - We can find them using calculators or trigonometric tables (sometimes called “log books”), then use them in our calculations.
   - Trigonometric tables give the trigonometric functions of angles to 4 decimal places.
6. Show pupils how to find $\sin 40^\text{o}$ using a trigonometric table if they are available.
   - Find 40° in the table. It is the first number given (0.6428), because there is no decimal on the degree.
7. Write 2 additional problems on the board:

   a. Find the measure of y:



   b. Find the measure of z:



8. Discuss:
   - Which trigonometric function will we use to solve a.? Why? (Answer: We will use cosine because it involves the adjacent side and hypotenuse.)
   - Which trigonometric function will we use to solve b.? Why? (Answer: We will use tangent because it involves the adjacent and opposite sides.)
9. Solve on the board, explaining each step to pupils. Show them how to use the trigonometric tables to find the trigonometric function of each angle.

   Solutions:

   a. Find the measure of y:

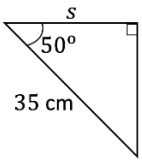
$$\cos\theta = \frac{\text{A}}{\text{H}}$$
$$\cos 45^\text{o} = \frac{y}{19}$$
$$19 \times \cos 45^\text{o} = y$$
$$9 \times 0.7071 = y$$
$$y = 13.4349$$
$$y = 13.4 \text{ cm to 1 d.p.}$$

   b. Find the measure of z:

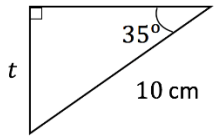
$$\tan\theta = \frac{\text{O}}{\text{A}}$$
$$\tan 38^\text{o} = \frac{z}{15}$$
$$15 \times \tan 38^\text{o} = z$$
$$15 \times 0.7813 = z$$
$$z = 11.7195$$
$$z = 11.7 \text{ cm to 1 d.p.}$$

10. Write the following problems on the board:

a. Find the measure of s:



b. Find the measure of t:



11. Discuss**:**
    - Which trigonometric function will we use for a.? (Answer: cosine)
    - Which trigonometric function will we use for b.? (Answer: sine)
12. Ask pupils to solve the problems with seatmates. If pupils do not have trigonometric tables or calculators, write on the board: $\cos 50^{\circ} = 0.6428$; $\sin 35^{\circ} = 0.5736$)
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solutions on the board.

Solutions:

a. Find the measure of s:

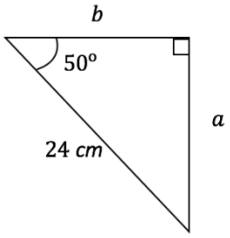
$$\cos \theta = \frac{\text{A}}{\text{H}}$$
$$\cos 50^{\circ} = \frac{s}{35}$$
$$35 \times \cos 50^{\circ} = s$$
$$35 \times 0.6428 = s$$
$$s = 22.498$$
$$s = 22.5 \text{ cm to 1 d.p.}$$

b. Find the measure of t:

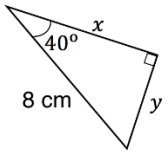
$$\sin \theta = \frac{\text{O}}{\text{H}}$$
$$\sin 35^{\circ} = \frac{t}{10}$$
$$10 \times \sin 35^{\circ} = t$$
$$10 \times 0.5736 = t$$
$$t = 5.736$$
$$t = 5.7 \text{ cm to 1 d.p.}$$

Practice *(15 minutes)*

1. Write on the board:
    a. Find the measures of sides a and b using the appropriate trigonometric ratios. Give your answers to 1 decimal place.



    b. Find the measures of sides x and y using the appropriate trigonometric ratios. Give your answers to 1 decimal place.



2. Ask pupils to work independently to solve the problems. Allow them to discuss with seatmates if needed.
3. Give pupils the trigonometric values needed to solve the problems if they do not have trigonometric tables:
    - $\sin 40^o = 0.6428$
    - $\sin 50^o = 0.7660$
    - $\cos 40^o = 0.7660$
    - $\cos 50^o = 0.6428$
4. Invite volunteers to write the solutions on the board and explain.
   Solutions:

a. Find the measure of a:

$$\sin \theta = \frac{\text{O}}{\text{H}}$$
$$\sin 50^o = \frac{a}{24}$$
$$24 \times \sin 50^o = a$$
$$24 \times 0.7660 = a$$
$$a = 18.384$$
$$a = 18.4 \text{ cm to 1 d.p.}$$

Find the measure of b:

$$\cos \theta = \frac{\text{A}}{\text{H}}$$
$$\cos 50^o = \frac{b}{24}$$
$$24 \times \cos 50^o = b$$
$$24 \times 0.6428 = b$$
$$b = 15.4272$$
$$b = 15.4 \text{ cm to 1 d.p.}$$

a. Find the measure of x:

$$\cos \theta = \frac{\text{A}}{\text{H}}$$
$$\cos 40^o = \frac{x}{8}$$
$$8 \times \cos 40^o = x$$
$$8 \times 0.7660 = x$$
$$x = 6.128$$
$$x = 6.1 \text{ cm to 1 d.p.}$$

Find the measure of y:

$$\sin \theta = \frac{\text{O}}{\text{H}}$$
$$\sin 40^o = \frac{y}{8}$$
$$8 \times \sin 40^o = y$$
$$8 \times 0.6428 = y$$
$$y = 5.1424$$
$$y = 5.1 \text{ cm to 1 d.p.}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L098 in the Pupil Handbook.

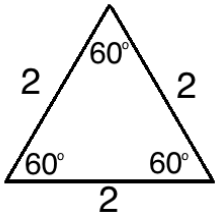
Lesson Title: Special angles (30°, 45°, 60°)	**Theme:** Trigonometry	
Lesson Number: M2-L099	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Derive and identify the trigonometric ratios of special angles 30°, 45°, and 60°. 2. Identify that $\tan\theta = \frac{\sin\theta}{\cos\theta}$.	**Preparation** None	

Opening *(3 minutes)*

1. Review equilateral and isosceles triangles.
2. Ask pupils to draw 1 equilateral triangle and 1 isosceles triangle in their exercise books.
3. Ask them to compare with seatmates.
4. Discuss:
    - What are the characteristics of an equilateral triangle? (Example answers: all 3 sides are equal, all 3 angles are equal, angles are all 60°.)
    - What are the characteristics of an isosceles triangle? (Example answers: 2 sides are equal, 2 angles are equal.)
5. Explain that today's lesson is on identifying the trigonometric ratios of some common angles using triangles.

Teaching and Learning *(25 minutes)*

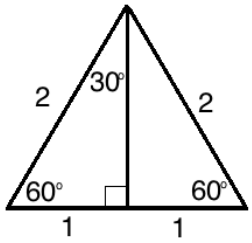
1. Explain:
    - It is useful to know the trigonometric ratios for some common angles, including 30°, 45°, and 60°.
    - Using triangles, we can find these ratios as numbers that are easy to remember. These will be easier to recall and use than long decimal numbers.
2. Draw an equilateral triangle on the board, and label its angles and sides as shown:



3. Discuss: How can I get a 30-degree angle from this triangle?
4. Allow pupils to discuss, then explain:

- We can bisect a 60-degree to get a 30-degree angle.
- If we bisect the angle of an equilateral triangle, this will form a 90-degree angle with the opposite side.
- This line also bisects the opposite side, so that it creates 2 equal segments.
- We will have a right-angled triangle that we can apply trigonometric ratios to.

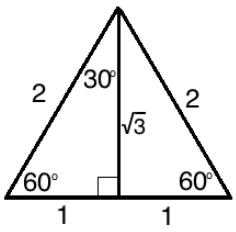
5. Sketch the angle bisector and relabel the equilateral triangle on the board so that it is as shown:



6. Discuss: How can we find the height of this triangle? (Answer: The easiest way is to use Pythagoras' theorem.)
7. Ask pupils to work with seatmates to find the height of the triangle, and leave their answer as a surd.
8. Invite a volunteer to write the solution on the board.
   Solution:

$$
\begin{aligned}
h^2 + 1^2 &= 2^2 && \text{Substitute 1 and 1 into the formula} \\
h^2 + 1 &= 4 && \text{Simplify} \\
h^2 &= 4 - 1 \\
h^2 &= 3 \\
\sqrt{h^2} &= \sqrt{3} && \text{Take the square root of both sides} \\
h &= \sqrt{3}
\end{aligned}
$$

9. Label the height of the triangle:



10. Discuss: How can we find the trigonometric ratios for 30° and 60° from this triangle?
11. Allow pupils to share ideas, then explain: Use the right-angled triangle on the left side, and apply the 3 trigonometric ratios to both the 30° angle and the 60° angle.
12. Find the ratios for the 30° angle as a class, and write them on the board:

$$
\begin{aligned}
\sin 30° &= \frac{O}{H} = \frac{1}{2} \\
\cos 30° &= \frac{A}{H} = \frac{\sqrt{3}}{2}
\end{aligned}
$$

$$\tan 30° = \frac{O}{A} = \frac{1}{\sqrt{3}}$$

13. Ask pupils to work with seatmates to find the ratios for the 60° angle.
14. Walk around to check for understanding and clear misconceptions.
15. Invite volunteers to write the ratios on the board.

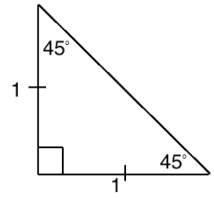
Solutions:

$$\sin 60° = \frac{O}{H} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{A}{H} = \frac{1}{2}$$

$$\tan 60° = \frac{O}{A} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

16. Draw the isosceles triangle shown below on the board:



17. Ask pupils to work with seatmates to find the length of the hypotenuse using Pythagoras' theorem.
18. Invite a volunteer to write the solution on the board.

Solution:

$$1^2 + 1^2 = c^2 \quad \text{Substitute 1 and 1 into the formula}$$

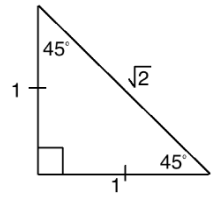
$$1 + 1 = c^2 \quad \text{Simplify}$$

$$2 = c^2$$

$$\sqrt{2} = \sqrt{c^2} \quad \text{Take the square root of both sides}$$

$$\sqrt{2} = c$$

19. Label the hypotenuse as shown:



20. Discuss: How can we find the trigonometric ratios for 45° from this triangle?
21. Allow pupils to share ideas, then explain: Use either 45° angle, and apply the trigonometric ratios.
22. Ask pupils to work with seatmates to write the 3 trigonometric ratios for 45°.
23. Invite volunteers to write the ratios on the board.

Solutions:

$$\sin 45° = \frac{O}{H} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{A}{H} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{O}{A} = \frac{1}{1} = 1$$

24. Explain:
    - We have now found the trigonometric ratios of 3 special angles.
    - From the special ratios on the board, we can observe a relationship between the trigonometric functions.
25. Write on the board: $\tan\theta = \frac{\sin\theta}{\cos\theta}$
26. Explain: For any angle, the tangent is equal to the sine divided by the cosine.
27. Show that this is true on the board using ratios for special angle 30°:

$$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

28. Explain: The result we found when we divided sine by cosine is the same as the result we found for $\tan 30°$ when using the triangle.

Practice *(10 minutes)*

1. Write on the board: Show the following by dividing the ratios of the special angles:
    a. $\tan 60° = \frac{\sin 60°}{\cos 60°}$
    b. $\tan 45° = \frac{\sin 45°}{\cos 45°}$
2. Ask pupils to work with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.
   Solutions:
    a. $\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \frac{\sqrt{3}}{1} = \sqrt{3}$
    b. $\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

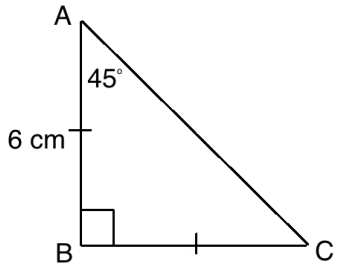
Closing *(2 minutes)*

1. Explain:
    - The ratios for special angles that you found today will be used in the next lesson to solve problems.
    - You will also be able to use the relationship of tangent to sine and cosine in future lessons.
2. For homework, have pupils do the practice activity PHM2-L099 in the Pupil Handbook.

Lesson Title: Applying special angles	**Theme:** Trigonometry	
Lesson Number: M2-L100	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use the special angles 30°, 45°, and 60° to solve problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the problem shown below on the board: Find the lengths of sides BC and AC:



2. Discuss: How can we find the measures of lengths AC and BC? (Examples: Use trigonometry; use the tangent and sine functions; use what we know about special angle 45°.)
3. Explain that today's lesson is on using special angles to solve problems.

Teaching and Learning *(20 minutes)*

1. Ask pupils to give the trigonometric functions for 45°. Allow them to look at their Pupil Handbook and notes.
2. Write the functions on the board:

$$\sin 45° = \frac{1}{\sqrt{2}}$$
$$\cos 45° = \frac{1}{\sqrt{2}}$$
$$\tan 45° = 1$$

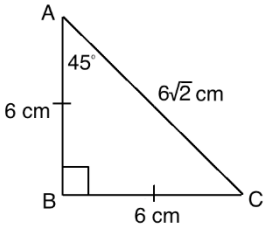
3. Explain: We will use these to find the measures of the missing sides.
4. Discuss:
   - Which trigonometric function can we use to find BC? (Answer: tangent)
   - Which trigonometric function can we use to find AC? (Answer: cosine)
5. Find the length of BC on the board, explaining each step:
   Step 1. Set $\tan 45°$ equal to the ratio from the triangle in the problem, and the ratio $\tan 45° = 1$ we found in the previous lesson:

$$\tan 45° = \frac{|BC|}{6} = 1$$

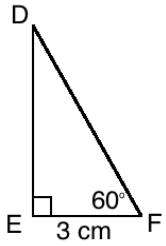
   Step 2. Multiply both sides by 6 to solve for $|BC|$:

$$|BC| = 6$$

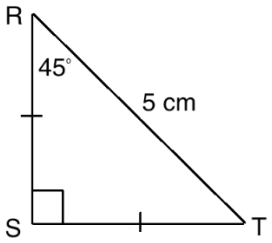
6. Find the length of AC on the board, explaining each step:
   Step 1. Set $\cos 45°$ equal to the ratio from the triangle in the problem, and the ratio of $\cos 45°$ we found in the previous lesson:
   $$\cos 45° = \frac{6}{|AC|} = \frac{1}{\sqrt{2}}$$
   Step 2. Cross-multiply to solve for $|AC|$:
   $$|AC| = 6\sqrt{2}$$
7. Label the sides of the triangle on the board:

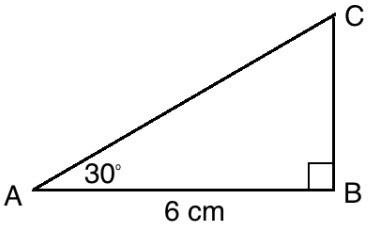


8. Write the problem shown below on the board: Find the measures of DE and DF:



9. Discuss:
   - How can we find the measure of DE? (Answer: Apply the tangent function to 60°.)
   - How can we find the measure of DF? (Answer: Apply the cosine function to 60°.)
10. Ask pupils to give the ratios for $\tan 60°$ and $\cos 60°$. Write them on the board. (Answers: $\tan 60° = \sqrt{3}$; $\cos 60° = \frac{1}{2}$)
11. Find the length of DE on the board, explaining each step:
    $$\tan 60° = \frac{|DE|}{3} = \sqrt{3}$$
    $$|DE| = 3\sqrt{3}$$
12. Find the length of DF on the board, explaining each step:
    $$\cos 60° = \frac{3}{|DF|} = \frac{1}{2}$$
    $$|DF| = 3 \times 2$$
    $$|DF| = 6$$
13. Write the following problems on the board:
    a. Find $|RS|$ and $|ST|$.
    b. Find $|AC|$ and $|BC|$.





14. Ask pupils to work with seatmates to find the solutions.
15. Invite volunteers to write the solutions on the board.

Solutions:

a. Find $|RS|$:

$$\cos 45° = \frac{|RS|}{5} = \frac{1}{\sqrt{2}}$$
$$|RS| \times \sqrt{2} = 5$$
$$|RS| = \frac{5}{\sqrt{2}}$$

Find $|ST|$:

$$\sin 45° = \frac{|ST|}{5} = \frac{1}{\sqrt{2}}$$
$$|ST| \times \sqrt{2} = 5$$
$$|ST| = \frac{5}{\sqrt{2}}$$

b. Find $|AC|$:

$$\cos 30° = \frac{6}{|AC|} = \frac{\sqrt{3}}{2}$$
$$|AC| \times \sqrt{3} = 6 \times 2$$
$$|AC| = \frac{12}{\sqrt{3}}$$

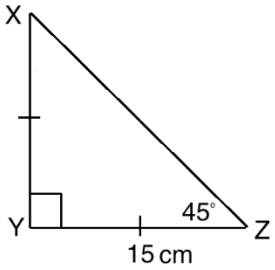
Find $|BC|$:

$$\tan 30° = \frac{|BC|}{6} = \frac{1}{\sqrt{3}}$$
$$|BC| \times \sqrt{3} = 6$$
$$|BC| = \frac{6}{\sqrt{3}}$$

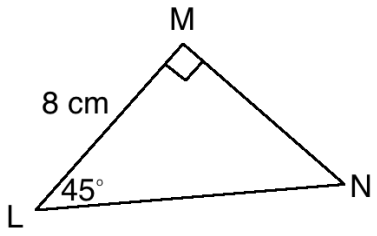
Practice *(15 minutes)*

1. Write on the board: Find the lengths of the missing sides of the triangles:

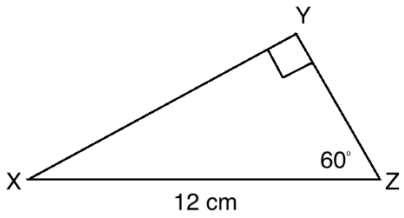
a.



b.



c.



2. Ask pupils to work with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.

4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.
   Solutions:

   a. Find $|XY|$:

   $$\tan 45° = \frac{|XY|}{15} = 1$$
   $$|XY| = 15$$

   Find $|XZ|$:

   $$\cos 45° = \frac{15}{|XZ|} = \frac{1}{\sqrt{2}}$$
   $$|XZ| = 15\sqrt{2}$$

   b. Find $|LN|$:

   $$\cos 45° = \frac{8}{|LN|} = \frac{1}{\sqrt{2}}$$
   $$|LN| = 8\sqrt{2}$$

   Find $|MN|$:

   $$\tan 45° = \frac{|MN|}{8} = 1$$
   $$|MN| = 8$$

   c. Find $|XY|$:

   $$\sin 60° = \frac{|XY|}{12} = \frac{\sqrt{3}}{2}$$
   $$2|XY| = 12\sqrt{3}$$
   $$|XY| = \frac{12\sqrt{3}}{2}$$
   $$= 6\sqrt{3}$$

   Find $|YZ|$:

   $$\tan 60° = \frac{|YZ|}{12} = \sqrt{3}$$
   $$|YZ| = 12\sqrt{3}$$

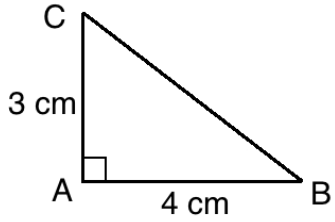
Closing *(2 minutes)*

1. Discuss: What did you notice about the triangles with 45-degree angles that we solved today?
2. Allow pupils to share ideas, then explain:
   - They are isosceles triangles, so 2 sides are the same length.
   - The hypotenuse is the length of the other sides multiplied by $\sqrt{2}$.
   - It is not always necessary to do calculations. Sometimes we can find the side lengths through observation.
3. For homework, have pupils do the practice activity PHM2-L100 in the Pupil Handbook.

Lesson Title: Inverse trigonometry	**Theme:** Trigonometry	
Lesson Number: M2-L101	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify that inverse trigonometric functions 'undo' the corresponding trigonometric functions 2. Apply inverse trigonometric functions to find unknown angles	**Preparation** Bring trigonometric tables and/or a calculator with inverse trigonometric functions if they are available. Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: Find the measure of angle B:



2. Discuss:
    a. Is there enough information to find the measure of B?
    b. Can we find the measure of an angle using trigonometry?
3. Allow pupils to share ideas, then explain: We have previously used trigonometry to find the measures of missing sides. Today you will see how we can use it to find the measures of missing angles using the lengths of 2 sides.
4. Explain that today's lesson is on inverse trigonometry.

Teaching and Learning *(30 minutes)*

1. Explain:
    - The inverse of a function is its opposite. It's another function that can undo the given function.
    - Inverse functions are shown with a power of -1.
2. Write on the board:
    Inverse sine: $\sin^{-1} x$
    Inverse sine 'undoes' sine: $\sin^{-1}(\sin\theta) = \theta$
3. Write the other inverse functions on the board: $\cos^{-1} x$, $\tan^{-1} x$
4. Explain: The inverse functions are also sometimes called "arcsine", "arccosine", and "arctangent".
5. Explain:
    - You can use inverse trigonometric functions to find the degree measure of an angle.
    - You can use the trigonometric tables (log books) or calculators.

- Using trigonometric tables, you will work backwards. Find the decimal number in the chart, and identify the angle that it corresponds to.

6. Write the following 2 problems on the board: Calculate the following:
   a. $\sin^{-1}(0.5015)$
   b. $\cos^{-1}(0.7891)$
7. If you have trigonometric tables available, demonstrate how to solve these problems using them. If you have a calculator available, you may solve using that too.
   a. Solution using a **sine table:**
      i. Find 0.5015 in the trigonometric table for sine.
      ii. It is in row 31, under the first column (.0). This means that the angle has measure 31.0°.
   b. Solution using a **cosine table:**
      - Look for 0.7891 in the cosine table. It is in row 37, under the column for .9.
      - This gives us the angle 37.9°.
8. Write the answers on the board:
   a. $\sin^{-1}(0.5015) = 31.0°$
   b. $\cos^{-1}(0.7891) = 37.9°$
9. Explain:
   - Recall that the trigonometric functions allow us to find missing sides of a right-angled triangle if we are given the angles.
   - The **inverse** trigonometric functions allow us to **find missing angles** of a right-angled triangle if we are given the sides.
10. Call pupils' attention to the problem on the board.
11. Solve for angle B, explaining each step:

Step 1. Identify which function to use. The opposite and adjacent sides are known, so we will use $\tan^{-1}$.

Step 2. Find the tangent ratio. This is the ratio that you will "undo" with $\tan^{-1}$ to find the angle:

$$\tan B = \frac{3}{4} = 0.75$$

Step 3. Find $\tan^{-1}$ of both sides to find the angle measure:

$$\begin{aligned} \tan B &= 0.75 \\ \tan^{-1}(\tan B) &= \tan^{-1}(0.75) \\ B &= \tan^{-1}(0.75) \end{aligned}$$

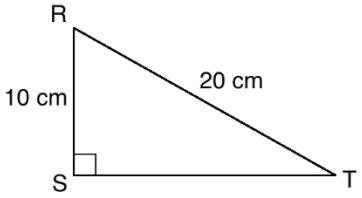
Calculate $\tan^{-1}(0.75)$ using the tangent table: Look for 0.75 in the table. It is not there, but 0.7481 is there. If we add 0.0018 to 0.7481, it will give us 0.75. Find 18 in the "add differences" table, and it corresponds to 7. Therefore, the angle is 36.87.

If you do not have trigonometric tables, solve using a calculator.

Write the answer on the board:

$$B = 36.87°$$

12. Write the following problem on the board: Find the measure of angle R:



13. Discuss: Which trigonometric function will we use? Why? (Answer: $\cos^{-1}$, because the known sides are adjacent to R, and the hypotenuse.)
14. Ask a volunteer to give the trigonometric ratio to be used, and write it on the board:

$$\cos R = \frac{10}{20} = \frac{1}{2} = 0.5$$

15. Ask pupils to work with seatmates to solve for R. They may use either a trigonometry table or calculator.
16. Walk around to support pupils and clear any misconceptions.
17. Ask a group of seatmates to share their solution and explain.

Solution:

$$\begin{aligned} \cos R &= 0.5 \\ \cos^{-1}(\cos R) &= \cos^{-1}(0.5) \\ R &= 60° \end{aligned}$$

18. Ask pupils to give the cosine of special angle 60°. Allow them to look at their notes. (Answer: $\cos 60° = \frac{1}{2}$)
19. Explain:
    - This is the same that we just saw in the solution. We took the inverse cosine of $\frac{1}{2}$, which gave us 60°.
    - If you recognize the common ratios in a problem, you may solve without using a calculator or trigonometry table.
20. Write the following problem on the board: Find x if $x = \sin^{-1}(\frac{\sqrt{3}}{2})$
21. Explain: Since the inverse sine is the opposite of sine, we can take the sine of both sides to eliminate it.
22. Solve on the board:

$$\begin{aligned} \sin x &= \sin(\sin^{-1}(\tfrac{\sqrt{3}}{2})) \\ \sin x &= \frac{\sqrt{3}}{2} \end{aligned}$$

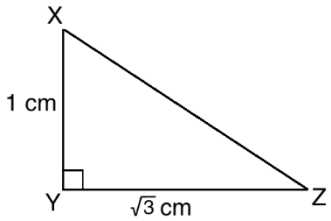
23. Ask pupils to give the value of x. Allow them to look at their notes and brainstorm about what x could be. (Answer: $x = 60°$; we know this from the lesson on special angles.)
24. Write the answer on the board: $x = 60°$
25. Write another problem on the board: Find y if $y = \tan^{-1}(\frac{1}{\sqrt{3}})$
26. Ask pupils to work with seatmates to find the answer.
27. Ask one group of seatmates to give their answer and explain how they found it.

Solution:

$$\begin{aligned} y &= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\ \tan y &= \tan(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)) \\ \tan y &= \frac{1}{\sqrt{3}} \\ \text{Therefore } y &= 30°, \text{ we know from the special angles lesson.} \end{aligned}$$

Practice *(6 minutes)*

1. Write the following problem on the board:
    a. Find the measure of angle X:



2. Ask pupils to work with independently to solve the problem. Allow discussion with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

    Solution:

    First note that we will use tangent, because we know the sides opposite and adjacent to X.

$$\begin{aligned} \tan X &= \frac{\sqrt{3}}{1} = \sqrt{3} \\ \tan^{-1}(\tan X) &= \tan^{-1}(\sqrt{3}) \\ X &= 60° \end{aligned}$$

    We know that $\tan^{-1}(\sqrt{3}) = 60°$ because $60°$ is a special angle.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L101 in the Pupil Handbook.

Lesson Title: Trigonometry and Pythagoras' Theorem	**Theme:** Trigonometry	
Lesson Number: M2-L102	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve right-angled triangles using trigonometric ratios and Pythagoras' Theorem.	**Preparation** Bring trigonometric tables if available.	

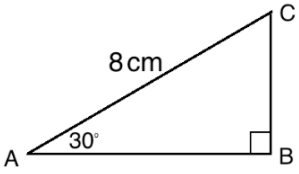
Opening *(3 minutes)*

1. Discuss:
    a. What does it mean to "solve" a triangle? (Answer: To "solve" means to find any missing side or angle measures.)
    b. What methods do you know for solving triangles? (Answer: trigonometric and inverse trigonometric functions; Pythagoras' theorem; finding angle measures by subtracting from 180°.)
2. Explain that today's lesson is on solving right-angled triangles using trigonometric ratios and Pythagoras' Theorem.

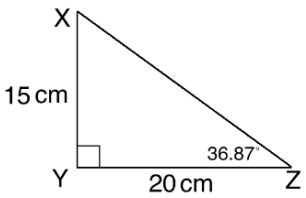
Teaching and Learning *(20 minutes)*

1. Explain:
    - When you have a triangle with missing sides and angles, you need to decide how to solve for them.
    - You can use a mix of Pythagoras' theorem and trigonometry to solve triangles.
    - In some cases, you could solve a problem using different methods. For example, in some cases the side of a right-angled triangle could be solved with Pythagoras' theorem or trigonometry. Choose the method you prefer, or the one that is best for the given problem.
2. Write the following problems on the board: Find the missing sides and angles of the triangles:

a.



b.



3. Discuss the best way to solve each problem:
    - Problem a.:
        - How can we find the missing sides? (Answer: We must use trigonometry. There is not enough information to use Pythagoras' theorem.)

- How can we find the missing angle C? (Answer: The easiest way is to subtract the known angles from 180°.)

- Problem b.:
  - How can we find the missing side? (Answer: Apply Pythagoras' theorem. We could use trigonometry, but when 2 sides are given it is generally easier to use Pythagoras' theorem.)
  - How can we find the missing angle X? (Answer: The easiest way is to subtract the known angles from 180°.)

4. Solve the problems as a class. Ask pupils to give the steps, and solve on the board as they explain.

   Solutions:

   a. Calculate $|AB|$:

   $\cos 30^{\circ} = \frac{|AB|}{8}$ Apply the cosine ratio

   $8 \times \cos 30^{\circ} = |AB|$ Multiply throughout by 8

   $8 \times \frac{\sqrt{3}}{2} = |AB|$ Use the special angle ratio

   $|AB| = 4\sqrt{3}$ cm

   Calculate $|BC|$:

   $\sin 30^{\circ} = \frac{|BC|}{8}$ Apply the sine ratio

   $8 \times \sin 30^{\circ} = |BC|$ Multiply throughout by 8

   $8 \times \frac{1}{2} = |BC|$ Use the special angle ratio

   $|BC| = 4$ cm

   Calculate $\angle C$: $180° - 90° - 30° = 60°$

   b. Calculate $|XZ|$:

   $15^2 + 20^2 = |XZ|^2$ Substitute the sides into the formula

   $225 + 400 = |XZ|^2$ Simplify

   $625 = |XZ|^2$

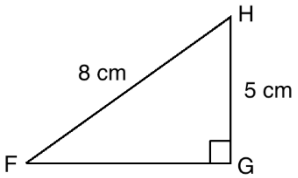
   $\sqrt{625} = \sqrt{|XZ|^2}$ Take the square root of both sides

   25 cm $= |XZ|$

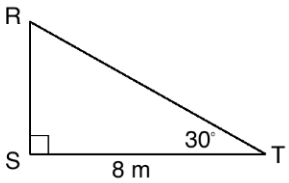
   Calculate $\angle X$: $180° - 90° - 36.87° = 53.13°$

5. Write the following problems on the board: Find the missing sides and angles of the triangles:

   a.



   b.



6. Discuss the best way to solve each problem:
   - Problem a.:

- Apply inverse trigonometry to find one missing angle (F or H).
- Subtract from 180° to find the other missing angle.
- Apply Pythagoras' theorem to find the missing side (FG)

- Problem b.:
  - Subtract from 180° to find the missing angle.
  - Apply trigonometry to find one missing side.
  - Apply either trigonometry or Pythagoras' theorem to find the other missing side.

7. Ask pupils to work with seatmates to solve the problems.
8. Walk around to check for understanding and clear misconceptions. Support pupils as needed.
9. Invite volunteers to write the solutions on the board.

Solutions:

a. Calculate $\angle F$:

$$\begin{aligned} \sin F &= \tfrac{5}{8} = 0.625 \\ \sin^{-1}(\sin F) &= \sin^{-1}(0.625) && \text{Find in the sine table} \\ F &= 38.68° \end{aligned}$$

Calculate $\angle H$: $180° - 90° - 38.68° = 51.32°$

Calculate $|FG|$:

$$\begin{aligned} |FG|^2 + 5^2 &= 8^2 && \text{Substitute the sides into the formula} \\ |FG|^2 + 25 &= 64 && \text{Simplify} \\ |FG|^2 &= 64 - 25 \\ |FG|^2 &= 39 && \text{Take the square root of both sides} \\ |FG| &= \sqrt{39}\ \text{cm} \end{aligned}$$

b. Calculate $\angle R$: $180° - 90° - 30° = 60°$

Calculate $|RS|$:

$$\begin{aligned} \tan 30° &= \tfrac{|RS|}{8} \\ 8\tan 30° &= |RS| \\ 8\left(\tfrac{1}{\sqrt{3}}\right) &= |RS| && \text{Apply the special angle ratio} \\ \tfrac{8}{\sqrt{3}} &= |RS| \end{aligned}$$

Calculate $|RT|$:

Pythagoras' theorem:

$$\begin{aligned} 8^2 + \left(\tfrac{8}{\sqrt{3}}\right)^2 &= |RT|^2 && \text{Substitute the sides into the formula} \\ 64 + \tfrac{64}{3} &= |RT|^2 && \text{Simplify} \\ \tfrac{192+64}{3} &= |RT|^2 \\ \tfrac{256}{3} &= |RT|^2 \\ \sqrt{\tfrac{256}{3}} &= \sqrt{|RT|^2} && \text{Take the square root of both sides} \end{aligned}$$

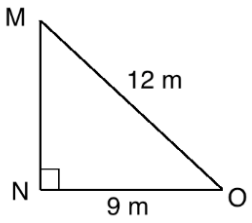
$$\frac{16}{\sqrt{3}} = |RT|$$

The same answer can be found using the cosine ratio.

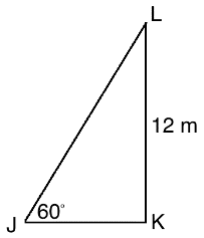
Practice *(16 minutes)*

1. Write on the board: Find the missing sides and angles of the triangles:

a.



b.



2. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Calculate $\angle M$:

$$\sin M = \frac{9}{12} = 0.75$$
$$\sin^{-1}(\sin M) = \sin^{-1}(0.75)$$
$$M = 48.6°$$

Calculate $\angle O$:

$$180° - 90° - 48.6° = 41.4°$$

Calculate $|MN|$:

$$|MN|^2 + 9^2 = 12^2$$
$$|MN|^2 + 81 = 144$$
$$|MN|^2 = 63$$
$$|MN| = \sqrt{63}$$
$$|MN| = 3\sqrt{7}$$

b. Calculate $|JK|$:

$$\tan 60° = \frac{12}{|JK|}$$
$$|JK| = \frac{12}{\tan 60°}$$
$$|JK| = \frac{12}{\sqrt{3}}$$

Calculate $|JL|$:

$$\sin 60° = \frac{12}{|JL|}$$
$$|JL| = \frac{12}{\sin 60°}$$
$$|JL| = \frac{12}{\frac{\sqrt{3}}{2}}$$
$$|JL| = \frac{24}{\sqrt{3}}$$

Pythagoras' theorem may also be used to find $|JL|$.

Calculate $\angle L$:

$$180° - 90° - 60° = 30°$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L102 in the Pupil Handbook.

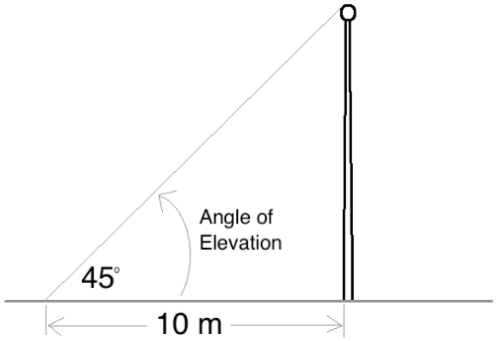
Lesson Title: Angles of elevation	**Theme:** Trigonometry	
Lesson Number: M2-L103	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Calculate angles of elevation. 2. Calculate height and distance associated with an angle of elevation.	**Preparation** Bring trigonometric tables if available.	

Opening *(3 minutes)*

1. Write the following problem on the board: At a point 10 metres away from a flag pole, the angle of elevation of the top of the pole is 45°. What is the height of the pole?
2. Discuss and allow pupils to share their ideas:
    a. What do you think the problem means by “angle of elevation”?
    b. What would this look like in a diagram?
3. Allow volunteers to try drawing a diagram for the problem on the board.
4. Explain that today’s lesson is angles of elevation.

Teaching and Learning *(20 minutes)*

1. Explain: “Elevation” is related to height. Problems on angles of elevation handle the angle that is associated with the height of an object.
2. Draw a diagram for the problem on the board:



3. Explain:
    - An angle of elevation is measured a certain distance away from an object.
    - Angle of elevation problems generally deal with 3 measures: the angle, the distance from the object, and the height of the object.
    - You may be asked to solve for any of these measures.
4. Discuss:
    - Looking at the diagram, how would you find the height of the flag pole? (Answer: Apply trigonometry; we can use the tangent ratio.)
5. Explain:
    - We apply trigonometric ratios to solve problems on angles of elevation.

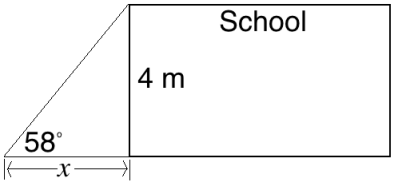
- Recall that we find missing sides with trigonometric ratios, missing angles with inverse trigonometry.

6. Solve the problem on the board, explaining each step:

$$\begin{aligned} \tan 45° &= \frac{h}{10} && \text{Set up the equation} \\ 1 &= \frac{h}{10} && \text{Substitute } \tan 45° = 1 \\ 10 \text{ m} &= h \end{aligned}$$

7. Explain: The height of the pole is 10 m. We were able to solve this using simple trigonometry.
8. Write the following problem on the board: A school building is 4 metres tall. At a point x metres away from the building, the angle of elevation is 58°. Find x.
9. Discuss: What does this problem ask us to find? (Answer: The distance of the school building from a certain point.)
10. Ask pupils to work with seatmates to draw a diagram for the problem.
11. Invite a volunteer to draw the diagram on the board.

Answer:



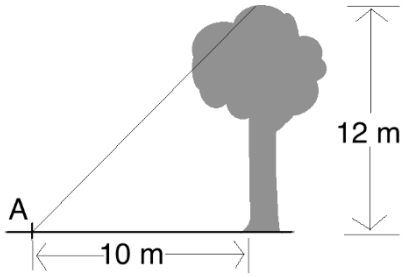
12. Discuss: What trigonometric ratio will you use to calculate x? (Answer: tangent)
13. Ask pupils to work with seatmates to solve the problem.
14. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} \tan 58° &= \frac{4}{x} && \text{Set up the equation} \\ 1.6 &= \frac{4}{x} && \text{Substitute } \tan 58° = 1.6 \text{ (from table)} \\ x &= \frac{4}{1.6} && \text{Change subject} \\ x &= 2.5 \text{ m} \end{aligned}$$

15. Explain: The point is 2.5 metres from the school.
16. Write the following problem on the board: Point A is 10 metres away from a tree. If the tree is 12 metres tall, what is the angle of elevation at point A?
17. Ask pupils to work with seatmates to draw a diagram for the problem.
18. Invite a volunteer to draw the diagram on the board.

Answer:



19. Discuss: How will we find the angle of elevation at point A? (Answer: Apply inverse trigonometry; use the inverse tangent.)
20. Ask pupils to find the measure of the angle of elevation with seatmates.
21. Invite a volunteer to write the solution on the board.

    Solution:

$$\begin{aligned} \tan A &= \frac{12}{10} = 1.2 \\ \tan^{-1}(\tan A) &= \tan^{-1}(1.2) \\ A &= 50.2° \qquad \text{Using the tangent table} \end{aligned}$$

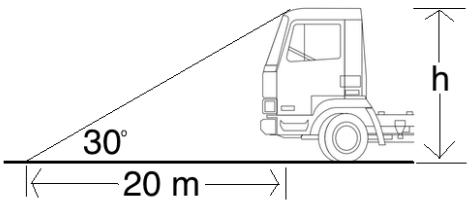
22. Explain: The angle of elevation of the tree from point A is 50.2°.

Practice *(16 minutes)*

1. Write the next problems on the board: Draw a diagram and solve each problem:
    a. At a point 20 metres away from a truck, the angle of elevation of the top of the truck is 30°. What is the height of the truck?
    b. A house is 2 metres tall. At a distance d metres away from the house, the angle of elevation is 50.2°. Find d.
    c. A point is 8 metres away from an elephant. If the elephant is 2.5 metres tall, what is the angle of elevation at the point?
2. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to draw the diagrams and write the solutions on the board. All other pupils should check their own work.

    Solutions:

    a. Diagram:



Solution:

$$\tan 30° = \frac{h}{20} \qquad \text{Set up the equation}$$

$\frac{\sqrt{3}}{3} = \frac{h}{20}$ Substitute $\tan 30° = \frac{\sqrt{3}}{3}$

$h = \frac{20\sqrt{3}}{3}$ m Cannot be simplified further

Alternatively, pupils could have found $\tan 30°$ in the tangent table and solved:

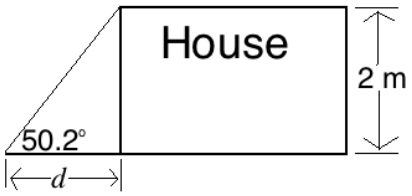
$\tan 30° = \frac{h}{20}$ Set up the equation

$0.5774 = \frac{h}{20}$ Substitute $\tan 30° = \frac{\sqrt{3}}{3}$

$h = 20 \times 0.5774$

$h = 11.548$ m

b. Diagram:



Solution:

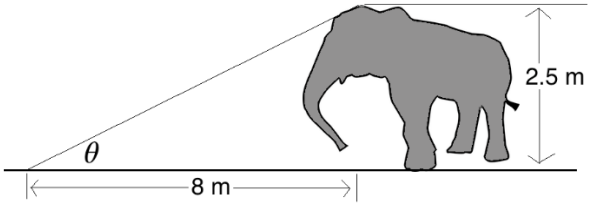
$\tan 50.2° = \frac{2}{d}$ Set up the equation

$1.2 = \frac{2}{x}$ Substitute $\tan 50.2° = 1.2$ (from table)

$x = \frac{2}{1.2}$ Change subject

$x = 1.7$ m

c. Diagram:



Solution:

$\tan\theta = \frac{2.5}{8} = 0.3125$

$\tan^{-1}(\tan\theta) = \tan^{-1}(0.3125)$

$\theta = 17.4°$ Using the tangent table

Closing *(1 minute)*

1. For homework, have pupils do activity PHM2-L103 in the Pupil Handbook.

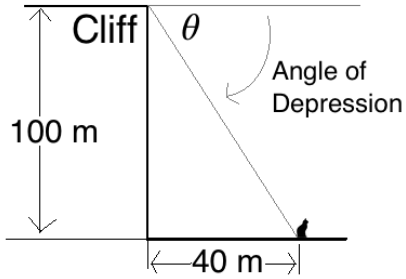
Lesson Title: Angles of depression	**Theme:** Trigonometry	
Lesson Number: M2-L104	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Calculate angles of depression. 2. Calculate depth and distance associated with an angle of depression.	**Preparation** Bring trigonometric tables if available. Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: A cliff is 100 metres tall. At a distance of 40 metres from the base of the cliff, there is a cat sitting on the ground. What is the angle of depression of the cat from the cliff?
2. Discuss and allow pupils to share their ideas:
    a. What do you think the problem means by “angle of depression”?
    b. What would this look like in a diagram?
3. Allow volunteers to try drawing a diagram for the problem on the board.
4. Explain that today’s lesson is angles of depression.

Teaching and Learning *(20 minutes)*

1. Explain:
    - “Depression” is the opposite of elevation.
    - “Depressed” means downward. So if there is an angle of depression, it is an angle in a downward direction.
2. Draw a diagram for the problem on the board:



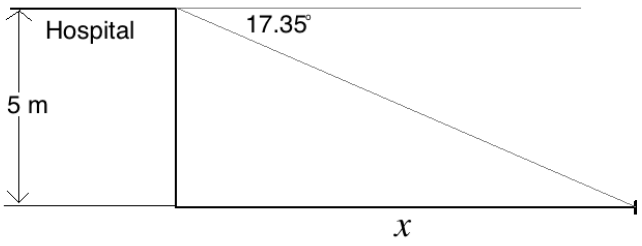
3. Explain:
    - The angle of depression is the angle made with the **horizontal** line. In this example, the horizontal line is at the height of the cliff.
    - Angle of depression problems generally deal with 3 measures: the angle, the horizontal distance, and the depth of the object.
    - Depth is the opposite of height. It is the distance downward.
    - You may be asked to solve for any of these measures.

4. Discuss: Looking at the diagram, how would you find the angle of depression of the cat? (Answer: Apply trigonometry; we can use the tangent ratio.)
5. Explain:
    - We apply trigonometric ratios to solve problems on angles of depression.
    - Recall that we find missing sides with trigonometric ratios, and missing angles with inverse trigonometry.
6. Solve the problem on the board, explaining each step:

$$\begin{aligned} \tan\theta &= \frac{100}{40} = 2.5 && \text{Set up the equation} \\ \tan^{-1}(\tan\theta) &= \tan^{-1}(2.5) && \text{Take the inverse tangent} \\ \theta &= 68.2 && \text{Use the tangent tables} \end{aligned}$$

7. Explain: The angle of elevation is 68.2°.
8. Write the following problem on the board: A hospital is 5 metres tall. A point is x metres away from the building, and the angle of depression is 17.35°. Find x.
9. Discuss: What does this problem ask us to find? (Answer: The horizontal distance between the hospital and the point on the ground.)
10. Ask pupils to work with seatmates to draw a diagram for the problem.
11. Invite a volunteer to draw the diagram on the board.

    Answer:



12. Discuss:
    - What trigonometric ratio will you use to calculate x? (Answer: tangent)
    - What is the length of the side of the triangle opposite 17.35°? How do you know? (Answer: It is 5 metres, because it is the same as the height of the hospital.)
13. Ask pupils to work with seatmates to solve the problem.
14. Invite a volunteer to write the solution on the board.

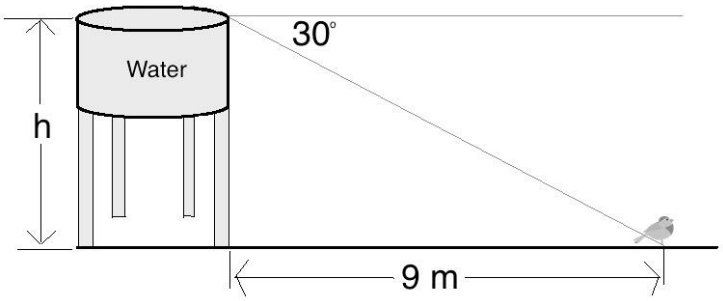
    Solution:

$$\begin{aligned} \tan 17.35° &= \frac{5}{x} && \text{Set up the equation} \\ 0.3125 &= \frac{5}{x} && \text{Substitute } \tan 17.35° = 0.3125 \text{ (from table)} \\ x &= \frac{5}{0.3125} && \text{Change subject} \\ x &= 16 \text{ m} \end{aligned}$$

15. Explain: The point is 16 metres from the hospital. That is the horizontal distance.
16. Write the following problem on the board: A bird is sitting 9 metres from the base of a water tank. The angle of depression of the bird from the top of the water tank is 30°. What is the height of the water tank?

17. Ask pupils to work with seatmates to draw a diagram for the problem.
18. Invite a volunteer to draw the diagram on the board.

Answer:



19. Discuss: How will we find the height of the water tank? (Answer: Use the tangent ratio)
20. Ask pupils to find the height of the water tank with seatmates.
21. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} \tan 30° &= \frac{h}{9} \\ \frac{\sqrt{3}}{3} &= \frac{h}{9} && \text{Substitute } \tan 30° = \frac{\sqrt{3}}{3} \\ \frac{9\sqrt{3}}{3} &= h && \text{Multiply throughout by 9} \\ h &= 3\sqrt{3} \text{ m} && \text{Simplify} \end{aligned}$$

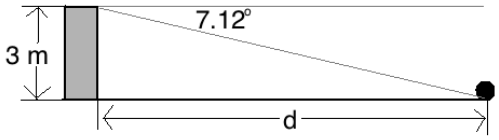
22. Explain: The height of the water tank is $3\sqrt{3}$ m.

Practice *(16 minutes)*

1. Write the following problems on the board: Draw a diagram and solve each problem:
    a. A child kicked a football off the top of a tower that is 3 metres tall. The ball landed on the ground. The angle of depression of the ball from the top of the tower is 7.12°. How far is the ball from the tower?
    b. A point X is on the same horizontal level as the base of a building. If the distance from X to the building is 10 m and the height of the building is 23 m, calculate the angle of depression of X from the top of the building. Give your answer to the nearest degree.
    c. A dog is sitting 60 metres from the base of a cliff. If the angle of depression of the dog from the cliff is 45°, how tall is the cliff?
2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to draw the diagrams and write the solutions on the board. All other pupils should check their own work.

Solutions:

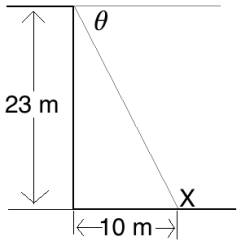
a. **Diagram:**



Solution:

$$\begin{aligned} \tan 7.12° &= \frac{3}{d} \\ 0.125 &= \frac{3}{d} \\ d &= \frac{3}{0.125} \\ d &= 24 \text{ m} \end{aligned}$$

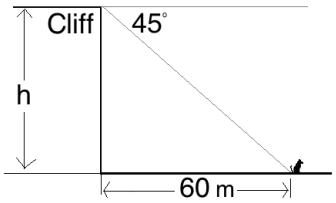
b. **Diagram:**



Solution:

$$\begin{aligned} \tan\theta &= \frac{23}{10} = 2.3 \\ \tan^{-1}(\tan\theta) &= \tan^{-1}(2.3) \\ \theta &= 66.5 \end{aligned}$$

c. **Diagram:**



Solution:

$$\begin{aligned} \tan 45° &= \frac{h}{60} \\ 1 &= \frac{h}{60} \\ h &= 60 \text{ m} \end{aligned}$$

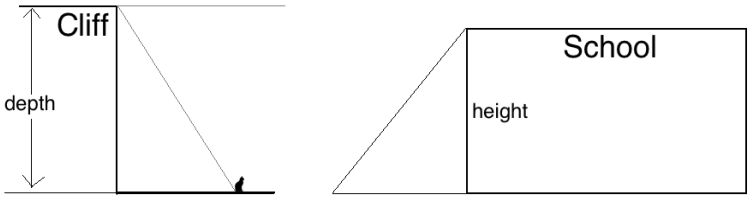
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L104 in the Pupil Handbook.

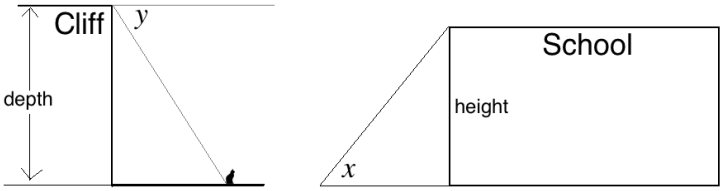
Lesson Title: Applications of angles of elevation and depression – Part 1	**Theme:** Trigonometry	
Lesson Number: M2-L105	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve practical problems related to angles of elevation and depression.	**Preparation** Bring trigonometric tables if available. Draw the diagrams in Opening on the board.	

Opening *(3 minutes)*

1. Draw the following diagrams on the board:



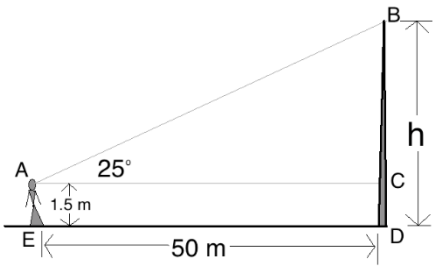
2. Invite a volunteer to come to the board and draw an x in the angle of elevation.
3. Invite another volunteer to draw a y in the angle of depression.
   Answers:



4. Remind pupils that angles of elevation and depression are always formed by a horizontal line, and another line. They are never formed by a vertical line.
5. Explain that today's lesson is on practical problems related to angles of elevation and depression.

Teaching and Learning *(20 minutes)*

1. Write on the board: A woman standing 50 metres from a flag pole observes that the angle of elevation of the top of the pole is 25°. Assuming her eye is 1.5 metres above the ground, calculate the height of the pole to the nearest metre.
2. Draw a diagram for the problem on the board:

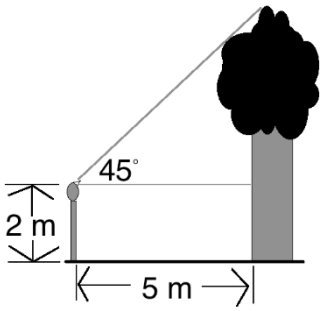


3. Explain: To find the height of the flag pole, we must find the length of BC, then add it to the height of the woman's eye, which is 1.5 metres.

4. Solve on the board, explaining each step:
   Step 1. Find $\overline{BC}$:

$\tan 25°$	$=$	$\frac{\overline{BC}}{50}$	Set up the equation
0.4663	$=$	$\frac{\overline{BC}}{50}$	Substitute $\tan 25° = 0.4663$ (from the table)
50×0.4663	$=$	$\overline{BC}$	Multiply throughout by 50
$\overline{BC}$	$=$	23.315	metres

   Step 2. Add: $h = BC + CD = 23.315 + 1.5 = 24.815$
   Rounded to the nearest metre, the height of the pole is 25 m.
5. Write the following problem on the board: A bird sits on a fence and looks up at a tree. It observes that the angle of elevation of the top of the tree is 45°. The bird's eye is 2 metres from the ground, and the bird is 5 metres from the tree. What is the height of the tree?
6. Ask pupils to work with seatmates to draw a diagram for the problem.
7. Invite one group of seatmates to volunteer to draw their diagram on the board. Correct any errors.
   Diagram:

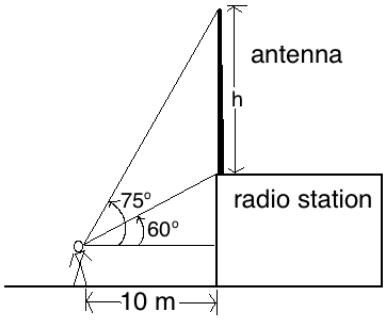


8. Ask volunteers to explain the process to find the height of the tree. (Answer: Find the portion of the height that is above the bird's eye using trigonometry. Then, add the 2-metre height of the bird's eye.)
9. Ask pupils to work with seatmates to find the height of the tree.
10. Invite a volunteer to write the solution on the board and explain.
    Solution:
    Step 1. Find the height of the portion of the tree above the bird's eye. Call this x:

$\tan 45°$	$=$	$\frac{x}{5}$	Set up the equation
1	$=$	$\frac{x}{5}$	Substitute $\tan 45° = 1$
5×1	$=$	x	Multiply throughout by 5
x	$=$	5 m	

    Step 2. Add this to the height of the bird's eye: $5 + 2 = 7$
    The height of the tree is 7 metres.
11. Write the following problem on the board: A man is standing 10 metres away from a radio station, looking at an antenna on top of a building. He notices that the angles of elevation of the top and bottom of the antenna are 60° and 75°, respectively. Find the height of the antenna.

12. Ask pupils to work with seatmates to draw a diagram for the problem.
13. Invite a volunteer with the correct diagram to draw it on the board. Diagram:
14. Ask pupils to share ideas about how to solve this problem.
15. Explain: To find the height of the antenna, first find the total height that the man observes with a 75° angle of elevation, then subtract the portion that is the building (using the 60° angle of elevation).



16. Ask volunteers to give each step to solve the problem. As they give them, solve the problem on the board:

Step 1. Find the height of the portion with an angle of elevation of 75°. Call it A:

$\tan 75°$	$=$	$\frac{A}{10}$	Set up the equation
3.732	$=$	$\frac{A}{10}$	Substitute $\tan 75° = 3.732$
3.732×10	$=$	A	Multiply throughout by 10
A	$=$	37.32	m

Step 2. Find the height of the portion with an angle of elevation of 60°. Call it B: Use the tangent table rather than the special angle, because the other angle we are working with (75°) is given as a decimal and we must subtract.

$\tan 60°$	$=$	$\frac{B}{10}$	Set up the equation
1.732	$=$	$\frac{B}{10}$	Substitute $\tan 60° = 1.732$
1.732×10	$=$	B	Multiply throughout by 5
B	$=$	17.32	m

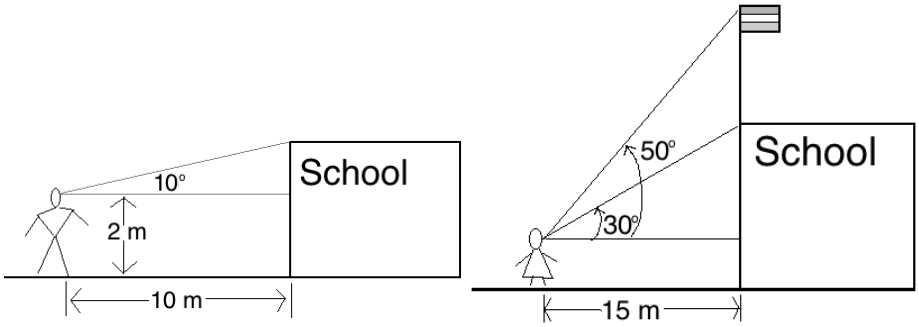
Step 3. Subtract: $h = A - B = 37.32 - 17.32 = 20.00$ m

Answer: The antenna is 20 metres tall.

Practice *(16 minutes)*

1. Write the following problems on the board: Draw a diagram and solve each problem:
   a. A teacher standing 10 metres from the school building observes that the angle of elevation of the top of the building is 10°. Assuming his eye is 2 metres above the ground, calculate the height of the school to the nearest metre.
   b. A school has a flag pole on top of the building. The principal is standing 15 metres away from the school building, looking at the flag pole on top. She notices that the angles of elevation of the top and bottom of the flag pole are 50° and 30°, respectively. Find the height of the flag pole to the nearest metre.
2. Ask pupils to work with seatmates to sketch diagrams for each problem.

3. Ask volunteers to sketch the diagrams on the board.
   Diagrams:



4. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
5. Walk around to check for understanding and clear misconceptions.
6. Invite volunteers to write the solutions on the board. All other pupils should check their own work.
   Solutions:
   a. Find the height of the portion of the building above the teacher's eye. Call this x:

$$\begin{aligned} \tan 10° &= \frac{x}{10} && \text{Set up the equation} \\ 0.1763 &= \frac{x}{10} && \text{Substitute } \tan 15° = 0.2679 \\ 0.1763 \times 10 &= x && \text{Multiply throughout by 5} \\ x &= 1.763 && \text{m} \end{aligned}$$

   Add this to the height of the teacher's eye: $1.763 + 2 = 3.763$
   After rounding, the height of the school is 4 metres.
   b. Find the height of the portion with an angle of elevation of 50°:

$$\begin{aligned} \tan 50° &= \frac{A}{15} && \text{Set up the equation} \\ 1.192 &= \frac{A}{15} && \text{Substitute } \tan 50° = 1.192 \\ A &= 17.88 && \text{m} \end{aligned}$$

   Find the height of the portion with an angle of elevation of 30°:

$$\begin{aligned} \tan 30° &= \frac{B}{15} && \text{Set up the equation} \\ 0.5774 &= \frac{B}{15} && \text{Substitute } \tan 30° = 0.5774 \\ B &= 8.661 && \text{m} \end{aligned}$$

   Subtract: $A - B = 17.88 - 8.661 = 9.219$ metres. To the nearest metre, the flag pole is 9 metres tall.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L105 in the Pupil Handbook.

Lesson Title: Applications of angles of elevation and depression – Part 2	**Theme:** Trigonometry	
Lesson Number: M2-L106	**Class:** SSS 2	**Time:** 40 minutes
 **Learning Outcome** By the end of the lesson, pupils will be able to solve practical problems related to angles of elevation and depression.	 **Preparation** 1. Bring trigonometric tables if available. 2. Review the question and answer bank at the end of this lesson plan. Choose problems to use in Teaching and Learning and Practice.	

Opening *(1 minute)*

1. Explain:
   a. Today we will solve practical problems on angles of elevation and depression.
   b. There are often problems on angles of elevation and depression on the WASSCE exam, and they take many different forms.
2. Remind pupils of the importance of drawing a diagram before solving.

Teaching and Learning *(19 minutes)*

1. Write a problem from the question bank on the board.
2. Ask pupils to draw the diagram for the problem.
3. Invite a volunteer to draw the diagram on the board.
4. Ask volunteers to describe the steps to solve the problem. As they give the steps, solve the problem on the board.
5. Write 2 more problems on the board.
6. Ask pupils to work with seatmates to solve the problems.
7. Walk around to check for understanding and clear misconceptions.
8. Invite volunteers to write the diagrams and solutions on the board.

Practice *(19 minutes)*

1. Write at least 3 problems on the board.
2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.
3. Invite volunteers to draw the diagrams on the board if needed to guide other pupils.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to write the solutions on the board.

Closing *(1 minute)*

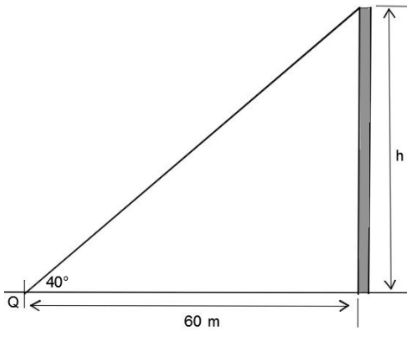
1. For homework, have pupils do the practice activity PHM2-L106 in the Pupil Handbook.

   [QUESTION BANK]
   Choose questions based on your pupils' understanding; choose topics that they need to review. See diagrams and solutions to the problems below.

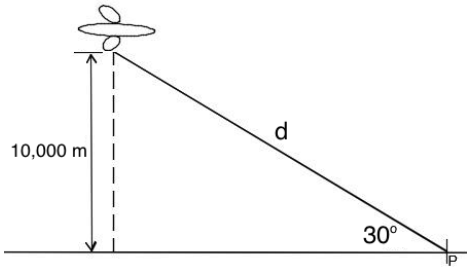
   1. A tower is 60 metres from a point Q, which is on the same horizontal level as the base of the tower. The top of the tower is at a 40° angle of elevation from point Q. Find the height of the tower. Give your answer to 1 decimal place.
   2. An airplane is flying over a forest. From a point P on the forest floor, the airplane is at a 30° angle of elevation. If the airplane is flying at a height of 10,000 metres, how far is it from point P?
   3. A girl is standing next to her teacher. The girl's height is 90 cm, and her shadow is 120 cm long. If the teacher's shadow is 200 cm long, what is the height of the teacher?
   4. A bird on top of a tree looks down at a piece of food laying 10 metres from the base of the tree. If the angle of depression from the bird's eye to the food is 50 degrees, find the height of the bird's eye from the ground.
   5. A man's eye level is 2 metres above the horizontal ground, and 36 metres from a building. If the building is 14 metres tall, calculate the angle of elevation of the top of the building from his eyes. Give your answer to the nearest degree.
   6. An object is 12 metres away from the base of a mast. The angle of depression of the object from the top of the mast is 48°. Calculate the height of the mast to 2 decimal places.
   7. Point A is 20 metres away from the foot of a building on the same horizontal ground. From point A, the angle of elevation to point P on the side of the building is 30°. The angle of elevation from A to the top of the building is 50°. Find the distance of point P from the top of the building. Give your answer to two decimal places.

[SOLUTIONS AND DIAGRAMS]

1.
$$\begin{aligned} \tan 40° &= \frac{h}{60} \\ 0.8391 &= \frac{h}{60} \\ 0.8391 \times 60 &= h \\ h &= 50.3 \text{ m} \end{aligned}$$



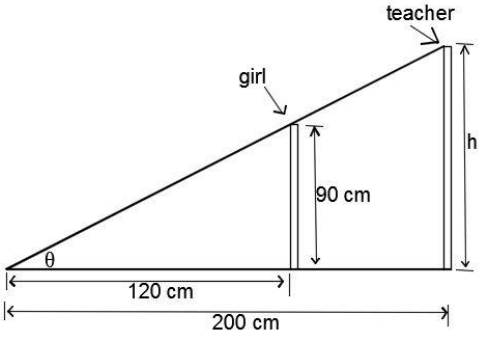
2. Use the sine ratio, because the distance between the airplane and the point is the hypotenuse:



$$\sin 30° = \frac{10{,}000}{d}$$
$$\frac{1}{2} = \frac{10{,}000}{d}$$
$$d = 10{,}000 \div \frac{1}{2}$$
$$d = 20{,}000 \text{ m}$$

3. First, find the angle of elevation using the pupil's height. Then, use the angle of elevation and the teacher's shadow to find her height.

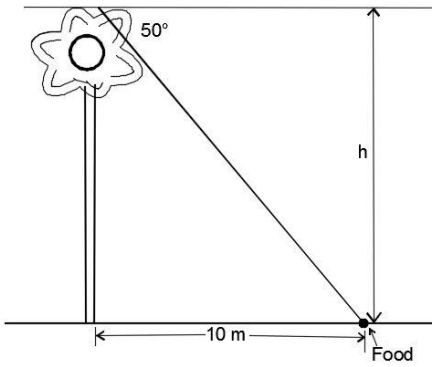
$$\tan\theta = \frac{90}{120} = 0.75$$
$$\tan^{-1}(\tan\theta) = \tan^{-1} 0.75$$
$$\theta = 36.87$$



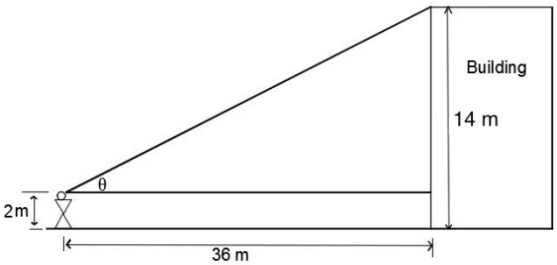
Find teacher's height:

$$\tan 36.87 = \frac{h}{200}$$
$$0.75 = \frac{h}{200}$$
$$h = 200 \times 0.75$$
$$h = 150 \text{ cm}$$

4. $$\tan 50° = \frac{h}{10}$$
$$1.192 = \frac{h}{10}$$
$$1.192 \times 10 = h$$
$$h = 11.92 \text{ m}$$



5. $$\tan\theta = \frac{14-2}{36} = 0.\overline{3}$$
$$\tan^{-1}(\tan\theta) = \tan^{-1} 0.\overline{3}$$
$$\theta = 18.43°$$
$$\theta = 18°$$

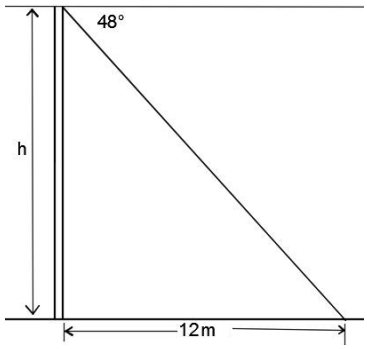


6. $\tan 48° = \frac{h}{12}$

$1.111 = \frac{h}{12}$

$1.111 \times 12 = h$

$h = 13.33 \text{ m}$



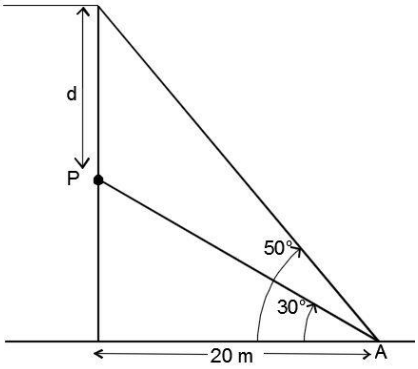
7. Find the total height of the building, and the height of point P. Subtract the height of P from the total height.

$\tan 30° = \frac{h_P}{20}$

$0.5774 = \frac{h_P}{20}$

$0.5774 \times 20 = h_P$

$h = 11.548 \text{ m}$



Height of the building:

$\tan 50° = \frac{h}{20}$

$1.192 = \frac{h}{20}$

$1.192 \times 20 = h$

$h = 23.84 \text{ m}$

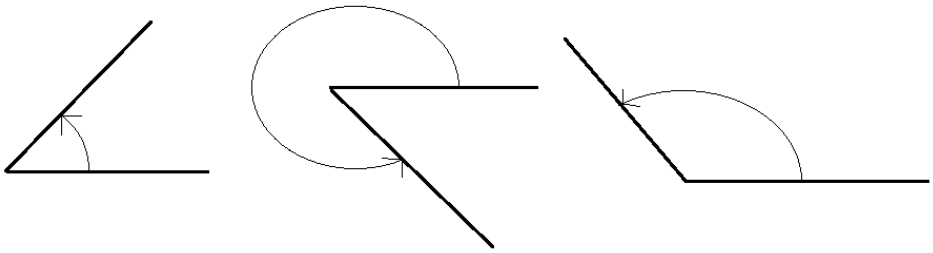
Distance of P from the top: $23.84 - 11.548 = 12.292$

Answer: 12.292 m

Lesson Title: The general angle – Part 1	**Theme:** Trigonometry	
Lesson Number: M2-L107	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to extend sine, cosine, and tangent ratios of acute angles to obtuse and reflex angles.	**Preparation** 1. Bring trigonometric tables if available. 2. Draw the diagrams in Opening on the board.	

Opening *(2 minutes)*

1. Draw on the board:

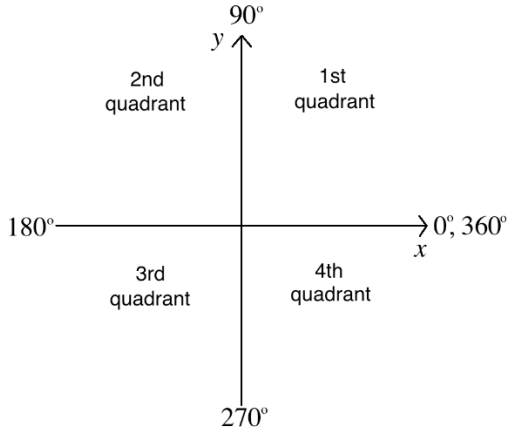


2. Ask pupils to determine the angle types with seatmates.
3. Invite volunteers to come to the board and label the angles with their types (Answer from left to right: acute, reflex, obtuse)
4. Explain that this lesson is on extending the sine, cosine, and tangent ratios to obtuse and reflex angles. Thus far, we have only found the trigonometric ratios of acute angles.

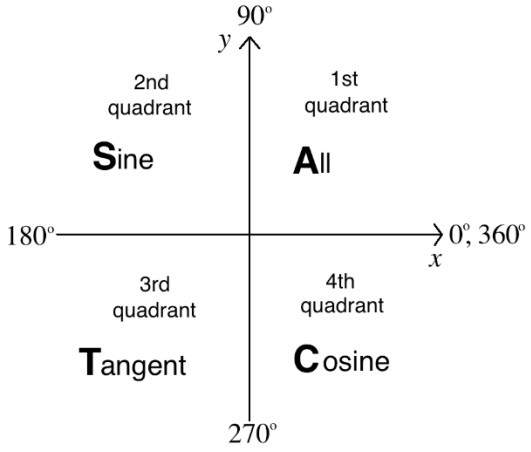
Teaching and Learning *(25 minutes)*

1. Write on the board: $\sin 100°$ $\cos 180°$ $\tan 240°$
2. Discuss:
    - Can you find the trigonometric functions of these angles?
    - Allow pupils to check the trigonometric tables, and notice that obtuse and reflex angles are not there.
    - If they find the answers with a calculator, accept these and write them on the board.
3. Explain: Today you will learn how to find the values of these using trigonometric tables.

4. Draw on the board:



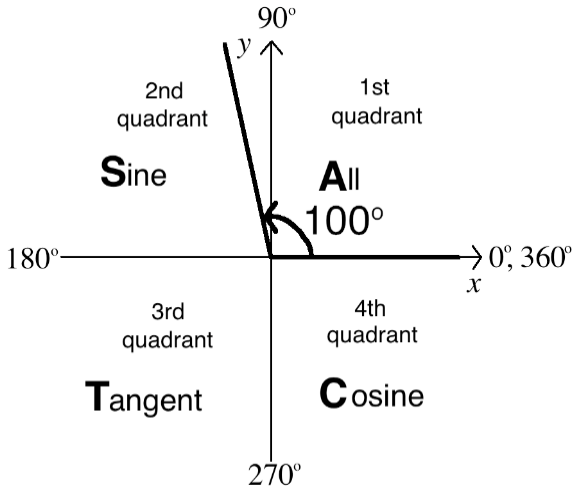
5. Explain:
    - Angles are centred at the origin, the point where the x- and y-axes cross. They open in a counterclockwise direction.
    - There are 4 quadrants that an angle could lie in.
    - An angle in the first quadrant is acute, and an angle in the second quadrant is obtuse. An angle in the third or fourth quadrant is a reflex angle.
    - The quadrant that an angle lies in tells you whether the result of the trigonometric ratio will be positive or negative.
6. Label the quadrants as shown:



7. Explain how to determine the sign of a ratio:
    - We use the word “ACTS” to remember which functions are **positive** in which quadrant. The word ACTS starts in the **first** quadrant and goes in a **clockwise** direction.
    - Remember that A stands for “all”, which means that all of the trigonometric ratios are positive in the first quadrant. The letters C, T, and S in other quadrants stand for trigonometric ratios, and tell us that they will be positive.

- All other ratios will be negative. For example, the second quadrant has an S. This means that the sine will be positive, while the cosine and tangent will be negative.

8. Explain how to determine the value of a ratio:
   - Each obtuse or reflex angle has an "associated acute angle". This is the acute angle that forms with the x-axis when it is laid on the 4 quadrants.
   - To find the ratio of an obtuse or reflex angle, find the ratio of the associated acute angle. Then, apply the correct sign for that quadrant.
9. Call pupils' attention to the first example you wrote on the board: $\sin 100°$
10. Draw 100° on the ACTS diagram:



11. Discuss: What is the associated acute angle for 100°?
12. Allow pupils to make their observations, then give the answer. (Answer: 80°; this is the angle formed by 100° and the x-axis. It can be found using subtraction: $180° - 100° = 80°$)
13. Ask pupils to find $\sin 80°$ in the sine table, and allow volunteers to call out the answer.
14. Write on the board: $\sin 80° = 0.9848$
15. Discuss: Is the sine ratio positive or negative in the second quadrant? (Answer: positive)
16. Write on the board: $\sin 100° = 0.9848$
17. Ask pupils to look at the second problem: $\cos 180°$
18. Discuss: What is the associated acute angle? (Answer: 0, because 180 lies on the x-axis)
19. Ask pupils to find $\cos 0°$, and allow volunteers to call out the answer.
20. Write on the board: $\cos 0° = 1$
21. Explain: The cosine of 180° will be negative, because the cosine is negative in the second quadrant.
22. Write on the board: $\cos 180° = -1$
23. Ask pupils to look at the third problem on the board: $\tan 240°$

24. Ask pupils to work with seatmates to draw a sketch of 240° on the ACTS diagram.
25. Discuss:
    - Which quadrant is 240° in? (Answer: third quadrant)
    - What angle does it make with the x-axis? (Answer: 60°)
    - Will the tangent function be positive or negative? (positive)
26. Ask pupils to work with seatmates to find $\tan 240°$.
27. Walk around to check for understanding and clear misconceptions.
28. Invite a group of seatmates to give their answer and explain how they found it. (Answer: 1.732 or $\sqrt{3}$; this is found with $\tan 60°$, the tangent of the associated acute angle)

Practice *(12 minutes)*

1. Write on the board: Find the trigonometric functions of the angles:
    a. $\tan 110°$
    b. $\sin 300°$
    c. $\cos 225°$
    d. $\sin 150°$
    e. $\tan 200°$
2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the answers on the board and explain how they found them. All other pupils should check their own work. (Answers: a. -2.747; b. -0.8660; c. -0.7071 or $-\frac{\sqrt{2}}{2}$, using special angle 45°; d. 0.5; e. 0.3640)

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L107 in the Pupil Handbook.

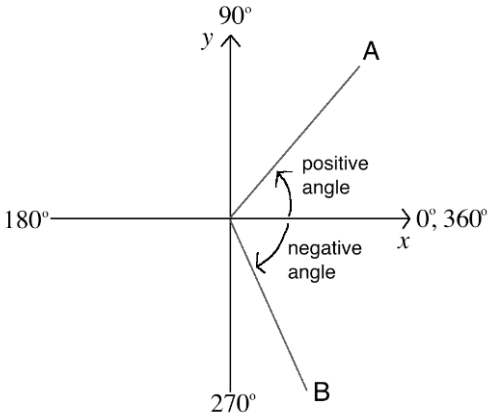
Lesson Title: The general angle – Part 2	**Theme:** Trigonometry	
Lesson Number: M2-L108	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to express a positive or negative angle of any size in terms of an equivalent positive angle between 0° and 360°, and find the trigonometric ratios.	**Preparation** 1. Bring trigonometric tables if available. 2. Write the problems in Opening on the board.	

Opening *(4 minutes)*

1. Write on the board: Find the value of: a. $\sin 290°$ b. $\tan 175°$
2. Ask pupils to find the solutions with seatmates.
3. Invite volunteers to come to the board, write their answers and explain how they found them (Answers: -0.9397, found using the corresponding acute angle 70°; b. -0.0875, found using the corresponding acute angle 5°).
4. Explain that this lesson is a continuation of the previous lesson. It will cover negative angles, and angles larger than 360°.

Teaching and Learning *(23 minutes)*

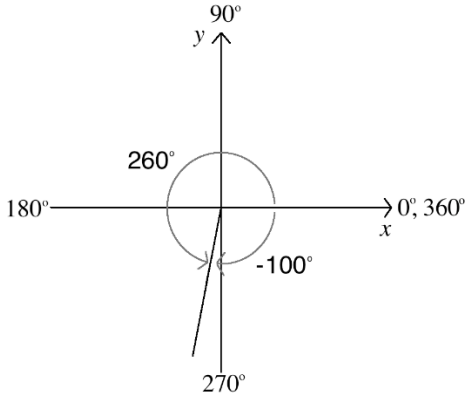
1. Draw on the board:



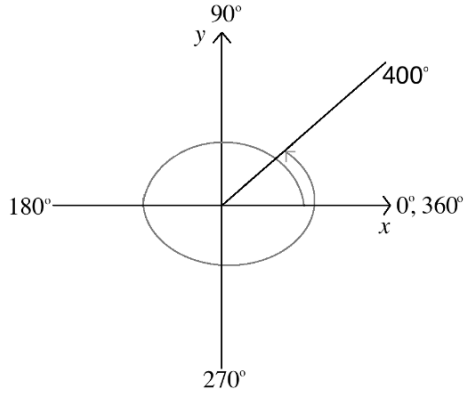
2. Explain:
    - Angles that open in a **counterclockwise** direction have **positive** values. These are the angles we have worked with so far. The angle formed by A is positive.
    - Angles that open in a **clockwise** direction have **negative** values. The angle formed by B is negative.
3. Write on the board: Find $\sin(-100°)$
4. Explain:
    - For each negative angle, there is a corresponding positive angle. The positive angle is the remainder of the full revolution (360°) that is not covered by the negative angle.

- The corresponding positive angle is found by subtracting the absolute value of the negative angle from 360°.
- The sine ratio of the negative angle is equal to the sine ratio of the corresponding positive angle.

5. Find the positive angle that corresponds to $-100°$: $360° - 100° = 260°$
6. Draw $-100°$ and $260°$ on the board as shown:



7. Write on the board: $\sin(-100°) = \sin 260°$
8. Find $\sin 260°$ using techniques from the previous lesson:
    - The corresponding acute angle is 80°.
    - $\sin 80° = 0.9848$
    - Because the angle ($260°$ or $-100°$) lies in the third quadrant, the sine ratio is negative.
    - $\sin 260° = -0.9848$
9. Write the answer on the board: $\sin(-100°) = \sin 260° = -0.9848$
10. Write the following problem on the board: Find $\cos 400°$
11. Explain: This angle is more than 360°. It is more than 1 full rotation.
12. Draw the following diagram on the board:



13. Discuss: How much bigger is 400° than 360°? (Answer: 40 degrees)
14. Explain:
    - To find a trigonometric ratio of an angle that is more than 360°, divide by 360° and find the remainder. The remainder will be a number less than 360°.
    - Find the trigonometric ratio of the remainder. This will be the answer.

15. Write on the board and make sure pupils understand: $400° \div 360 = 1 \text{ remainder } 40°$
16. Write on the board: $\cos 400° = \cos 40°$
17. Ask pupils to find the answer with seatmates.
18. Ask a volunteer to give the answer and explain. (Answer: $\cos 40° = 0.7660$, from the cosine table)
19. Write the answer on the board: $\cos 400° = \cos 40° = 0.7660$
20. Write the following problems on the board: a. $\tan(-75°)$ b. $\sin 700°$
21. Ask volunteers to give the steps to solve each problem. As they give the steps, work the problems on the board.

    Solutions:

    a.

$\tan(-75°)$	$=$	$\tan(285°)$	Corresponding positive angle
	$=$	$-\tan(75°)$	Corresponding acute angle 4th quadrant is negative
	$=$	-3.732	From tangent table

    b.

$\sin(700°)$	$=$	$\sin(340°)$	Remainder after dividing $700° \div 360°$
	$=$	$-\sin(20°)$	Corresponding acute angle 4th quadrant is negative
	$=$	-0.3420	From tangent table

22. If a calculator is available, use it to show pupils how to check their answers.

Practice *(12 minutes)*

1. Write the following problems on the board: Find the trigonometric functions of the angles:
    a. $\tan(-25°)$
    b. $\sin 800°$
    c. $\cos 550°$
    d. $\sin(-120°)$
    e. $\cos(-340°)$
    f. $\tan 365°$
2. Ask pupils to work independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the answers on the board and explain how they found them. All other pupils should check their own work. (Answers: a. -0.4663; b. 0.9848; c. -0.9848; d. -0.8660; e. 0.9397; f. 0.0875)

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L108 in the Pupil Handbook.

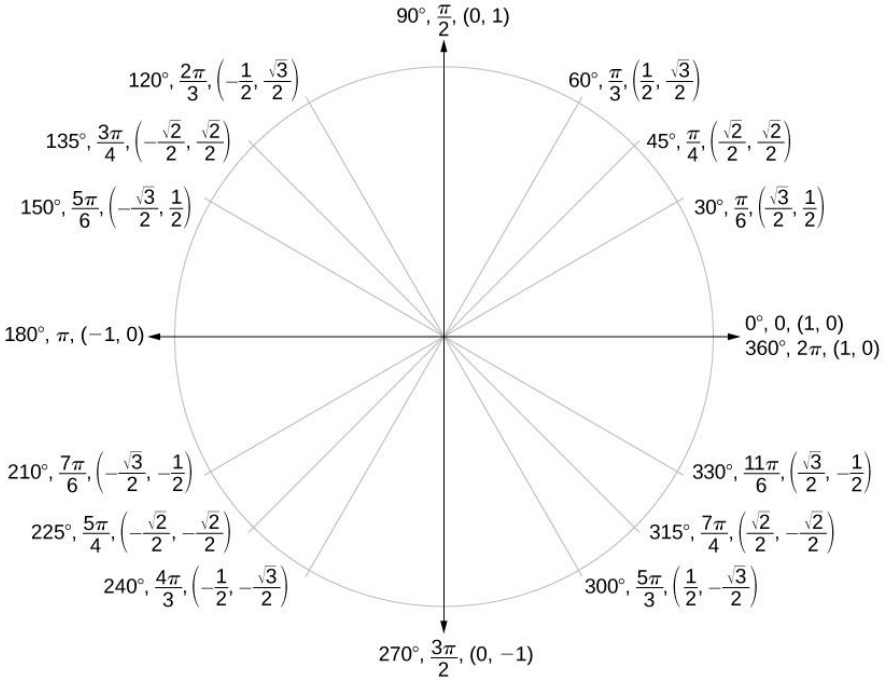
Lesson Title: The unit circle	**Theme:** Trigonometry	
Lesson Number: M2-L109	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to define $\sin\theta$ and $\cos\theta$ as ratios within a unit circle.	**Preparation** 1. Bring trigonometric tables if available. 2. Draw the unit circle on the board, or prepare the drawing on vanguard to be used in multiple lessons.	

Opening *(4 minutes)*

1. Write on the board: Find the value of: a. $\sin(-250°)$ b. $\cos 305°$
2. Ask pupils to find the solutions with seatmates.
3. Invite volunteers to come to the board, write their answers and explain how they found them (Answers: 0.9397, found using the corresponding positive angle 110°, and corresponding acute angle 70°; b. 0.5736, found using the corresponding acute angle 55°).
4. Explain that this lesson is on the unit circle, which is a diagram that helps us to remember the sine and cosine ratios of common angles.

Teaching and Learning *(20 minutes)*

1. Draw the unit circle on the board or prepare it on vanguard:



Unit Circle[2]

[2] Licensed under a Creative Commons Attribution 4.0 International License. OpenStax College, Precalculus. OpenStax CNX. http://cnx.org/contents/fd53eae1-fa23-47c7-bb1b-972349835c3c@.

2. Explain:
    - This is a unit circle. It is drawn on the Cartesian plane so that the length of its radius is 1 unit.
    - Any point P on the circle forms an angle where each side of the angle is a radius of the circle.
    - Each point P on the circle has coordinates that are an ordered pair.
    - The x-value of the ordered pair is the cosine of the angle formed by P.
    - The y-value of the ordered pair is the sine of the angle formed by P.
3. Write on the board: $x = \cos\theta$, $y = \sin\theta$
4. Discuss:
    - Ask pupils to look at the unit circle and make observations about the angles that are labeled.
    - Encourage them to notice that the special angles (30°, 45°, 60°) are there. Angles that correspond to these are also there, in each quadrant.
5. Explain:
    - The angles of the unit circle are the special angles, and angles in each quadrant that correspond to them.
    - For example, look at angle 150°. Its corresponding acute angle (the acute angle it forms with the x-axis) is 30°, which is a special angle.
    - Notice that the cosine and sine functions for 150° and 30° are the same, except that the cosine function of 150° is negative. That is because it is in the 2nd quadrant.
    - Remember ACTS, the rule for deciding whether trigonometric ratios are positive or negative.
6. Write on the board: Find: a. $\sin 60°$ b. $\cos 240°$
7. Explain: We can find these without doing any calculation.
8. Solve a. using the unit circle:
    - Identify 60° on the unit circle.
    - Identify the y-coordinate of the point, $\frac{\sqrt{3}}{2}$.
    - Write on the board: $\sin 60° = \frac{\sqrt{3}}{2}$.
9. Solve b. using the unit circle:
    - Identify 240° on the unit circle.
    - Identify the x-coordinate of the point, $-\frac{1}{2}$.
    - Write on the board: $\cos 240° = -\frac{1}{2}$.
10. Write the following problems on the board: Find: a. $\sin 90°$ b. $\cos 315°$
11. Ask pupils to work with seatmates to solve the problems using the unit circle.
12. Walk around to check for understanding and clear misconceptions.
13. Invite volunteers to write the answers on the board and explain. (Answers: a. 1; b. $\frac{\sqrt{2}}{2}$)

Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Find $\cos 30°$
    b. Find $\sin 45°$
    c. Find $\cos 0°$
    d. Find $\sin 300°$
    e. Find $\sin 180°$
    f. Find the cosine and sine ratios for 225°.
    g. Write the following in ascending order: $\cos 120°$, $\cos 45°$, $\cos 330°$, $\cos 180°$
2. Ask pupils to work independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board and identify each answer for problems a. through f. on the unit circle and give the answer (Answers: a.$\frac{\sqrt{3}}{2}$; b.$\frac{\sqrt{2}}{2}$; c. 1; d. $-\frac{\sqrt{3}}{2}$; e. 0; f. $\cos 225° = -\frac{\sqrt{2}}{2}$, $\sin 225° = -\frac{\sqrt{2}}{2}$)
5. Invite another volunteer to share the solution to g.
   Solution:
   Find the cosine of each angle: $\cos 120° = -\frac{1}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$, $\cos 330° = \frac{\sqrt{3}}{2}$, $\cos 180° = -1$
   Order them from least to greatest: $-1, -\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}$
   Which gives: $\cos 180°, \cos 120°, \cos 45°, \cos 330°$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L109 in the Pupil Handbook.

Lesson Title: Problem solving with trigonometric ratios	**Theme:** Trigonometry	
Lesson Number: M2-L110	**Class:** SSS 2	**Time:** 40 minutes
 **Learning Outcome** By the end of the lesson, pupils will be able to solve various problems using the sine, cosine, and tangent ratios of any angle between 0° and 360°.	 **Preparation** 1. Bring trigonometric tables if available. 2. Write the problems in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: Find the value using the unit circle:
   a. $\sin 225°$ b. $\cos 315°$
2. Ask pupils to find the solutions with seatmates.
3. Invite volunteers to come to the board, write their answers and explain how they found them (Answers: a. $-\frac{\sqrt{2}}{2}$; b. $\frac{\sqrt{2}}{2}$).
4. Explain that this lesson is on problem solving using the trigonometric ratios. Pupils will use information that they have learned in previous lessons.

Teaching and Learning *(20 minutes)*

1. Write the following problem on the board: Without using a calculator, find the value of $\tan x$ if $x = 240°$.
2. Discuss: How would you solve this problem? What information would you use?
3. Allow pupils to respond, then explain:
   - We know that we can find a tangent if we have the sine and cosine, because $\tan x = \frac{\sin x}{\cos x}$.
   - We know $\sin 240°$ and $\cos 240°$ from the unit circle.
4. Invite volunteers to write the values of $\sin 240°$ and $\cos 240°$ on the board. (Answer: $\sin 240° = -\frac{\sqrt{3}}{2}$ and $\cos 240° = -\frac{1}{2}$)
5. Ask pupils to work with seatmates to find the answer to the problem.
6. Invite a volunteer to write the solution on the board.
   Solution:
   $\tan 240° = \frac{\sin 240°}{\cos 240°} = \left(-\frac{\sqrt{3}}{2}\right) \div \left(-\frac{1}{2}\right) = \left(-\frac{\sqrt{3}}{2}\right) \times \left(-\frac{2}{1}\right) = \sqrt{3}$
7. Write the following problem on the board: Without using a calculator or trigonometric table, find the value of $y = 6 \sin x + 2 \cos x$ if $x = 300°$.
8. Ask volunteers to explain how to solve the problem (Answer: substitute $x = 300°$ into the formula and solve for y. Use the values from the unit circle.)
9. Ask pupils to solve the problem with seatmates.
10. Ask a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} y &= 6\sin x + 2\cos x \\ &= 6\sin 300° + 2\cos 300° \\ &= 6\left(-\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right) && \text{Substitute the ratios} \\ &= -3\sqrt{3} + 1 && \text{Simplify} \end{aligned}$$

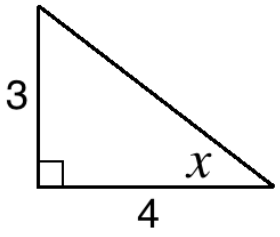
11. Write the following problem on the board: Without using trigonometry tables or calculators, simplify $\frac{3\tan 60°+2\cos 30°}{\sin 30°}$.
12. Ask volunteers to explain how to solve the problem. (Answer: These are special angles; substitute the value for each and simplify.)
13. Ask pupils to solve the problem with seatmates.
14. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} \frac{3\tan 60°+2\cos 30°}{\sin 30°} &= \frac{3\sqrt{3}+2\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} && \text{Substitute the special angle ratios} \\ &= (3\sqrt{3} + \sqrt{3}) \times 2 && \text{Simplify} \\ &= (4\sqrt{3}) \times 2 \\ &= 8\sqrt{3} \end{aligned}$$

15. Write the following problem on the board: Given that $\tan x = \frac{3}{4}$ where $0° \le x \le 90°$, find the value of $3\cos x$.
16. Ask volunteers to explain how to solve the problem. (Answer: We can draw a triangle and use the lengths of the sides to solve. We could also use inverse trigonometry to find the value of angle x, then substitute angle x in $3\cos x$.)
17. Solve on the board, explaining each step:

    Draw the triangle where $\tan x = \frac{3}{4}$. →



    Use Pythagoras' theorem to find the hypotenuse:

$$\begin{aligned} 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \\ \sqrt{25} &= \sqrt{c^2} \\ c &= 5 \end{aligned}$$

    Find $\cos x$: $\cos x = \frac{A}{H} = \frac{4}{5}$

    Therefore, $3\cos x = 3 \times \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$

Practice *(16 minutes)*

1. Write the following problems on the board:
   a. Find the value of $\tan 150°$ without using a calculator.
   b. If $\sin x = \frac{4}{5}$ and $0° \le x \le 90°$, find the value of $\tan x - \cos x$.

c. Given that $\tan x = \sqrt{3}$ where $0° \leq x \leq 90°$, find the value of $\frac{1+\sin^2 x}{\cos x}$.

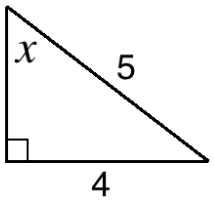
d. Without using a calculator or trigonometric table, find the value of $y = 4\sin x - \cos x$ if $x = 120°$.

2. Ask pupils to work independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board and write the solutions. Support them as needed.

Solutions:

a. $\tan 150° = \frac{\sin 150°}{\cos 150°} = \frac{1}{2} \div \left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{2} \times \left(-\frac{2}{\sqrt{3}}\right) = -\frac{1}{\sqrt{3}}$

b. Draw the triangle and find the missing side length, then solve:
Triangle:



Use Pythagoras' theorem to find the missing side:

$$\begin{aligned} a^2 + 4^2 &= 5^2 \\ a^2 + 16 &= 25 \\ a^2 &= 9 \\ \sqrt{a^2} &= \sqrt{9} \\ a &= 3 \end{aligned}$$

Find $\tan x$: $\tan x = \frac{O}{A} = \frac{4}{3}$

Find $\cos x$: $\cos x = \frac{A}{H} = \frac{3}{5}$

Therefore, $\tan x - \cos x = \frac{4}{3} - \frac{3}{5} = \frac{20}{15} - \frac{9}{15} = \frac{11}{15}$

c. Note that $\tan 60° = \sqrt{3}$. We can identify this because it is a special angle. Substitute $x = 60°$ into the formula:

$$\frac{1+\sin^2 x}{\cos x} = \frac{1+\sin^2 60°}{\cos 60°} = \frac{1+\left(\frac{\sqrt{3}}{2}\right)^2}{\frac{1}{2}} = \frac{1+\frac{3}{4}}{\frac{1}{2}} = \frac{\frac{7}{4}}{\frac{1}{2}} = \frac{7}{4} \div \frac{1}{2} = \frac{7}{2} \times \frac{2}{1} = 7$$

d. Substitute $x = 120°$ into the formula and simplify:

$$y = 4\sin 120° - \cos 120° = 4\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2}\right) = 2\sqrt{3} + \frac{1}{2}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L110 in the Pupil Handbook.

Lesson Title: Graph of $\sin\theta$	**Theme:** Trigonometry	
Lesson Number: M2-L111	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use the unit circle to draw the graphs of $\sin\theta$ for $0 \leq \theta \leq 360°$ and solve related trigonometric problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the following problem on the board: Draw the graph of $y = \sin x$ for values of x from 0° to 360°, using intervals of 45°.
2. Discuss and let pupils share their ideas:
    a. How do you think we would go about graphing a trigonometric function?
    b. What steps would you take to graph this?
3. Explain that this lesson is on graphing the sine function. The next 2 lessons will also be on graphing sine and cosine.

Teaching and Learning *(18 minutes)*

1. Explain:
    - We will graph the sine function using a table of values, just as we do for other types of functions.
    - The x-values in our table of values will be degrees between 0° and 360°. We want intervals of 45°, so add 45° to each x-value to get the next value for the table.
2. Draw the empty table of values on the board:

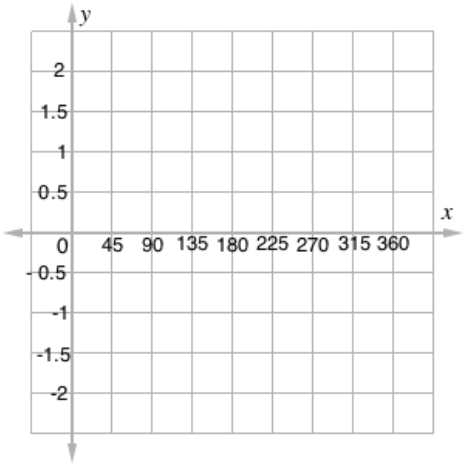
x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$\sin x$									

3. Ask volunteers to give the value of $\sin x$ for each angle in the table. Remind them to look at the unit circle.
4. As they give the values, write them in the table:

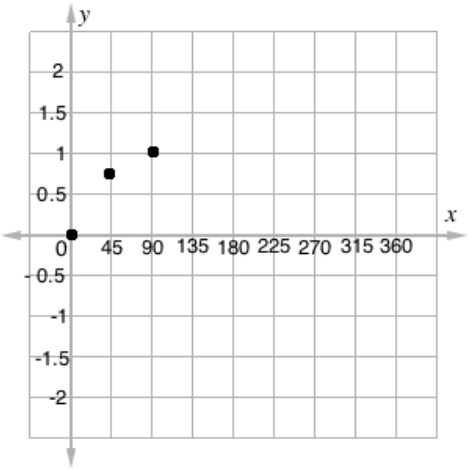
x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$\sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0

5. Explain: Now we have many points on the sine curve. Each of these sets of values can be written as an ordered pair and plotted on the Cartesian plane.
6. Write a few of the ordered pairs on the board: $(0°, 0), (45°, \frac{\sqrt{2}}{2}), (90°, 1)$
7. Ask a volunteer to give $\frac{\sqrt{2}}{2}$ as a decimal number. They can use a calculator or look for $\sin 45°$ in the sine table. (Answer: 0.7071)
8. Explain: Notice that all of our y-values in the table are between -1 and 1. Keep this in mind.

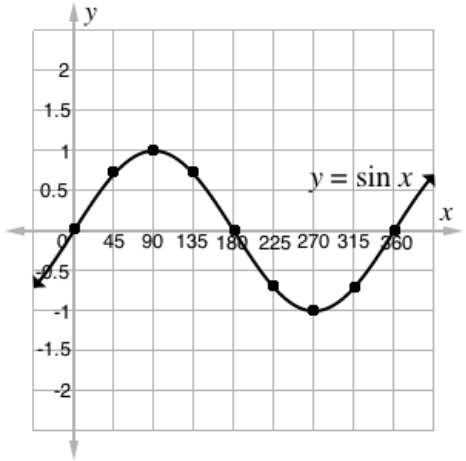
9. Draw an empty Cartesian plane on the board, labeling the axes as shown below:



10. Plot the first 3 points from the table on the Cartesian plane:

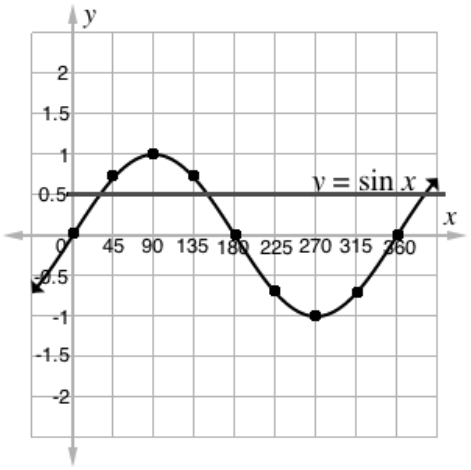


11. Invite volunteers to come to the board and plot the other points. Support them as needed.
12. Connect all the points in the curve, and label it as shown:



13. Explain:
    - This curve represents the sine function.
    - It goes on forever in both directions, but the same shape repeats. It always stays between -1 and 1 on the y-axis.
14. Write on the board: Use the graph to solve $\sin x = 0$

15. Explain:
    - We want to find the places on the graph of sine where $y = 0$.
    - This is similar to solving a quadratic equation. We have graphed the function, and we want to find where it crosses the x-axis.
16. Ask pupils to give the solutions. As they give them, write them on the board. (Answers: $x = 0°, 180°, 360°$)
17. Write on the board: Find the truth set of the equation $\sin x = \frac{1}{2}$.
18. Explain:
    - To find the truth set, we find all points in the given interval where this equation is true. This equation tells us that $y = \frac{1}{2}$.
    - Draw a horizontal line at $y = \frac{1}{2}$, and find all the points at which the line intersects the curve of $y = \sin x$.
19. Draw the horizontal line:



20. Ask pupils to identify the approximate x-values (within $0° \leq x \leq 360°$) at which the line and curve intersect. Write the answers on the board. (Answers: 30°, 150°)
21. Explain: These can also be observed in the unit circle. You can see that the sine function is $\frac{1}{2}$ at angles 30° and 150°.

Practice *(19 minutes)*

1. Write on the board:
    a. Draw the graph of $y = -2\sin x$ for values of x from 0° to 360°, using intervals of 45°.
    b. From the graph, find y when $x = 170°$.
    c. Find the truth set of the equation $-2\sin x = \frac{1}{2}$
2. Draw an empty table on the board for question a. Explain to pupils that the middle row ($\sin x$) is just to help them calculate correctly. They can fill this row first, then multiply by -2 and fill the last row.

x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$\sin x$									
$-2\sin x$									

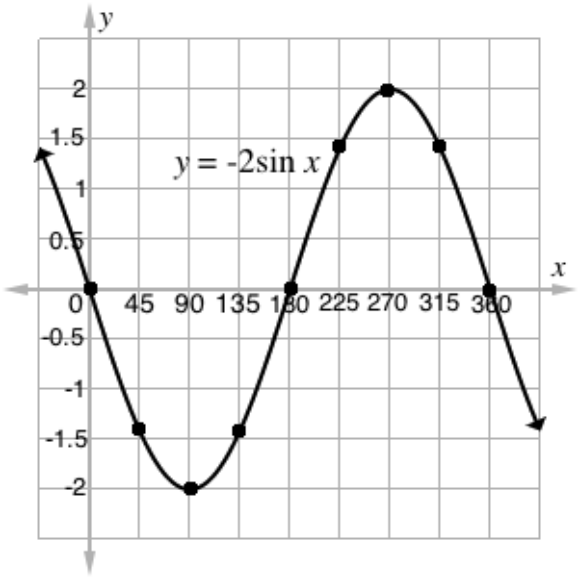
3. Work as a class if needed to complete a few columns in the table and plot the first few points on the graph.
4. Ask pupils to work with seatmates to complete the problems.
5. Walk around to check for understanding and clear misconceptions.
6. Invite a few volunteers to come to the board to complete the table of values and graph, and solve problems b. and c. Support them as needed.

   Solutions:

   a. Completed table:

x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$\sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0
$-2\sin x$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2	$\sqrt{2}$	0

   Graph:



   b. Identify 170 on the x-axis, and find the y-value at that point. It is slightly below the x-axis, at approximately $y = -0.3$.

   c. Draw a horizontal line at $y = \frac{1}{2}$ and identify the points of intersection with the curve $y = -2\sin x$. These are at approximately $x = 195°, 345°$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L111 in the Pupil Handbook.

Lesson Title: Graph of $\cos\theta$	**Theme:** Trigonometry	
Lesson Number: M2-L112	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use the unit circle to draw the graphs of $\cos\theta$ for $0 \leq \theta \leq 360°$ and solve related trigonometric problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the following problem on the board: Draw the graph of $y = \cos x$ for values of x from 0° to 360°, using intervals of 45°.
2. Discuss and let pupils share their ideas:
    a. How do you think we would go about graphing the cosine function?
    b. What steps would you take to graph this?
3. Explain that this lesson is on graphing the cosine function. This follows the same process as graphing the sine function in the previous lesson, but the graphs will look different.

Teaching and Learning *(20 minutes)*

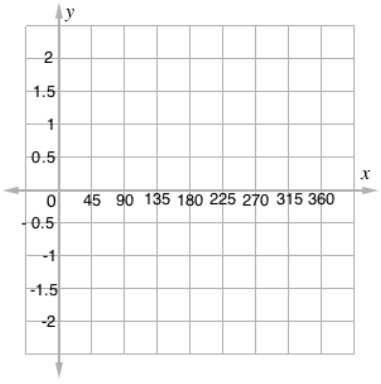
1. Draw the empty table of values on the board:

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$\cos x$									

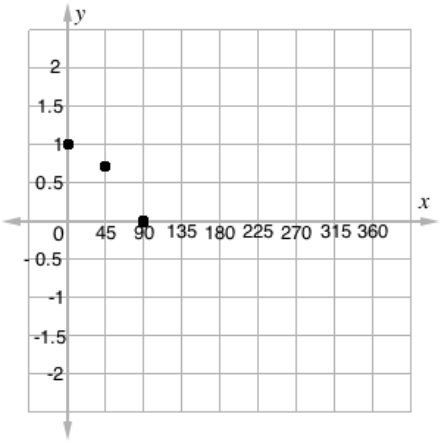
2. Ask volunteers to give the value of $\cos x$ for each angle in the table. Remind them to look at the unit circle.
3. As they give the values, write them in the table:

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$\cos x$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1

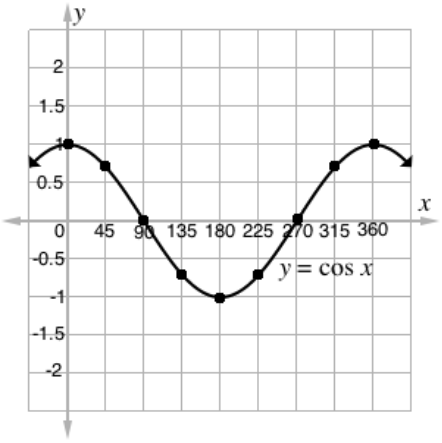
4. Explain: As with the sine function, all of our y-values in the table are between -1 and 1.
5. Draw an empty Cartesian plane on the board, labeling the axes as shown below:



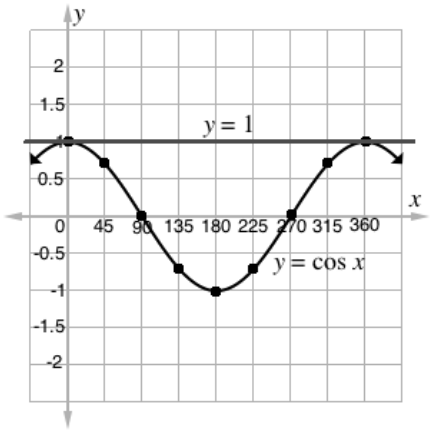
6. Plot the first 3 points from the table on the Cartesian plane:



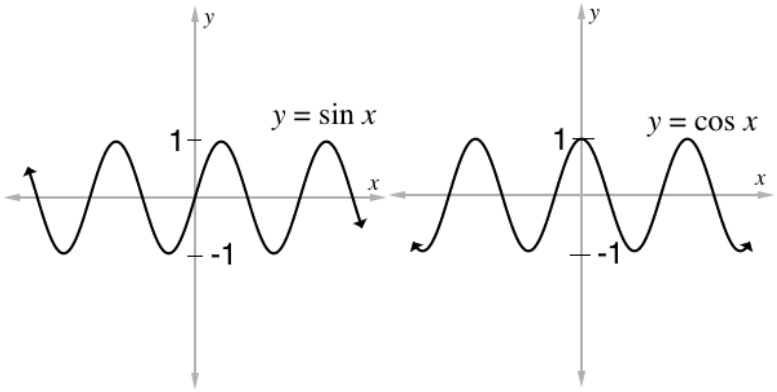
7. Invite volunteers to come to the board and plot the other points. Support them as needed.
8. Connect all of the points in the curve, and label it as shown:



9. Explain:
    - This curve represents the cosine function.
    - It goes on forever in both directions, but the same shape repeats. It always stays between -1 and 1 on the y-axis.
10. Write on the board: Use the graph to solve $\cos x = 0$
11. Explain: We want to find the places on the graph of the cosine where $y = 0$, or the places where the curve crosses the x-axis.
12. Ask pupils to give the solutions. As they give them, write them on the board. (Answers: $x = 90°, 270°$)
13. Write on the board: Find the truth set of the equation $\cos x = 1$.
14. Explain:
    - To find the truth set, we find all points where this equation is true. This equation tells us that $y = 1$.
    - Draw a horizontal line at $y = 1$, and find all the points at which the line intersects the curve of $y = \cos x$.
15. Draw the horizontal line:



16. Ask pupils to identify the approximate x-values at which the line and curve intersect. Write the answers on the board. (Answers: $x = 0°, 360°$)
17. Explain: These can also be observed in the unit circle. You can see that the cosine function is 1 at angles 0° and 360°.
18. Discuss and allow pupils to share their ideas:
    - Do the functions for sine and cosine look the same or different?
    - What are the differences? (Example answer: The sine function passes through the origin (0, 0), but the cosine function does not.)
19. Draw sketches of the sine and cosine functions side-by-side on the board, showing them stretching in both directions:



20. Explain:
    - All values for $\sin x$ and $\cos x$ lie between -1 and +1.
    - The sine and cosine curves have the same shape, but different starting points.

Practice *(17 minutes)*

1. Write on the board:
    a. Draw the graph of $y = 2\cos x$ for values of x from 0° to 180°, using intervals of 30°.
    b. From the graph, find y when $x = 100°$.
    c. Find the truth set of the equation $2\cos x = -1$.
2. Draw an empty table on the board for question a. Explain to pupils that the middle row ($\cos x$) is just to help them calculate correctly. They can fill this row first, then

multiply by 2 and fill the last row. Note that the x-values in this table count by 30s, because that is the interval we were given.

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$\cos x$							
$2\cos x$							

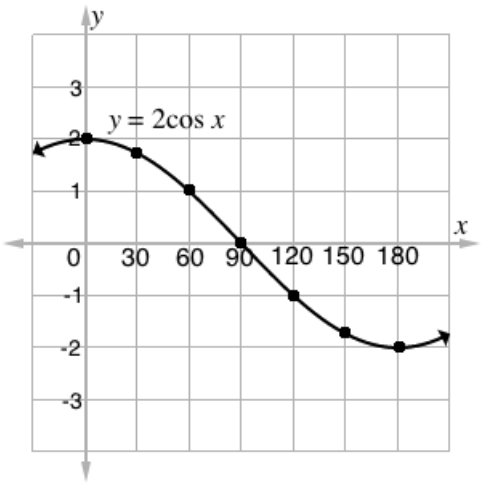
3. Work as a class if needed to complete a few columns in the table and plot the first few points on the graph.
4. Ask pupils to work with seatmates to complete the problems.
5. Walk around to check for understanding and clear misconceptions.
6. Invite a few volunteers to come to the board to complete the table of values and graph, and solve problems b. and c. Support them as needed.

   Solutions:

   a. Completed table:

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$2\cos x$	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2

   Graph:



   b. Identify 100 on the x-axis, and find the y-value at that point. It is slightly below the x-axis, at approximately $y = -0.3$.

   c. Draw a horizontal line at $y = -1$ and identify the points of intersection with the curve $y = 2\cos x$. The only intersection shown is at $x = 120°$. Therefore, the truth set is $x = 120°$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L112 in the Pupil Handbook.

Lesson Title: Graphs of $\sin\theta$ and $\cos\theta$	**Theme:** Trigonometry	
Lesson Number: M2-L113	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw the graphs of functions of the form $y = a\sin\theta + b\cos\theta$ for $0° \leq \theta \leq 360°$ and solve related trigonometric problems.	**Preparation** 1. Bring trigonometric tables if available. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the following problem on the board: Draw the graph of $y = \sin x + 2\cos x$ for values of x from 0° to 180°, using intervals of 30°.
2. Discuss and let pupils share their ideas:
    a. Is it possible to graph this function on the Cartesian plane?
    b. What steps would you take to graph this?
3. Explain that this lesson is on graphing functions where the sine and cosine functions appear together. It is possible to graph a function that contains both sine and cosine. This type of question is often on the WASSCE exam.

Teaching and Learning *(20 minutes)*

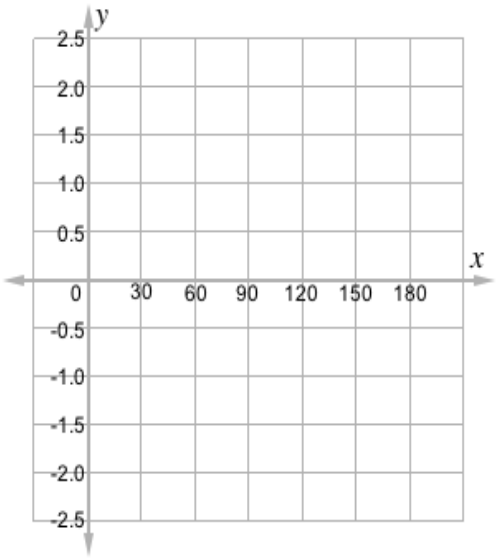
1. Draw the empty table of values on the board:

x	0°	30°	60°	90°	120°	150°	180°
$\sin x$							
$2\cos x$							
$\sin x + 2\cos x$							

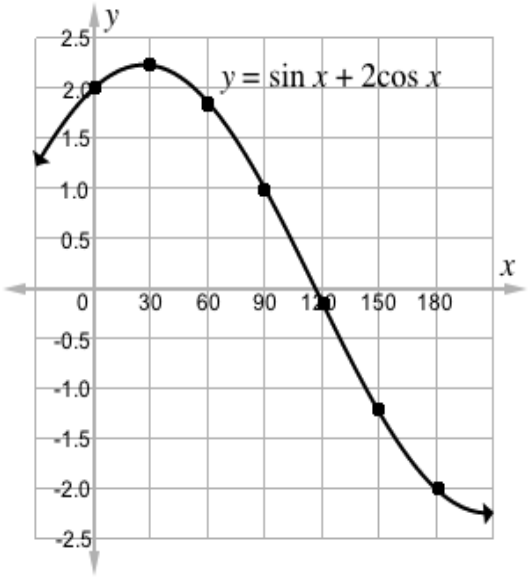
2. Explain:
    - To help us stay organized while doing calculations, we have a row for each trigonometric function.
    - In the last row, we will add the sine and cosine terms together and get the value of our function y.
3. Ask volunteers to give the values of $\sin x$ and $2\cos x$ for each angle in the table, correct to **1 decimal place**. They may use calculators or trigonometric tables to find the values.
4. As they give the values, write them in the table:

x	0°	30°	60°	90°	120°	150°	180°
$\sin x$	0	0.5	0.9	1	0.9	0.5	0
$2\cos x$	2.0	1.7	1.0	0	-1.0	-1.7	-2.0
$\sin x + 2\cos x$	2.0	2.2	1.9	1.0	-0.1	-1.2	-2.0

5. Draw an empty Cartesian plane on the board, labeling the axes as shown below:



6. Invite volunteers to come to the board and plot the points. Support them as needed.
7. Connect all of the points in a curve, and label it as shown:



8. Write on the board: Use the graph to solve $\sin x + 2\cos x = 0$
9. Ask pupils to give the solution and explain how they found it. Accept approximate answers. (Answer: $x = 116°$, because that is where the function crosses the x-axis.)
10. Write on the board: Find the truth set of the equation $\sin x = 2 - 2\cos x$.
11. Explain: To find the truth set, change the equation so that it has the same form as the function we graphed.
12. Write on the board and explain:

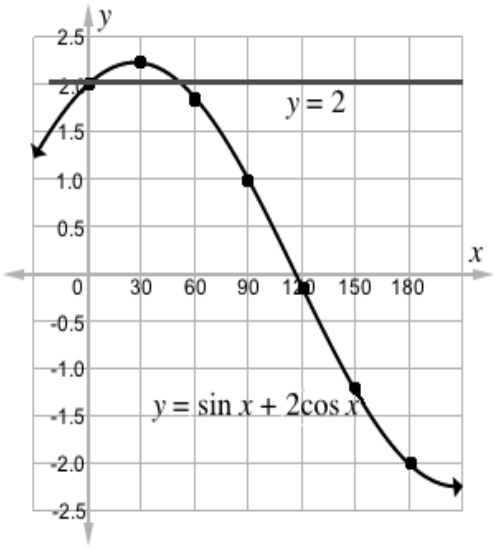
$$\sin x = 2 - 2\cos x$$

$$\sin x + 2\cos x = 2$$

13. Explain:

- The equation can be rewritten so that it is our original function set equal to 2.
- Draw a horizontal like at $y = 2$, and find all the points at which the line intersects the curve of $y = \sin x + 2\cos x$.

14. Invite a volunteer to draw the horizontal line on the board:



15. Ask pupils to identify the x-values at which the line and curve intersect. Write the answers on the board. (Answers: $x = 0°, 50°$ (approximately))

Practice *(17 minutes)*

1. Write on the board:
   a. Copy and complete the table of values, correct to one decimal place, for the relation $y = 3\sin x - \cos x$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$3\sin x - \cos x$	-1.0	0.6		3.0				-0.6	-2.1	-3.0		-2.4	

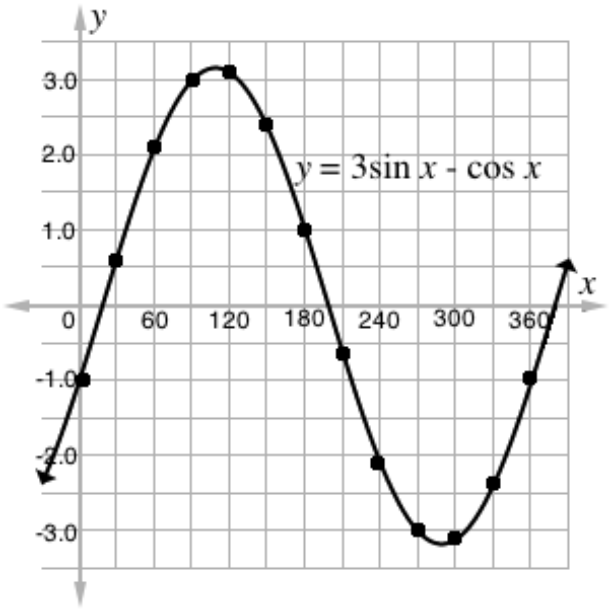
   b. Using scales of 2 cm to 30° on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of the relation $y = 3\sin x - \cos x$ for $0° \leq x \leq 360°$.
   c. Use the graph to solve $3\sin x - \cos x = 0$
   d. Find the truth set of the equation $\cos x + 3 = 3\sin x$
2. Explain: If you have a ruler, use it to make the marks on your x- and y-axes 2 centimetres apart. If you do not have a ruler, estimate 2 cm. What is important is that the tick marks on your axes are the same distance apart from one another.
3. Work as a class if needed to complete a few values in the table and plot the first few points on the graph.
4. Ask pupils to work with seatmates to complete the problems.
5. Walk around to check for understanding and clear misconceptions.
6. Invite a few volunteers to come to the board to write the solutions. Support them as needed.

Solutions:

a. Completed table:

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$3\sin x - \cos x$	-1.0	0.6	2.1	3.0	3.1	2.4	1.0	-0.6	-2.1	-3.0	-3.1	-2.4	-1.0

b. Graph:



c. Identify the points where the function intersects the x-axis. Approximate values are $x = 20°, 200°$

d. Rewrite the function: $3\sin x - \cos x = 3$. Draw a horizontal line at $y = 3$. It intersects the function at $x = 90°, 130°$ (approximately), so that is the truth set.

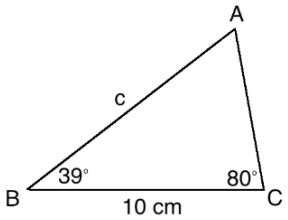
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L113 in the Pupil Handbook.

Lesson Title: The sine rule	**Theme:** Trigonometry	
Lesson Number: M2-L114	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to derive the sine rule and use it to calculate lengths and angles in triangles.	**Preparation** 1. Bring trigonometric tables and calculators if available. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

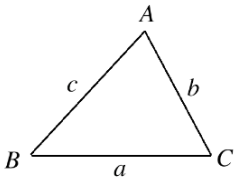
1. Write the following problem on the board: Find the length of missing side c:



2. Discuss and let pupils share their ideas:
    a. Is it possible to find the length of c with the information given?
    b. What steps would you take to solve this?
3. Explain that this lesson is on the sine rule. The sine rule allows us to solve for missing angles and sides in a triangle.

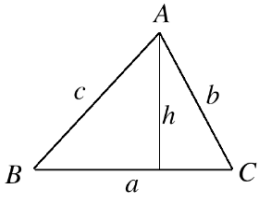
Teaching and Learning *(25 minutes)*

1. Explain:
    - We have previously solved for side lengths of right-angled triangles. We did this using Pythagoras' theorem and trigonometry.
    - We also used similar triangles and theorems to solve for the sides of certain triangles.
    - The sine rule allows us to solve for the missing side of any triangle, as long as we have enough information.
2. Write on the board: Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ for a triangle:



3. Explain:
    - For triangle ABC, the angles are usually labeled with capital letters, while the sides are labeled with lower case letters.
    - Using the sine rule, we can solve a triangle if we are given 2 angles and 1 side in the problem, or if we are given 2 sides and the angle opposite 1 of them.

4. Prove that the sine rule is true on the board. Explain each step to pupils.
   Step 1. Draw the perpendicular (height) from A to BC:



   Step 2. Write the sine ratio for angles B and C on the board:

$$\sin B = \frac{h}{c} \text{ and } \sin C = \frac{h}{b}$$

   Step 3. Solve each equation for h:

$$h = c \sin B \text{ and } h = b \sin C$$

   Step 4. Set the 2 formulae for h equal to one another:

$$c \sin B = b \sin C$$

   Step 5. Divide throughout by $\sin B$ and $\sin C$:

$$c = \frac{b \sin C}{\sin B}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

5. Explain:
   - We have shown part of the sine rule. We have shown that it is true for c and b.
   - We could draw an additional perpendicular from C to AB to show that the same is true for a. and b.
   - Thus, we have the sine rule, which says that the 3 fractions are equal.
6. Call pupils attention to the problem on the board from Opening.
7. Discuss: Now how do you think we will solve this? (Answer: Apply the sine rule; substitute the 2 known angles and the known side to find the unknown side.)
8. Solve on the board, explaining each step:

   Use two fractions from the sine rule: $\frac{a}{\sin A} = \frac{c}{\sin C}$

   Substitute the known values (a and C) into the formula:

$$\frac{10}{\sin A} = \frac{c}{\sin 80°}$$

   There are 2 unknowns. Find A by subtracting the known angles of the triangle from 180: $A = 180° - (39° + 80°) = 61°$

   Substitute $A = 61°$ into the formula:

$$\frac{10}{\sin 61°} = \frac{c}{\sin 80°}$$

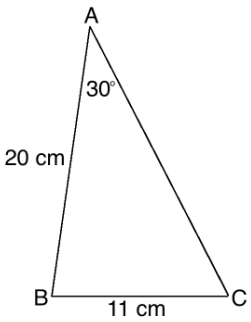
$$10 \times \sin 80° = c \times \sin 61°$$

$$c = \frac{10 \times \sin 80°}{\sin 61°} \quad \text{Solve for } c$$

$$c = \frac{10 \times 0.9848}{0.8746} \quad \text{Substitute values from the sine table}$$

$$c = 11.26 \text{ cm} \quad \text{Simplify}$$

9. Write the following problem on the board: Find angles B and C in the triangle below:



10. Ask pupils to explain how to solve the problem. As they give the steps, solve it on the board:

    We have enough info to find C with the formula: $\frac{a}{\sin A} = \frac{c}{\sin C}$.

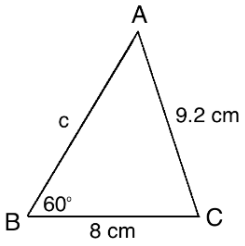
    Substitute the values and solve:

$$\frac{11}{\sin 30°} = \frac{20}{\sin C} \quad \text{Substitute in the formula}$$
$$11 \times \sin C = 20 \times \sin 30° \quad \text{Cross multiply}$$
$$\sin C = \frac{20 \times \sin 30°}{11} \quad \text{Solve for } C$$
$$\sin C = \frac{20 \times 0.5}{11} = \frac{10}{11}$$
$$\sin C = 0.9091$$
$$C = \sin^{-1} 0.9091 \quad \text{Take the inverse sine of both sides}$$
$$C = 65.38° \quad \text{Use the sine table}$$

    Subtract A and C from 180 to find B: $B = 180° - (30° + 65.38°) = 84.62°$

11. Write the following problem on the board: Find the remaining angles of ΔABC if $a = 8$ cm, $b = 9.2$ cm, and $\angle B = 60°$.
12. Ask pupils to work with seatmates to draw a sketch of the triangle.
13. Invite a volunteer to come to the board to draw and label the triangle.

    Answer:



14. Ask pupils to solve the problems with seatmates.
15. Walk around to check for understanding and clear misconceptions.
16. Invite a group of seatmates to write the solution on the board. All other pupils should check their work.

    Solution:

    Step 1. Find the measure of A.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{8}{\sin A} = \frac{9.2}{\sin 60°} \quad \text{Substitute in the formula}$$
$$9.2 \times \sin A = 8 \times \sin 60° \quad \text{Cross multiply}$$

$$\sin A = \frac{8 \times \sin 60°}{9.2}$$ Solve for A

$$\sin A = \frac{20 \times 0.8660}{11} = \frac{6.928}{9.2}$$

$$\sin A = 0.7530$$

$$A = \sin^{-1} 0.7530$$ Take the inverse sine of both sides

$$A = 48.85°$$ Use the sine table

Step 2. Find the measure of C.

$$C = 180° - (60° + 48.85°) = 71.15°$$

Practice *(12 minutes)*

1. Write on the board: In ΔABC, $a = 12$ cm, $b = 20$ cm, and $\angle B = 120°$. Solve the triangle completely.
2. Explain that "solving completely" means to find all of the missing sides and angles.
3. Ask pupils to work individually or with seatmates to complete the problem.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to come to the board to draw the triangle and write the solution.

Solution:

Step 1. Find A:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{12}{\sin A} = \frac{20}{\sin 120°}$$

$$\sin A = \frac{12 \times \sin 120°}{20}$$

$$\sin A = \frac{12 \times 0.8660}{20}$$

$$\sin A = 0.5196$$

$$A = \sin^{-1} 0.5196$$

$$A = 31.30°$$

Step 2. Find C:

$$C = 180° - (120° + 31.30°) = 28.7°$$

Step 3. Find c:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

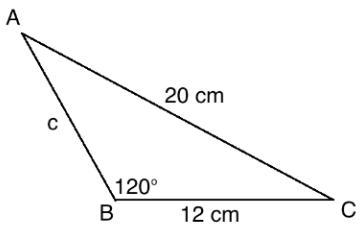
$$\frac{20}{\sin 120°} = \frac{c}{\sin 28.7°}$$

$$c = \frac{20 \times \sin 28.7°}{\sin 120°}$$

$$c = \frac{20 \times 0.4802}{0.8660}$$

$c = 11.1$ cm (to 1 d.p.)

Diagram:



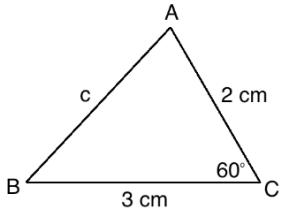
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L114 in the Pupil Handbook.

Lesson Title: The cosine rule	**Theme:** Trigonometry	
Lesson Number: M2-L115	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to derive the cosine rule and use it to calculate lengths and angles in triangles.	**Preparation** 1. Bring trigonometric tables and calculators if available. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

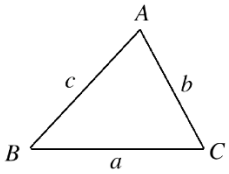
1. Write on the board: Find the length of missing side c:



2. Discuss and let pupils share their ideas:
    a. Is it possible to find the length of c with the information given?
    b. Can we use the sine rule to solve this?
3. Explain that this lesson is on the cosine rule. The cosine rule also allows us to solve for missing angles and sides in a triangle.

Teaching and Learning *(25 minutes)*

1. Explain:
    - We cannot use the sine rule to solve the problem on the board, because we do not have enough information. To use the sine rule, we must have at least one ratio of the formula.
    - The cosine rule allows us to solve for the missing side of other triangles, where the sine rule cannot be used.
    - We can use the cosine rule if two sides and the angle between them are given, as in the problem on the board.
2. Write on the board: Cosine rule: For a triangle:

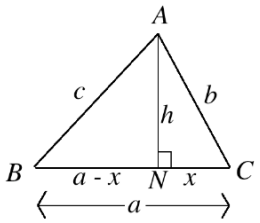


The following are true:

$$c^2 = a^2 + b^2 - 2ab\ \cos C$$
$$b^2 = a^2 + c^2 - 2ac\ \cos B$$
$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

3. Prove that the cosine rule is true on the board. Explain each step to pupils.

Step 1. Draw the perpendicular (height) from A to BC. Label the triangle as shown:



Step 2. Apply Pythagoras' theorem to the right-angled triangle ABN on the left-hand side of triangle ABC, then solve as follows:

$$\begin{aligned} c^2 &= (a-x)^2 + h^2 && \text{Pythagoras' theorem} \\ &= a^2 - 2ax + x^2 + h^2 && \text{Expand the binomial} \end{aligned}$$

In the triangle, $x^2 + h^2 = b^2$ (Pythagoras' theorem on triangle ACN). Substitute this in the equation above:

$$c^2 = a^2 - 2ax + b^2$$

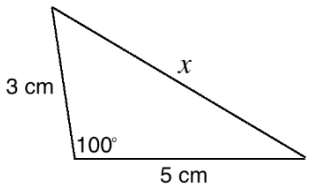
In triangle ACN, we can find the cosine ratio of C as: $\cos C = \frac{x}{b}$. Solving for x, we have $x = b\cos C$. Substitute this in the formula:

$$\begin{aligned} c^2 &= a^2 - 2ab\cos C + b^2 \\ &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

4. Explain: We have shown part of the cosine rule. We have shown that it is true for c^2. We could also show that it is true for a^2 and b^2 using a similar method.
5. Call pupils' attention to the problem on the board from Opening.
6. Discuss:
   - Now how do you think we will solve this? (Answer: Apply the cosine rule; substitute the known angle and the 2 known sides to find the unknown side.)
   - Which formula for cosine rule will we use? (Answer: $c^2 = a^2 + b^2 - 2ab\ \cos C$)
7. Solve on the board, explaining each step:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\ \cos C && \text{Formula} \\ &= 3^2 + 2^2 - 2(3)(2)\ \cos 60° && \text{Substitute values from triangle} \\ &= 3^2 + 2^2 - 2(3)(2)(0.5) && \text{Substitute } \cos 60° = 0.5 \\ &= 9 + 4 - 12\ (0.5) && \text{Simplify} \\ &= 13 - 6 \\ c^2 &= 7 \\ c &= \sqrt{7} = 2.65 \text{ cm to 2 d.p.} && \text{Take the square root of both sides} \end{aligned}$$

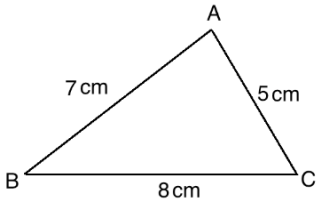
8. Write the following problem on the board: Find the length of x in the triangle below:



9. Ask pupils to explain how to solve the problem. As they give the steps, solve it on the board:

$$\begin{aligned} x^2 &= 3^2 + 5^2 - 2(3)(5)\cos 100° && \text{Substitute values from triangle} \\ &= 3^2 + 5^2 - 2(3)(5)(-0.1736) && \text{Substitute } \cos 100° = -0.1736 \\ &= 9 + 25 + 5.208 && \text{Simplify} \\ x^2 &= 39.208 \\ x &= \sqrt{39.208} && \text{Take the square root of both sides} \\ x &= 6.26 \text{ cm to 2 d.p.} \end{aligned}$$

10. Write the following problem on the board: Find the measures of angles A, B, and C:



11. Discuss: Can we use the cosine rule to solve this problem? If so, how?
12. Explain: We can change the subject of each formula for the cosine rule, and use the new formulae to find the angles.
13. Write the formulae on the board, with the cosine functions as the subject of the formulae:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

14. Solve the problem on the board, explaining each step:

$$\begin{aligned} \cos A &= \frac{5^2+7^2-8^2}{2(5)(7)} = \frac{25+49-64}{70} = \frac{10}{70} = 0.1429 \\ \cos A &= 0.1429 \\ A &= \cos^{-1} 0.1429 \\ A &= 81.8° \text{ to 1 d.p.} \end{aligned}$$

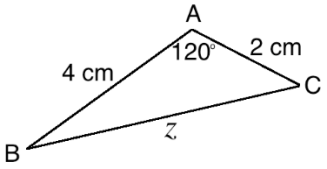
$$\begin{aligned} \cos B &= \frac{7^2+8^2-5^2}{2(7)(8)} = \frac{49+64-25}{112} = \frac{88}{112} = 0.7857 \\ \cos B &= 0.7857 \\ B &= \cos^{-1} 0.7857 \\ B &= 38.2° \text{ to 1 d.p.} \end{aligned}$$

$$\cos C = \frac{5^2+8^2-7^2}{2(5)(8)} = \frac{25+64-49}{80} = \frac{40}{80} = 0.5$$
$$\cos C = 0.5$$
$$C = \cos^{-1} 0.5$$
$$C = 60°$$

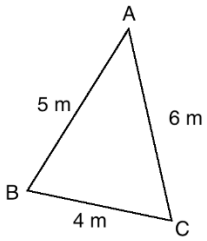
15. Check the work by adding the angles together:
$$A + B + C = 81.8° + 38.2° + 60° = 180°$$
16. Explain: After using the formulae, it is always a good idea to check your work by adding. Remember that the angles of any triangle sum to 180°.

Practice *(12 minutes)*

1. Write the following problems on the board:
   a. Find the length of z:



   b. Find the measure of angle A:



2. Ask pupils to work individually or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to draw the triangle and write the solution.
   Solutions:
   a.

z^2	$=$	$4^2 + 2^2 - 2(4)(2)\cos 120°$	Substitute values from triangle
	$=$	$4^2 + 2^2 - 2(4)(2)(-0.5)$	Substitute $\cos 120° = -0.5$
	$=$	$16 + 4 + 8$	Simplify
z^2	$=$	28	
z	$=$	$\sqrt{28}$	Take the square root of both sides
z	$=$	5.29 cm to 2 d.p.	

   b.
$$\cos A = \frac{6^2+5^2-4^2}{2(6)(5)} = \frac{36+25-16}{60} = \frac{45}{60} = 0.75$$
$$A = \cos^{-1} 0.75$$
$$A = 41.41° \text{ to 2 d.p.}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L115 in the Pupil Handbook.

Lesson Title: Application of sine and cosine rules	**Theme:** Trigonometry	
Lesson Number: M2-L116	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use the sine and cosine rules to solve triangles.	**Preparation** 1. Bring trigonometric tables and calculators if available. 2. Ask pupils to bring calculators, or allow them to use calculators on their mobile phones, if either are available.	

Opening *(4 minutes)*

1. Ask volunteers to simultaneously come to write the sine and cosine rules on the board. They may look at their notes or PH.

   Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

   Cosine rule:

   To solve for sides:

$$c^2 = a^2 + b^2 - 2ab\ \cos C$$
$$b^2 = a^2 + c^2 - 2ac\ \cos B$$
$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

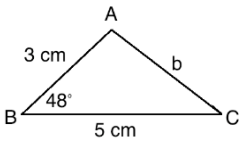
   To solve for angles:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

2. Explain that this lesson is on using the sine and cosine rules to solve problems. Pupils will need to decide which rule to use for a given problem.

Teaching and Learning *(16 minutes)*

1. Ask volunteers to explain when to use the sine rule, and when to use the cosine rule. Allow discussion. Write the correct answers on the board:
   - **Sine rule:** Use when given 2 angles and any side, or 2 sides and the angle opposite 1 of them.
   - **Cosine rule:** Use when given two sides and the included angle.
2. Write the problem on the board: In the triangle below, find b, A, and C.



3. Discuss:
   - How would you solve this problem? Which rule or rules would you use?

- What would you do first?

4. Allow pupils to share their ideas, then explain:
   - We can use the cosine rule to find the missing side, b.
   - After finding b., we can use either the sine rule or the cosine rule for finding angles to find A and C. It will be easier to use the sine rule to find the angles.
   - Find the value of the smaller angle (C) first. Finding the sine of an acute angle is less ambiguous than finding the sine of an obtuse angle.
   - Once we have 2 angles, we can solve for the 3rd by subtracting from 180°.

5. Solve on the board, explaining each step:

Step 1. Use the cosine rule to find b:

b^2	$=$	$a^2 + c^2 - 2ac\ \cos B$	Formula
	$=$	$3^2 + 5^2 - 2(3)(5)\ \cos 48°$	Substitute values from triangle
	$=$	$3^2 + 5^2 - 2(3)(5)(0.6691)$	Substitute $\cos 48° = 0.6691$
	$=$	$9 + 25 - 30\ (0.6691)$	Simplify
	$=$	$34 - 20.073$	
b^2	$=$	13.927	
b	$=$	$\sqrt{13.927} = 3.73$ cm to 2 d.p.	Take the square root of both sides

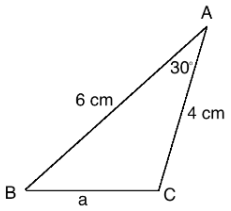
Step 2. Use the sine rule to find C:

	$\frac{3.73}{\sin 48°}$	$=$ $\frac{3}{\sin C}$	
$3.73 \times \sin C$	$=$	$3 \times \sin 48°$	Cross-multiply
$\sin C$	$=$	$\frac{3 \times \sin 48°}{3.73}$	
$\sin C$	$=$	$\frac{3 \times 0.7431}{3.73}$	Substitute values from the sine table
$\sin C$	$=$	0.5977	Simplify
C	$=$	$\sin^{-1} 0.5977$	
C	$=$	$36.71°$	

Step 3. Subtract to find A:

$$A = 180° - (48° + 36.71°) = 95.29°$$

6. Write the following problem on the board: Find all of the missing sides and angles in the triangle below:



7. Ask pupils to work with seatmates to solve the problem.
8. Invite a volunteer to write the solution on the board.

Solution:

Step 1. Use the cosine rule to find a:

$$
\begin{aligned}
a^2 &= 6^2 + 4^2 - 2(6)(4)\ \cos 30° && \text{Substitute values from triangle}\\
&= 6^2 + 4^2 - 2(6)(4)(0.8660) && \text{Substitute } \cos 30° = 0.8660\\
&= 36 + 16 - 48\ (0.8660) && \text{Simplify}\\
&= 52 - 41.568\\
a^2 &= 10.432\\
a &= \sqrt{10.432} = 3.23 \text{ cm to 2 d.p.} && \text{Take the square root of both sides}
\end{aligned}
$$

Step 2. Use the sine rule to find B:

$$
\begin{aligned}
\frac{3.23}{\sin 30°} &= \frac{4}{\sin B}\\
3.23 \times \sin B &= 4 \times \sin 30° && \text{Cross-multiply}\\
\sin B &= \frac{4 \times \sin 30°}{3.23}\\
\sin B &= \frac{4 \times 0.5}{3.23} && \text{Substitute } \sin 30° = 0.5\\
\sin B &= 0.6192 && \text{Simplify}\\
B &= \sin^{-1} 0.6192\\
B &= 38.26°
\end{aligned}
$$

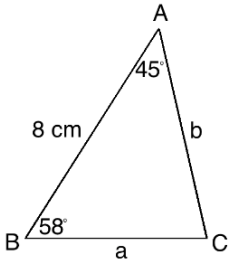
Step 3. Subtract to find C:

$$C = 180° - (30° + 38.26°) = 111.74°$$

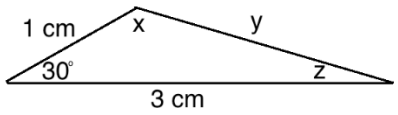
Practice *(19 minutes)*

1. Write the following problems on the board: Solve the triangles for all of the unknown sides and angles.

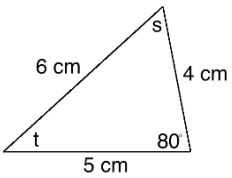
a.



b.



c.



2. Explain: You must choose the rule, or rules, to use for each problem. You may discuss the best strategy with your neighbor before solving.
3. Allow pupils to work individually or with seatmates to solve the problems.
4. Walk around to check for understanding and clear misconceptions. If needed, involve pupils in a class discussion to determine which rule or rules to use.
5. Invite volunteers to come to the board to write the solutions.

   Solutions:

   a. **Step 1.** Subtract to find C: $C = 180° - (58° + 45°) = 77°$

   Step 2. Use the sine rule to find a:

$$
\begin{aligned}
\frac{a}{\sin 45°} &= \frac{8}{\sin 77°}\\
a &= \frac{8 \times \sin 45°}{\sin 77°} && \text{Solve for } c
\end{aligned}
$$

$$a = \frac{8 \times 0.7071}{0.9744}$$ Substitute values from the sine table

$$a = 5.81 \text{ cm}$$ Simplify

Step 3. Use the sine rule to find b:

$$\frac{b}{\sin 58°} = \frac{8}{\sin 77°}$$

$$b = \frac{8 \times \sin 58°}{\sin 77°}$$ Solve for c

$$b = \frac{8 \times 0.8480}{0.9744}$$ Substitute values from the sine table

$$b = 6.96 \text{ cm}$$ Simplify

b. **Step 1.** Use the cosine rule to find y:

$$y^2 = 1^2 + 3^2 - 2(1)(3)\ \cos 30°$$

$$= 1 + 9 - 6\ (0.8660)$$

$$= 10 - 5.196$$

$$y^2 = 4.804$$

$$y = \sqrt{4.804} = 2.19 \text{ cm to 2 d.p.}$$

Step 2. Use the sine rule to find z:

$$\frac{1}{\sin z} = \frac{2.19}{\sin 30°}$$

$$2.19 \times \sin z = \sin 30°$$ Cross-multiply

$$\sin z = \frac{\sin 30°}{2.19}$$

$$\sin z = \frac{0.5}{2.19}$$ Substitute $\sin 30° = 0.5$

$$\sin z = 0.2283$$ Simplify

$$z = \sin^{-1} 0.2283$$

$$z = 13.2°$$

Step 3. Subtract to find x: $x = 180° - (30° + 13.2°) = 136.8°$

c. Either the sine or cosine rule can be used to find one of the angles, then subtract from 180° to find the other angle. The sine rule is shown:

Step 1. Use the sine rule to find t:

$$\frac{4}{\sin t} = \frac{6}{\sin 80°}$$

$$6 \times \sin t = 4 \sin 80°$$ Cross-multiply

$$\sin t = \frac{4 \sin 80°}{6}$$

$$\sin t = \frac{4(0.9848)}{6}$$ Substitute $\sin 80° = 0.9848$

$$\sin t = 0.6565$$ Simplify

$$t = \sin^{-1} 0.6565$$

$$t = 41.03°$$

Step 2. Subtract to find s: $s = 180° - (41.03° + 80°) = 58.97°$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L116 in the Pupil Handbook.

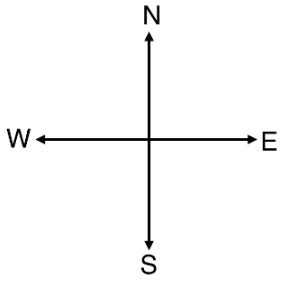
Lesson Title: Compass bearings	**Theme:** Bearings	
Lesson Number: M2-L117	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Interpret bearings in terms of compass directions. 2. Draw diagram representations of bearing statements.	**Preparation** Bring a protractor, and ask pupils to bring protractors if available.	

Opening *(4 minutes)*

1. Discuss:
    a. What do you think of when I say the word “compass”?
    b. What types of compasses do you know of?
2. Allow pupils to share their ideas and discuss. Remind them that there are 2 types of compasses: the tool for geometry construction, and the navigational tool.
3. Explain that this lesson is on compass bearings. Compass bearings are related to the navigational compass.

Teaching and Learning *(20 minutes)*

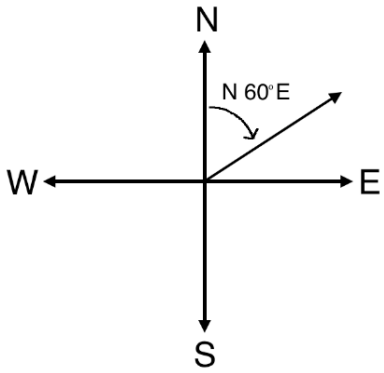
1. Discuss: What are navigational compasses used for? What types of situations or professions would need navigational compasses?
2. Encourage various responses, then explain:
    - Navigational compasses are used by ships to determine the correct direction or route to travel. They can also be used on land for the same purpose.
    - Information from compasses can also be used to determine distances on land or on water.
    - Now we have GPS and other technology that helps us to navigate. Although we use compass bearings less often in modern times, they are still an accurate way to navigate.
3. Draw the compass directions on the board:



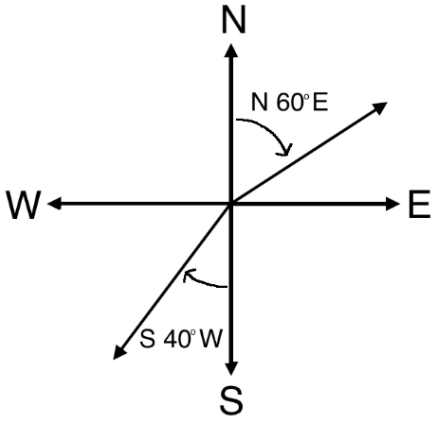
4. Explain: Compass bearings are measured from north or south.
5. Write on the board: N 60° E
6. Explain:

- This means 60° east of north.
- To find this direction, we will use a protractor to measure 60° from north, toward east.

7. Use a protractor to measure 60° from north on the compass on the board. Mark the bearing with an arrow, as shown below.

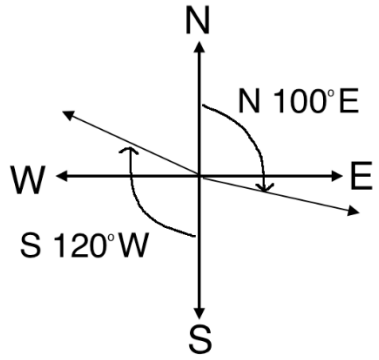


8. Write on the board: S 40° W
9. Explain:
    - This means 40° west of south.
    - To find this direction, use a protractor to measure 40° from south, toward west.
10. Draw and label this on the board, as shown below:

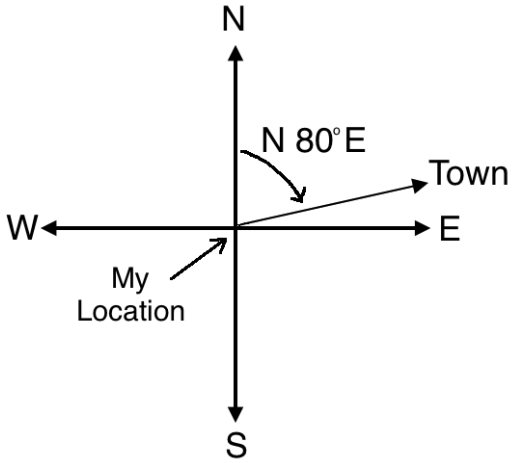


11. Write on the board: Draw and label the bearings:
    a. N 100° E
    b. S 120° W
12. Ask pupils to work with seatmates to draw a compass and these 2 bearings.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to draw the solutions on the board. They may do so on one compass.

Solutions:



15. Write on the board: From our location, a certain town is on a bearing N 80° E.
16. Explain: This means that if we face north, we will find the town at a bearing of 80° toward the east.
17. Stand and face north in your classroom. If you do not know which direction is north, choose a direction that you think could be north, and tell pupils that you will assume you are facing north.
18. Discuss: I want to turn in the direction of the town. Which way should I turn? How far should I turn?
19. Allow pupils to share their ideas, and turn in the directions they tell you. Then, explain: From north, we want to turn 80° toward the east. Remember that 90° is a right angle, so I will turn to my right side slightly less than that.
20. Turn your body and face a direction that is approximately 80° from north.
21. Explain: If I walked in this direction, eventually I would reach the town.
22. Draw this in a diagram on the board:



Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Draw a compass showing north, east, south, and west.
    b. On your compass, draw:
        i. N 25° E

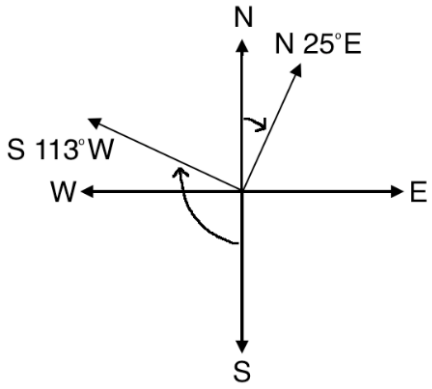
ii. S 113° W

c. A ship captain sees another ship at a bearing of N 60° E from his location. Draw this in a diagram.

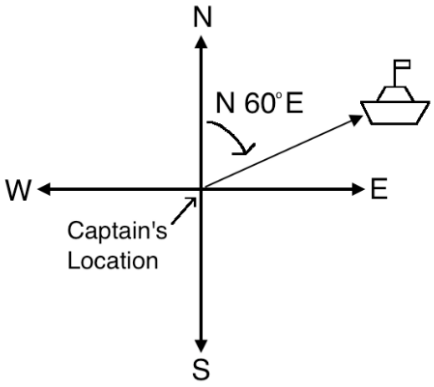
2. Ask pupils to work individually to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to draw the solutions.

Solutions:

a. and b.:



c. Accept accurate drawings. Example:



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L117 in the Pupil Handbook.

Lesson Title: Three figure bearings	**Theme:** Bearings	
Lesson Number: M2-L118	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify angles measured clockwise from the geographic north. 2. Represent bearings as angles in three digits. 3. Solve simple problems involving three figure bearings.	**Preparation** Bring a protractor, and ask pupils to bring protractors if available.	

Opening *(3 minutes)*

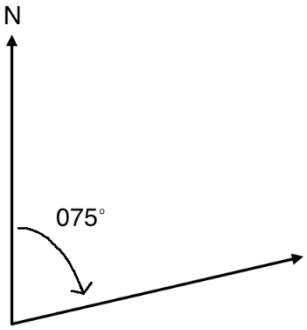
1. Discuss:
    a. What are bearings used for? (Example answers: navigation, determining direction and distance)
    b. How many degrees are there in a full revolution? (Answer: 360°)
2. Allow pupils to share their ideas and discuss.
3. Explain that this lesson is on three figure bearings. These are used for the same purpose as compass bearings, but are represented differently.

Teaching and Learning *(21 minutes)*

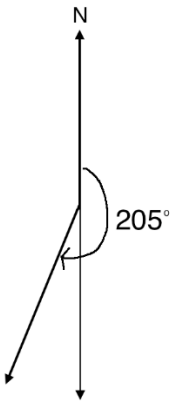
1. Explain: Three-figure bearings are bearings given in 3 digits. These 3 digits give the angle of the bearing from geographic north.
2. Draw an arrow pointing north on the board:

3. Explain:
    - Three-figure bearings give the angle in the clockwise direction.
    - The angles range from 000° to 360°. They must always have 3 digits, even when they're actually less than 100 degrees.
4. Write on the board: 000° - 360°
5. Use a protractor to draw and label 009° on the board:

6. Use a protractor to draw and label 075° on the board:

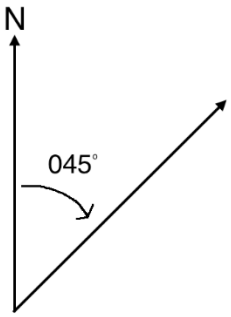


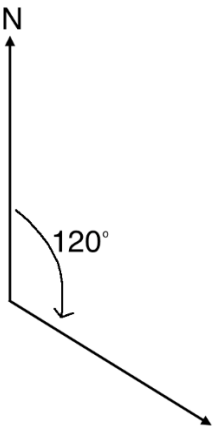
7. Use a protractor to draw and label 205° on the board. Extend the vertical line and use it to measure 25° clockwise from south (because $205° - 180° = 25°$).

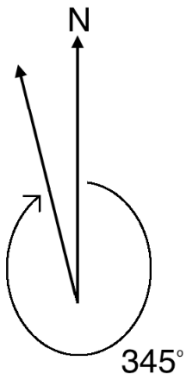


8. Write on the board: Draw diagrams for the following three point bearings:
    a. 045°
    b. 120°
    c. 345°
9. Ask pupils to work with seatmates to draw the diagrams.
10. Walk around to check for understanding and clear misconceptions.
11. Invite volunteers to share their drawings, or draw sketches on the board. For the sake of time, they do not need to do the work again with a protractor.

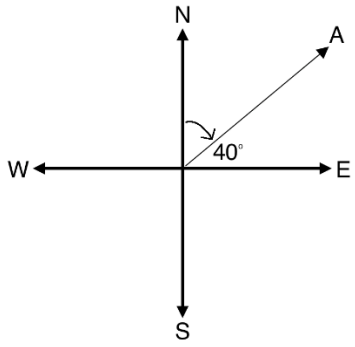
Answers:

a. 

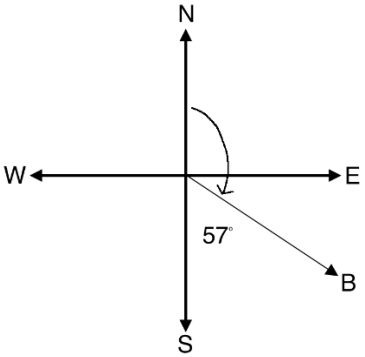
b. 

c. 

12. Write the following problem on the board: Find the three point bearing of A:



13. Discuss: How can we find the bearing of A? (Answer: the 4 directions are given, and we know east is 90° from north. Subtract the given angle from 90°.)
14. Solve on the board: $A = 90° - 40° = 50°$; $A = 050°$
15. Draw the following problem on the board: Find the three point bearing of B:

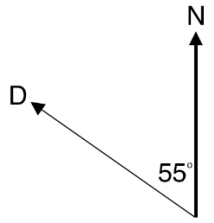


16. Discuss: How can we find the bearing of B? (Answer: We know south is 180° from north. Subtract the given angle from 180°.)
17. Ask pupils to solve the problem with seatmates.
18. Invite a volunteer to write the solution on the board. (Answer: $B = 180° - 57° = 123°$)

Practice *(15 minutes)*

1. Write the following problems on the board:

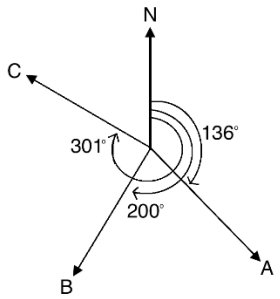
a. Draw points with the following bearings from north (N) on one diagram:
   A: 136°
   B: 200°
   C: 301°
b. A ship at sea is on a bearing of 068° from your current location. Draw a diagram for this.
c. Find the three point bearing of D in the diagram:



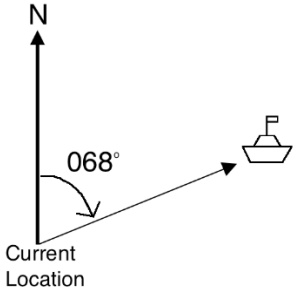
5. Ask pupils to work individually to solve the problems.
6. Walk around to check for understanding and clear misconceptions.
7. Invite volunteers to come to the board to write the solutions.

Solutions:

a.



b. Accept accurate drawings. Example:



c. Subtract the given angle from 360° to find the angle that D makes when the line rotates clockwise from N: $360° - 55° = 305°$

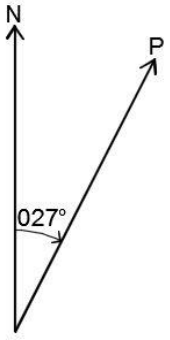
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L118 in the Pupil Handbook.

Lesson Title: Reverse bearings	**Theme:** Bearings	
Lesson Number: M2-L119	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to find the reverse bearing of a given bearing.	**Preparation** 1. Bring a protractor, and ask pupils to bring protractors if available. 2. Write the problem in Opening on the board.	

Opening *(3 minutes)*

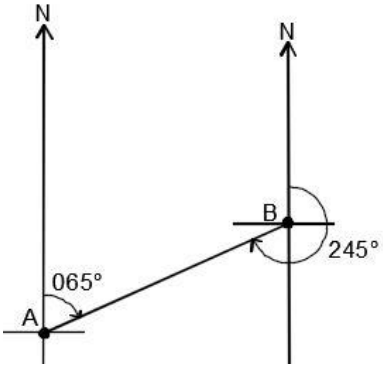
1. Write a revision problem on the board: Point P is at a bearing of 027° from north. Draw a diagram showing P.
2. Ask pupils to work with seatmates to draw the diagram.
3. Invite a volunteer to share their drawing with the class.
   Solution:



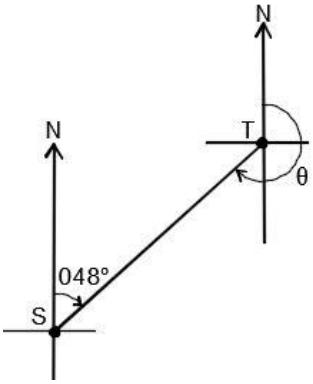
4. Explain that this lesson is on reverse bearings.

Teaching and Learning *(21 minutes)*

1. Discuss: What is the meaning of “reverse”? (Example answers: Opposite, to go backwards or in the opposite direction.)
2. Explain:
   - When we talk about “reverse” bearings, we must have 2 points.
   - Consider 2 points A and B. We have the bearing from A to B, and we have the bearing from B to A.
   - These are different, because bearings are about direction. A to B is a different direction than B to A. They are reverse.
   - Reverse bearings are sometimes called back bearings.
3. Draw the following diagram on the board:

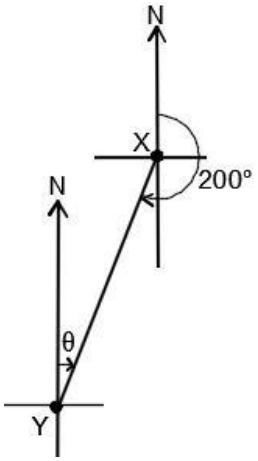


4. Explain:
    - The bearing from A to B is 065°.
    - The bearing from B to A is 245°.
    - For both bearings, we use the line that joins them and the north direction. We find the bearing of the line joining them from north.
5. Write on the board:
    Reverse bearing $= \theta + 180°$ if θ is less than $180°$
    Reverse bearing $= \theta - 180°$ if θ is more than $180°$
6. Explain: Depending on the size of the first bearing, you will add or subtract 180° to find the reverse bearing.
7. Show that this is true for the example given on the board.
8. Explain:
    - The bearing of B from A is 065°, which is less than 180°.
    - We will add 180° to find the reverse bearing.
9. Write on the board: $65° + 180° = 245°$
10. Write the following problem on the board: If the bearing of T from S is 048°, find the bearing of S from T.
11. Explain: There are 2 ways that bearings can be described in problems. “The bearing of T from S” is the same as “the bearing from S to T”.
12. Write each statement on the board if needed, and ensure that pupils understand.
13. Ask pupils to work with seatmates to draw a diagram for the problem. Remind them to show north, both points, and the known angle.
14. Invite a volunteer to share their drawing with the class.
    Diagram:



15. Ask pupils to work with seatmates to solve the problem.

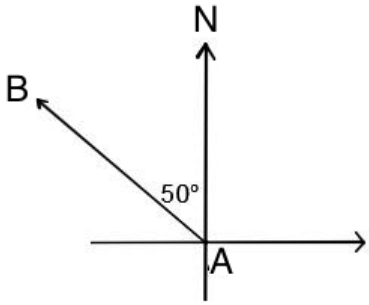
16. Invite a volunteer to write the solution on the board. (Solution: $\theta = 48° + 180° = 228°$)
17. Write the following problem on the board: If the bearing of X from Y is 200°, Find the bearing from Y to X.
18. Ask pupils to work with seatmates to draw a diagram for the problem. Remind them to show north, both points, and the known angle.
19. Invite a volunteer to share their drawing with the class.
    Diagram:



20. Ask pupils to work with seatmates to solve the problem.
21. Invite a volunteer to write the solution on the board. (Solution: $\theta = 200° - 180° = 020°$)
22. Remind pupils that three point bearings have 3 digits. They should write $020°$ instead of $20°$.

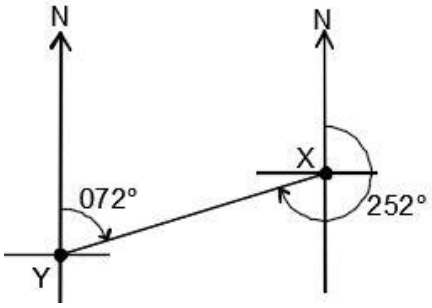
Practice *(15 minutes)*

1. Write the following problems on the board:
    a. The bearing of X from Y is 072°. Draw a diagram and find the bearing of Y from X.
    b. In the diagram below, find:
        i. The bearing of B from A
        ii. The bearing of A from B

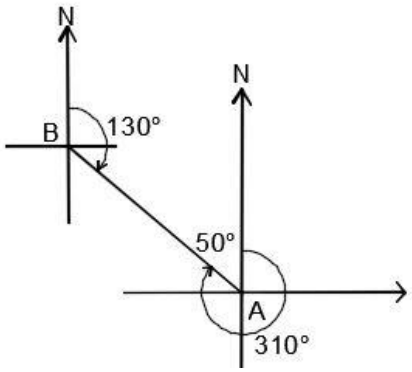


2. Ask pupils to work individually to solve the problems. Allow discussion with seatmates if needed.

3. Walk around to check for understanding and clear misconceptions. If needed, remind pupils of the information from the previous lesson that is needed to solve problem b part i.
4. Invite volunteers to come to the board to write the solutions.
   Solutions:
   a. Diagram:



   Solution: $72° + 180° = 252°$
   b. i. Find the bearing from north. Subtract the given angle (50°) from 360°: $360° - 50° = 310°$.
   ii. Find the reverse bearing using the result from part i: $310° - 180° = 130°$.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L119 in the Pupil Handbook.

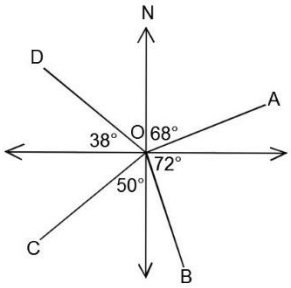
Lesson Title: Bearing problem solving – Part 1	**Theme:** Bearings	
Lesson Number: M2-L120	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw diagrams and solve bearings problems that do not involve distances.	**Preparation** 1. Bring a protractor, and ask pupils to bring protractors if available. 2. Write the problems in Opening and Teaching and Learning on the board.	

Opening *(3 minutes)*

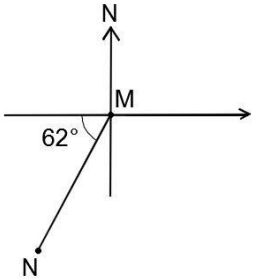
1. Write a revision problem on the board: If the bearing from A to B is 017°, find the bearing from B to A.
2. Ask pupils to work with seatmates to solve the problem.
3. Invite a volunteer to write the solution on the board. (Solution: $17° + 180° = 197°$)
4. Explain that this lesson is on solving bearings problems. Pupils will use information from the previous lessons on bearings and reverse bearings to solve problems.

Teaching and Learning *(21 minutes)*

1. Write the following problems on the board:
    a. Find the three-figure bearings of the points A, B, C, and D from point O:

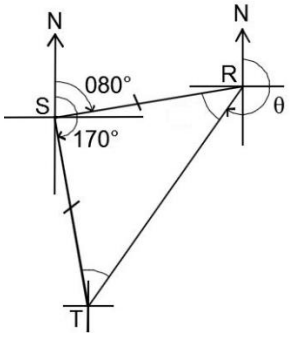


    b. In the diagram below, find:
        i. The bearing of N from M
        ii. The bearing of M from N



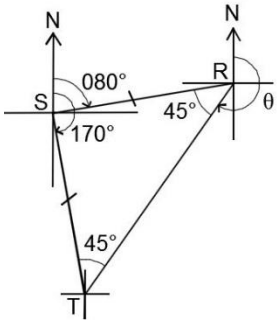
2. Discuss: How would you solve each problem? What steps would you take?

3. Allow pupils to discuss as a class for a few minutes. Guide them to the correct methods. Accept accurate responses, including:
    a. Add the given angles to 90° or 180° to get the full rotation of each point from north.
    b. i. Use the given angle to solve for the angle formed by MN and the south direction, and add this to 180° to find the full rotation.
    ii. Find the reverse bearing of that in part i.
4. Ask pupils to work with seatmates to solve the problems.
5. Walk around to check for understanding and clear misconceptions.
6. Invite volunteers to write the solutions on the board.
    Solutions:
    a. Point A: $068°$
    Point B: $90° + 72° = 162°$
    Point C: $180° + 50° = 230°$
    Point D: $180° + 90° + 38° = 308°$
    b. i. The angle formed by MN and the south direction: $90° - 62° = 28°$
    Bearing of N from M: $180° + 28° = 208°$
    ii. Subtract to find the reverse bearing: $208° - 180° = 028°$
7. Write the following problem on the board: The bearing of R from S is 080°, and the bearing of T from S is 170°, where R, S, and T are 3 points on the plane. If S is equidistant from R and T, find the bearing of T from R.
8. Explain:
    - We need to draw a diagram first to help us visualise this problem.
    - Let's go through each piece of information and draw our diagram.
9. Ask pupils to give information from the problem that can be drawn. Discuss as a class.
10. Explain:
    - We first draw the points in the problem using the bearings given.
    - We can then use other information to label the diagram.
    - The problem tells us that S is equidistant from R and T. This means that the 3 points make an isosceles triangle. We can use facts we know about isosceles triangles to find more angles.
11. Draw the diagram on the board:

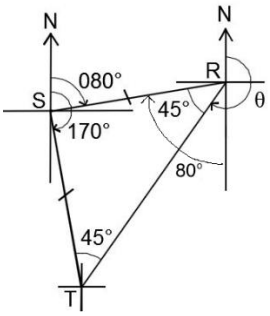


12. Ask pupils if they can find any of the angles of the triangle. (Answer: We can find the angle at point S by subtracting the bearing of R from the bearing of T.)

13. Find the angle of S on the board: $170° - 80° = 90°$
14. Explain: Since the triangle is isosceles, the other 2 angles are equal. Subtract the known angle (90°) from 180°, then divide by 2.
15. Find the angles that R and T form in the triangle on the board: $R = T = \frac{90°}{2} = 45°$.
16. Label the angles of the triangle:



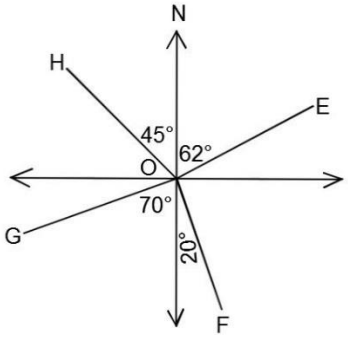
17. Explain: We know that the angle formed by RS and the south line at R is 80°. We know this because the north lines are parallel, so opposite interior angles are equal.
18. Label the 80° angle at R on the board:



19. Explain:
    - The bearing from R to T is formed by 180° and the angle formed by RT and the south line.
    - We find the angle formed by RT and the south line by subtracting 45° from 80° (point out these angles at point R on the board).
    - We add the result to 180°.
20. Solve on the board:
    $80° - 45° = 35°$
    Bearing of T from R: $180° + 35° = 115°$
21. Check for understanding and clear any misconceptions.

Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Find the three-figure bearings of the points E, F, G, and H from point O:



b. If the bearing of Y from X is 281°, find the bearing of X from Y.
c. The bearing from X to Y is 075°, and the bearing from X to Z is 153°, where X, Y, and Z are 3 points on the plane. If X is equidistant from Y and Z, find the bearing from Y to Z.

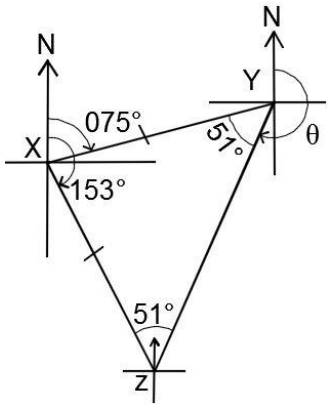
2. Ask pupils to work individually or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

Solutions:

a. Diagram:
Point E: $062°$
Point F: $180° - 20° = 160°$
Point G: $180° + 70° = 250°$
Point H: $360° - 45° = 315°$

b. Calculate the reverse bearing: $281° - 180° = 101°$

c. Diagram:



Find the equal angles of the isosceles triangle:
Angle of triangle at $X = 153° - 75° = 78°$
Solve for other angles: $180° - 78° = 102°$ → $102° \div 2 = 51°$
Find the angle formed by line YZ and the south direction: $75° - 51° = 24°$
Bearing from Y to Z: $180° + 24° = 104°$

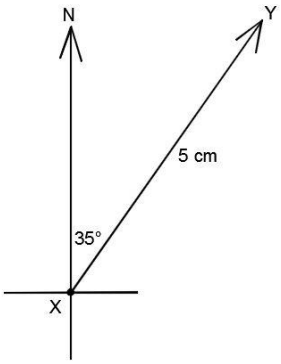
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L120 in the Pupil Handbook.

Lesson Title: Distance-bearing form and diagrams	**Theme:** Bearings	
Lesson Number: M2-L121	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Write the distance and bearing of one point from another as (r, θ). 2. Interpret a distance-bearing problem and draw a corresponding diagram.	**Preparation** 1. Bring a protractor, and ask pupils to bring protractors if available. 2. Draw the diagram in Opening on the board.	

Opening *(3 minutes)*

1. Draw on the board:

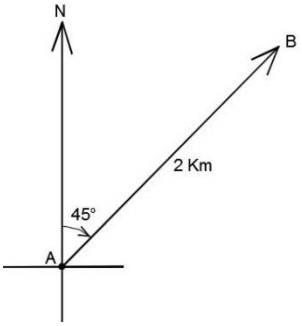


2. Discuss:
    a. What is the three-point bearing from X to Y? (Answer: 035°)
    b. What do you notice about this diagram? How is it different than the bearings diagrams we saw before? (Answer: The distance between X and Y is given.)
3. Explain that this lesson is on distance-bearing form. This is another way to describe bearings that use the distances between points.

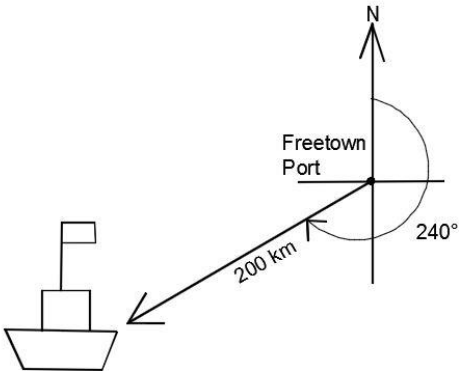
Teaching and Learning *(21 minutes)*

1. Write on the board: $\overrightarrow{XY} = (5\text{ cm}, 035°)$
2. Explain:
    - The position of point Y from point X is described by these 2 numbers.
    - To describe the relationship between two points, give the distance and then the three-point bearing in brackets.
3. Write on the board: The position of a point Q from another point P can be represented by $\overrightarrow{PQ} = (r, \theta)$, where r is the distance between the 2 points, and θ is the three-point bearing from P to Q.
4. Write on the board: A hunter starts at point A and travels through the bush 2 km in the direction 045° to point B. Give the bearing and draw a diagram.
5. Ask a volunteer to give the bearing for this problem. (Answer: $\overrightarrow{AB} = (2\text{ km}, 045°)$)

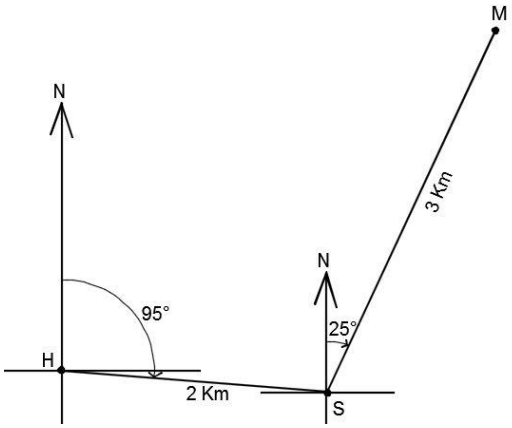
6. Ask pupils to describe what the diagram will look like in their own words. Encourage discussion.
7. Draw the diagram on the board:



8. Write the following problem on the board: A boat sailed from Freetown port at a bearing of 240°. It is now 200 km from Freetown. Write the ship's bearing and draw a diagram.
9. Ask pupils to work with seatmates to draw the diagram.
10. Walk around to check for understanding and clear misconceptions.
11. Invite a volunteer to write the bearing on the board. (Answer: $(200\ \text{km}, 240°)$)
12. Ask volunteers to share their drawings with the class. Accept accurate diagrams.
    Diagram:



13. Write the following problem on the board: A pupil walked 2 km in the 095° direction from home (point H) to school (point S). She then walked 3 km in the 025° direction from school to the market (point M).
    a. Give the bearing from H to S.
    b. Give the bearing from S to M.
    c. Draw the diagram.
14. Ask pupils to give the answers to a and b, then write them on the board. (Answers: a. $\overrightarrow{HS} = (2\ \text{km}, 095°)$; b. : $\overrightarrow{SM} = (3\ \text{km}, 025°)$)
15. Ask pupils to describe what the diagram will look like in their own words. Encourage discussion.
16. Draw the diagram on the board:

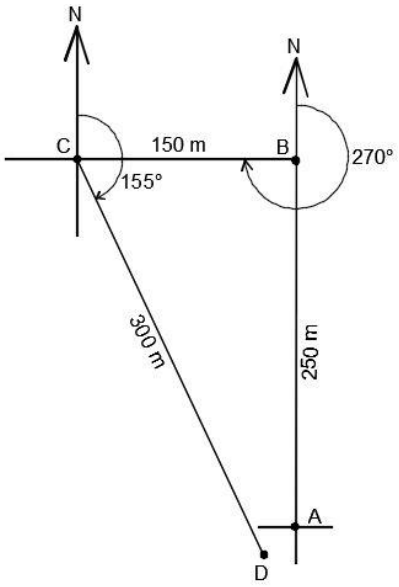


17. Write the following problem on the board: Sia walked 250 metres due north (point A to B.), then 150 metres due west (point B to C). She then walked 300 metres on a bearing of 155° (point C to D).
    a. Write the bearings for each of her 3 walks.
    b. Draw a diagram of her movement.
18. Ask pupils to work with seatmates to solve the problem.
19. Walk around to check for understanding and clear misconceptions.
20. Invite volunteers to write the solution on the board.

Solutions:

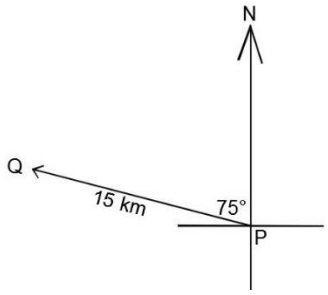
a. $(250\ \text{m}, 000°), (150\ \text{m}, 270°), (300\ \text{m}, 155°)$

b.



Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Write the bearing from point P to Q:



b. A driver starts at point A and travels 10 km in the direction 048° to point B. He then travels 7 km south to point C.
    i. Write the bearing from A to B.
    ii. Write the bearing from B to C.
    iii. Draw a diagram.

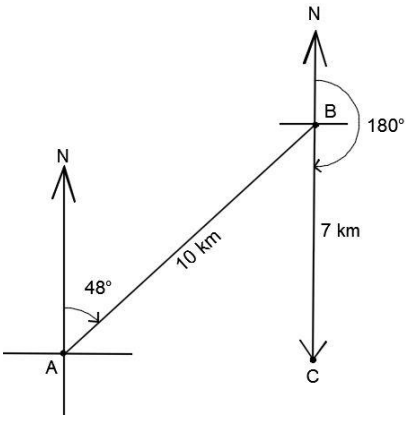
c. A ship travels 60 km from point R in the direction 300° to point S. It then travels 75 km from point S in the direction 160° to point T.
    i. Write the bearing from R to S.
    ii. Write the bearing form S to T.
    iii. Draw a diagram.

2. Ask pupils to work individually to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

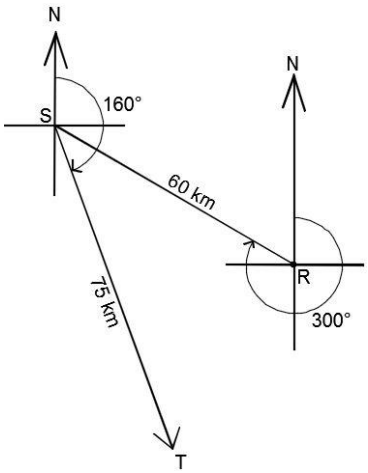
Solutions:

a. Find the three point bearing: $360° - 75° = 285°$;
Bearing: $\overrightarrow{PQ} = (15\text{ km}, 285°)$

b. i. $\overrightarrow{AB} = (10\text{ km}, 048°)$
ii. $\overrightarrow{BC} = (7\text{ km}, 180°)$
iii. Diagram:



c. i. $\overrightarrow{RS} = (60\text{ km}, 300°)$
ii. $\overrightarrow{ST} = (75\text{ km}, 160°)$
iii. Diagram:



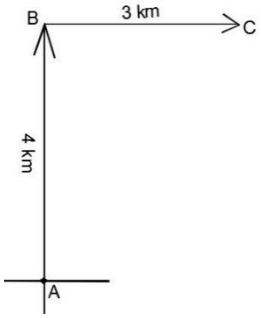
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L121 in the Pupil Handbook.

Lesson Title: Bearing problem solving – Part 2	**Theme:** Bearings	
Lesson Number: M2-L122	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve bearings problems with right triangles. 2. Apply Pythagoras' theorem and trigonometric ratios to calculate distance and direction.	**Preparation** 1. Bring a protractor and trigonometry tables, and ask pupils to bring them if available. 2. Write the problem in Opening on the board.	

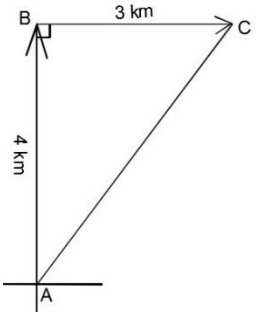
Opening *(3 minutes)*

1. Write on the board: Hawa walked 4 km from point A to B in the north direction, then 3 km from point B to C in the east direction. Draw a diagram.
2. Ask pupils to work with seatmates to draw the diagram.
3. Invite a volunteer to draw the diagram on the board. Diagram:
4. Explain that this lesson is on solving bearing problems. Pupils will use Pythagoras' theorem and trigonometric ratios to solve for distance and direction.



Teaching and Learning *(21 minutes)*

1. Discuss:
   - How far is point C from point A?
   - Look at the diagram on the board and determine how to find this distance.
2. Allow discussion, then explain: The points A, B and C form a right-angled triangle. We can use Pythagoras' theorem to find the distance from C to A.
3. Draw a line connecting A to C, and the lines to show that B is a right angle:



4. Solve on the board, explaining each step:

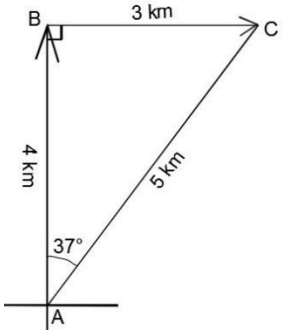
$$|AB|^2 + |BC|^2 = |AC|^2 \quad \text{Apply Pythagoras' theorem}$$
$$4^2 + 3^2 = |AC|^2 \quad \text{Substitute known lengths}$$
$$16 + 9 = |AC|^2 \quad \text{Simplify}$$
$$25 = |AC|^2$$
$$\sqrt{25} = \sqrt{|AC|^2} \quad \text{Take the square root of both sides}$$

$$5 \text{ km} = |AC|$$

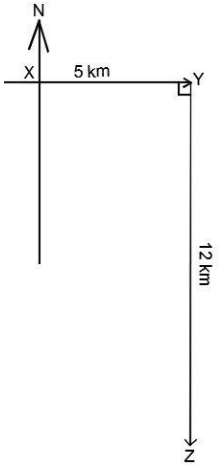
5. Discuss:
   - What is the bearing of C from A?
   - Look at the diagram on the board and determine how to find this bearing.
6. Allow discussion, then explain:
   - We can use trigonometry to find the angle of the triangle at point A.
   - We know the lengths of all sides, so we can use any trigonometric ratio.
7. Solve on the board, explaining each step:

$$\begin{aligned} \tan A &= \tfrac{3}{4} = 0.75 && \text{Apply tangent ratio} \\ \tan^{-1}(\tan A) &= \tan^{-1}(0.75) && \text{Take inverse tangent of both sides} \\ A &= \tan^{-1}(0.75) \\ A &= 36.87° && \text{From the tangent table} \end{aligned}$$

8. Explain: We will round the angle to the nearest degree, because that is how degrees are usually given in bearing form.
9. Label the diagram with the length and angle you have calculated:



10. Write in distance-bearing form: $\overrightarrow{AC} = (5 \text{ km}, 037°)$)
11. Write the following problem on the board: A ship traveled 5 km due east from point X to point Y, then 12 km due south from point Y to point Z.
    a. Draw a diagram for the problem.
    b. Find the distance from point X to point Z.
    c. Find the bearing from point X to point Z.
12. Ask pupils to work with seatmates to draw the diagram.
13. Walk around to check for understanding and clear misconceptions.
14. Invite a volunteer to draw the diagram on the board. **Diagram:** a.
15. Ask pupils to work with seatmates to solve b. and c.
16. Walk around to check for understanding and clear misconceptions.
17. Invite volunteers to write the solutions on the board. As they write the solutions, ask them to also label the diagram on the board with the answers they found.



Solutions:

b. Use Pythagoras' theorem:

$$\begin{aligned} |XY|^2 + |YZ|^2 &= |XZ|^2 && \text{Apply Pythagoras' theorem} \\ 5^2 + 12^2 &= |XZ|^2 && \text{Substitute known lengths} \end{aligned}$$

$$
\begin{aligned}
25 + 144 &= |XZ|^2 && \text{Simplify} \\
169 &= |XZ|^2 \\
\sqrt{169} &= \sqrt{|XZ|^2} && \text{Take the square root of both sides} \\
13 \text{ km} &= |XZ|
\end{aligned}
$$

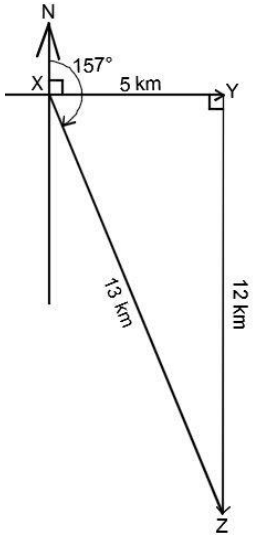
c. The angle of the bearing from X to Z is more than 90°. Find the angle of X in the triangle XYZ, and add this to 90°.

$$
\begin{aligned}
\tan X &= \frac{12}{5} = 2.4 && \text{Apply tangent ratio} \\
\tan^{-1}(\tan X) &= \tan^{-1}(2.4) && \text{Take inverse tangent of both sides} \\
X &= \tan^{-1}(2.4) \\
X &= 67.38° && \text{From the tangent table}
\end{aligned}
$$

Round to the nearest degree, and add to 90°: $90° + 67° = 157°$

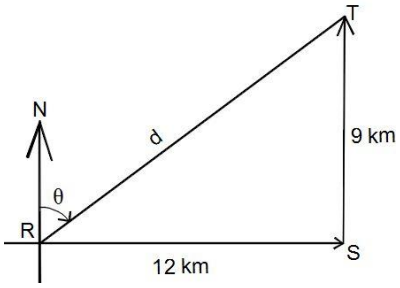
The bearing from X to Z is $\overrightarrow{XZ} = (13 \text{ km}, 157°)$)

Labelled diagram:



Practice *(15 minutes)*

1. Write the following problems on the board:
   a. Find the bearing from R to T.



   b. A farmer travels 10 km due north to reach his land. He then travels 24 km due east to bring his harvest to a market.
      i. Draw a diagram for the problem.
      ii. Find the distance from his starting point to the market.
      iii. Find the bearing from his starting point to the market.
2. Ask pupils to work individually or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.

4. Invite volunteers to come to the board to write the solutions. As they write the solutions, ask them to also label the diagram on the board with the answers they found.

Solutions:

a. Use Pythagoras' theorem to find RT:

$$\begin{aligned} |RS|^2 + |ST|^2 &= |RT|^2 \\ 12^2 + 9^2 &= |RT|^2 \\ 144 + 81 &= |RT|^2 \\ 225 &= |RT|^2 \\ \sqrt{225} &= \sqrt{|RT|^2} \\ 15 \text{ km} &= |RT| \end{aligned}$$

Bearing: $\overrightarrow{RT} = (15\text{ km}, 053°)$)

Find angle R inside the triangle:

$$\begin{aligned} \tan R &= \frac{9}{12} = 0.75 \\ \tan^{-1}(\tan R) &= \tan^{-1}(0.75) \\ R &= \tan^{-1}(0.75) \\ R &= 36.87° \end{aligned}$$

Subtract from 90◦: $90° - 37° = 53°$

b. i. Diagram: See below. Points may be labeled with any letter of the pupil's choice. In the example diagram, O, F and M are used.

Find OM:

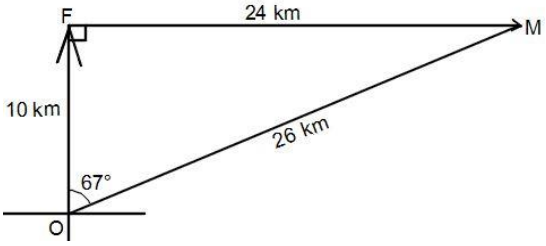
$$\begin{aligned} |OF|^2 + |FM|^2 &= |OM|^2 \\ 10^2 + 24^2 &= |OM|^2 \\ 100 + 576 &= |OM|^2 \\ 676 &= |OM|^2 \\ \sqrt{676} &= \sqrt{|OM|^2} \\ 26 \text{ km} &= |OM| \end{aligned}$$

Bearing: $\overrightarrow{OM} = (26\text{ km}, 067°)$

Find angle O inside the triangle:

$$\begin{aligned} \tan O &= \frac{24}{10} = 2.4 \\ \tan^{-1}(\tan O) &= \tan^{-1}(2.4) \\ O &= \tan^{-1}(2.4) \\ O &= 67.38° \end{aligned}$$

Diagram:



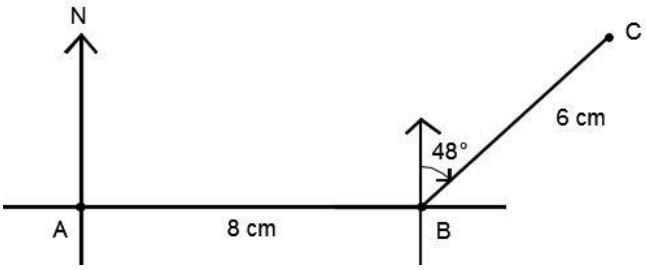
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L122 in the Pupil Handbook.

Lesson Title: Bearing problem solving – Part 3	**Theme:** Bearings	
Lesson Number: M2-L123	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve bearings problems with acute and obtuse triangles. 2. Apply the sine and cosine rules to calculate distance and direction.	**Preparation** 1. Bring a protractor, trigonometry tables, and a calculator. Ask pupils to bring them if available. 2. Write the problem in Opening on the board.	

Opening *(3 minutes)*

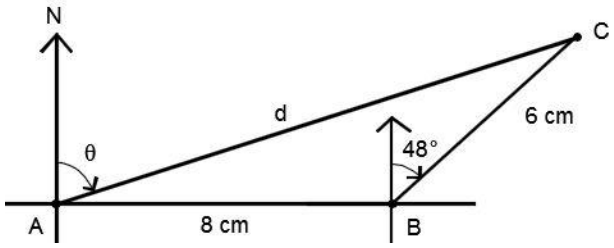
1. Write on the board: A woman walks due east from point A to point B, a distance of 8 kilometres. She then changes direction and walks 6 km to point C on a bearing of 048°.
2. Ask pupils to work with seatmates to draw a diagram for this story.
3. Ask a volunteer to draw the diagram on the board.
   Answer:



4. Explain that this lesson is on solving bearing problems. Pupils will use the sine and cosine rules to calculate distance and direction.

Teaching and Learning *(21 minutes)*

1. Write the following on the board:
   a. What is the distance from A to C?
   b. What is the bearing of C from A?
2. Label the diagram on the board as shown:



3. Discuss:

- What steps would you take to solve question a., the distance? Why? (Answer: Use the cosine rule, because we know 2 sides and the angle between them.)
- What steps would you take to find the bearing of C from A? (Answer: Find angle A in the triangle using the sine rule, and subtract from 90°)

4. Allow discussion, then explain:
   - When you draw a bearings diagram and find a triangle that is not a right-angled triangle, you can use the sine and/or cosine rule.
   - The sine and cosine rules can be used to find the angles and sides of the triangle. Remember that the angle inside the triangle does not always give the bearing. For example, in this problem the bearing is the angle θ, which can be found by subtracting the angle of the triangle from 90°.
5. Solve on the board, explaining each step:

   a. Use cosine rule to find $|AC|$:

$$
\begin{aligned}
|AC|^2 &= |AB|^2 + |BC|^2 - 2|AB||BC|\cos B && \text{Formula}\\
&= 8^2 + 6^2 - 2(8)(6)\ \cos(90+48)° && \text{Substitute values from triangle}\\
&= 64 + 36 - 96\ \cos 138°\\
&= 100 - 96\ (-0.7431) && \text{Substitute } \cos 138° = -0.7431\\
&= 100 + 71.3376\\
|AC|^2 &= 171.3376\\
|AC| &= \sqrt{171.3376} = 13.09 \text{ km to 2 d.p.} && \text{Take the square root of both sides}
\end{aligned}
$$

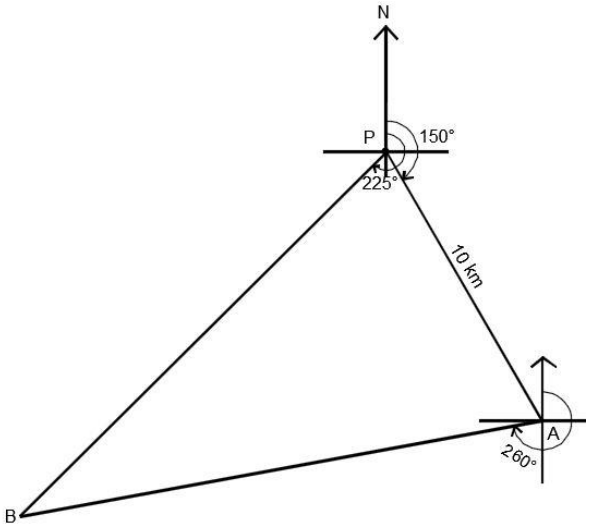
   b. Use the sine rule to find the angle inside the triangle at A:

$$
\begin{aligned}
\frac{6}{\sin A} &= \frac{13.09}{\sin 138°} && \text{Substitute in the formula}\\
\sin A &= \frac{6 \sin 138°}{13.09} && \text{Solve for } A\\
\sin A &= \frac{6 \times 0.6691}{13.09}\\
\sin A &= 0.3067\\
A &= \sin^{-1} 0.3067 && \text{Take the inverse sine of both sides}\\
A &= 17.86° && \text{Use the sine table}
\end{aligned}
$$

   Round to $18°$, and subtract from $90°$ to find the bearing: $90° - 18° = 72°$

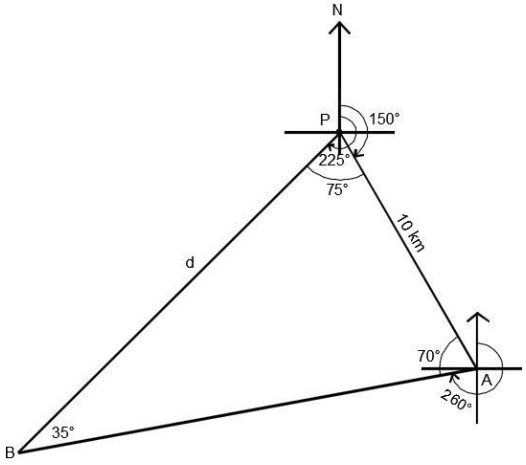
   The bearing of C from A is $\overrightarrow{AC} = (13.09\ \text{km}, 72°)$.
6. Write the following problem on the board: Two ships A and B left a port P at the same time. Ship A travels on a bearing of 150°, and ship B travels on a bearing of 225°. After some time, ship A is 10 km from the port and the bearing of B from A is 260°.
   d. Draw a diagram for the problem.
   e. Find the distance of ship B from the port.
7. Ask pupils to work with seatmates to draw the diagram.
8. Walk around to check for understanding and clear misconceptions.
9. Invite a volunteer to draw the diagram on the board.

Diagram:



10. Discuss: How can we find the distance of ship B from the port? What steps would you take? (Answer: The angles of the triangle can all be found using the properties of triangles and subtraction. We can then apply the sine rule to find the side of the triangle, PB.)

11. Solve the problem on the board. Involve pupils in each step.
Step 1. Solve for missing angles. Label them on the diagram as you find them (see below):
    - Find angle P in the triangle using subtraction: $P = 225° - 150° = 75°$.
    - To find the angle of A in the triangle, first find the other missing angle at point A. It is an opposite interior angle with an angle at point P. The angle at P can be found using subtraction: $180° - 150° = 30°$. Subtract the known angles at A from $360°$: $A = 360° - 260° - 30° = 70°$.
    - Find angle B in the triangle by subtracting angles P and A from $180°$: $B = 180° - 75° - 70° = 35°$.



Step 2. Apply the sine rule:

With angle B, there is enough information to apply the sine rule.

$$\frac{10}{\sin 35°} = \frac{d}{\sin 70°}$$ Substitute in the formula

$$d = \frac{10 \sin 70°}{\sin 35°} \quad \text{Solve for } d$$
$$d = \frac{10 \times 0.9397}{0.5736} \quad \text{Use the sine table}$$
$$d = 16.38 \text{ km}$$

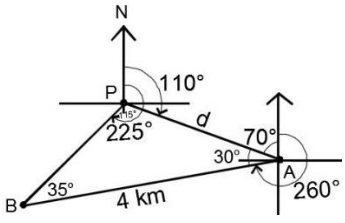
Practice *(15 minutes)*

1. Write the following problem on the board:
    a. The bearings of ships A and B from port P are 225° and 110°, respectively. Ship A is 4 km from ship B on a bearing of 260°. Calculate the distance of ship A from the port.
2. Ask pupils to work with seatmates to solve the problem. They will work independently to solve similar problems in the next lesson.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions. As they write the solutions, ask them to also label the diagram on the board with the answers they found.
   Solution:
   Step 1. Solve for the missing angles. Label them on the diagram as you find them (see below):
    - Find angle P in the triangle using subtraction: $P = 225° - 110° = 115°$
    - Find the opposite interior angles of A and P: $180° - 110° = 70°$. Subtract the known angles at A from $360°$: $A = 360° - 260° - 70° = 30°$.
    - Find angle B in the triangle by subtracting angles P and A from $180°$: $B = 180° - 115° - 30° = 35°$.

   Step 2. Apply the sine rule:
    - With angle B, there is enough information to apply the sine rule.

$$\frac{d}{\sin 35°} = \frac{4}{\sin 115°} \quad \text{Substitute in the formula}$$
$$d = \frac{4 \sin 35°}{\sin 115°} \quad \text{Solve for } d$$
$$d = \frac{4 \times 0.5736}{0.9063} \quad \text{Use the sine table}$$
$$d = 2.53 \text{ km}$$

Labelled diagram:



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L123 in the Pupil Handbook.

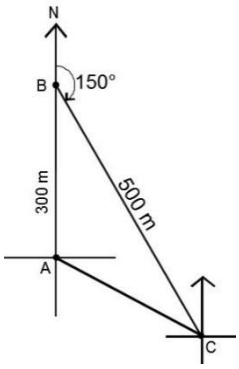
Lesson Title: Bearing problem solving – Part 4	**Theme:** Bearings	
Lesson Number: M2-L124	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve bearing problems using the appropriate rules or theorems.	**Preparation** Bring a protractor, trigonometry tables, and a calculator. Ask pupils to bring them if available.	

Opening *(2 minutes)*

1. Discuss: What are the Maths techniques that you can use to solve bearings problems? (Example answers: Triangle properties, Pythagoras' theorem, trigonometric ratios, sine and cosine rules.)
2. Explain that this lesson is on solving bearing problems. Pupils will need to decide what techniques to use to solve given problems.

Teaching and Learning *(22 minutes)*

1. Write the following problem on the board: A man walks 300 metres due north, then 500 metres at a bearing of 150°. How far is he from his original location?
2. Ask pupils to work with seatmates to draw a diagram for the problem. They should not solve for the distance yet.
3. Invite a volunteer with a correct diagram to draw it on the board:



4. Discuss: How can we solve this problem? (Answer: We can use the cosine rule, since we have 2 known sides and the angle between them.)
5. Ask pupils to work with seatmates to solve the problem.
6. Walk around to check for understanding and clear misconceptions.
7. Invite a group of seatmates to write the solution on the board and explain.
   Solution:
   Step 1. Find the angle inside the triangle at B: $B = 180° - 150° = 30°$
   Step 2. Apply the cosine rule:

$$\begin{aligned} |AC|^2 &= |AB|^2 + |BC|^2 - 2|AB||BC|\cos B && \text{Formula} \\ &= 300^2 + 500^2 - 2(300)(500)\ \cos 30° && \text{Substitute values from triangle} \\ &= 90{,}000 + 250{,}000 - 300{,}000\ \cos 30° && \text{simplify} \end{aligned}$$

$= 340{,}000 - 300{,}000(0.8660)$ Substitute $\cos 30° = 0.8660$

$= 340{,}000 - 259{,}800$

$|AC|^2 = 80{,}200$

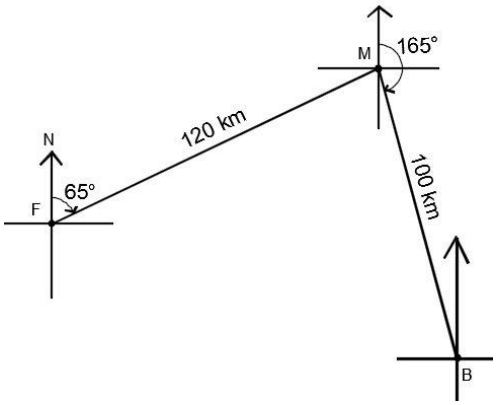
$|AC| = \sqrt{80{,}200} = 283.2$ m to 2 d.p. Take the square root of both sides

Answer: The man is 283.2 metres from his original location.

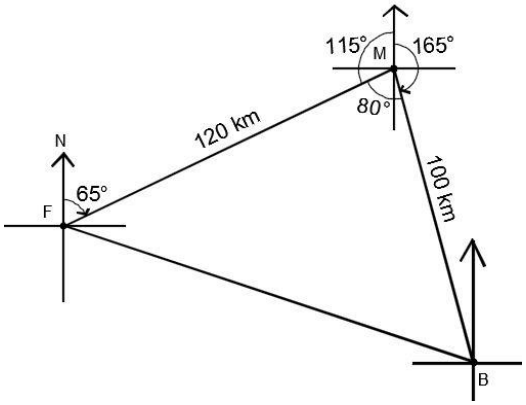
8. Write the following problem on the board: An airplane flies from Freetown (point F) on a bearing of 65° to Makeni (point M), a distance of 120 km. It arrives in Makeni, but changes course and flies to Bo (point B), a distance of 100 km, on a bearing of 165°.

    i. What is the distance from Freetown to Bo?
    ii. What is the bearing of Bo from Freetown?

9. Ask pupils to work with seatmates to draw the diagram.
10. Invite a volunteer with a correct diagram to draw it on the board:



11. Discuss: How can we solve this problem? (Answer: First find the missing angles in the triangle. If we solve for the angle M, we can use the cosine rule.)
12. Work as a class to find the angle of the triangle at M.
    - Note that there are opposite interior angles with point F, which has a known angle of $65°$.
    - The unknown angle outside of the triangle at M is $180° - 65° = 115°$.
    - Subtract the known angles at M from 360: $360° - 115° - 165° = 80°$



13. Ask pupils to work with seatmates to solve part i. Remind them to use the cosine rule.

14. Invite a volunteer to write the solution on the board.

$$
\begin{aligned}
|FB|^2 &= |FM|^2 + |MB|^2 - 2|FM||MB|\cos M && \text{Formula}\\
&= 120^2 + 100^2 - 2(120)(100)\ \cos 80° && \text{Substitute values from triangle}\\
&= 14{,}400 + 10{,}000 - 24{,}000\cos 80°\\
&= 24{,}400 - 24{,}000\ (0.1736) && \text{Substitute } \cos 80° = 0.1736\\
&= 24{,}400 - 4166.4\\
|FB|^2 &= 20{,}233.6\\
|FB| &= \sqrt{20{,}233.6} = 142.24 \text{ km to 2 d.p.} && \text{Take the square root of both sides}
\end{aligned}
$$

15. Ask volunteers to explain how to find the bearing. (Answer: We can use the sine rule to find the missing angle of F inside the triangle. We will add this to 65°.)
16. Ask pupils to work with seatmates to solve part ii.
17. Invite a volunteer to write the solution on the board.

$$
\begin{aligned}
\frac{100}{\sin F} &= \frac{142.24}{\sin 80°} && \text{Substitute in the formula}\\
\sin F &= \frac{100\sin 80°}{142.24} && \text{Solve for } F\\
\sin F &= \frac{100\times 0.9848}{142.24}\\
\sin F &= 0.6924\\
F &= \sin^{-1} 0.6924 && \text{Take the inverse sine of both sides}\\
F &= 43.82° && \text{Use the sine table}
\end{aligned}
$$

Add: Bearing $= 43.82° + 65° = 108.82°$

The bearing from Freetown to Bo is $\overrightarrow{FB} = (142.24\text{ km}, 108.82°)$.

Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Two hunters left from point P at the same time. Hunter A walked 3 kilometres at a bearing of 60°, and hunter B walked 4 kilometres at a bearing of 150°. How far are they from each other?
    b. Village X is 10 km from the nearest hospital on a bearing of 70°. Village Y is 8 km from the same hospital on a bearing of 145°. Calculate:
        i. The distance of village Y from village X, to the nearest kilometre.
        ii. The bearing of Y from X, to the nearest degree.
2. Ask pupils to work individually or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions. Other pupils should check their work.

    Solutions: (see diagrams below)

    a. **Step 1.** Find the angle of P in the triangle: $150° - 60° = 90°$

    Step 2. Note that this is a right-angled triangle, so the distance between A and B can be found with Pythagoras' theorem:

$$
\begin{aligned}
|AP|^2 + |PB|^2 &= |AB|^2 && \text{Apply Pythagoras' theorem}\\
3^2 + 4^2 &= |AB|^2 && \text{Substitute known lengths}
\end{aligned}
$$

$$9 + 16 = |AB|^2 \quad \text{Simplify}$$
$$25 = |AB|^2$$
$$\sqrt{25} = \sqrt{|AB|^2} \quad \text{Take the square root of both sides}$$
$$5 \text{ km} = |AB|$$

The hunters are 5 km from each other.

b. **Step 1.** Find the angle of H in the triangle: $145° - 70° = 75°$

Step 2. Use the cosine rule to find $|XY|$:

$$|XY|^2 = |HX|^2 + |HY|^2 - 2|HX||HY| \cos H \quad \text{Formula}$$
$$= 10^2 + 8^2 - 2(10)(8) \cos 75° \quad \text{Substitute values from triangle}$$
$$= 100 + 64 - 160 \cos 75°$$
$$= 164 - 160\,(0.2588) \quad \text{Substitute } \cos 75° = 0.2588$$
$$= 164 - 41.408$$
$$|XY|^2 = 122.592$$
$$|XY| = \sqrt{122.592} = 11 \text{ km} \quad \text{Take the square root of both sides}$$

Step 3. To find the bearing of Y from X, first find the other angles at X:

Find the angle **inside** the triangle at X using the sine rule:

$$\frac{8}{\sin X} = \frac{11}{\sin 75°} \quad \text{Substitute in the formula}$$
$$\sin X = \frac{8 \sin 75°}{11} \quad \text{Solve for } X$$
$$\sin X = \frac{8 \times 0.9659}{11}$$
$$\sin X = 0.7025$$
$$X = \sin^{-1} 0.7025 \quad \text{Take the inverse sine of both sides}$$
$$X = 44.63° \quad \text{Use the sine table}$$

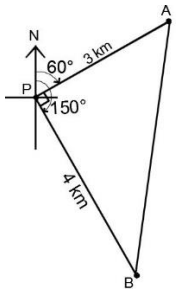
Find the small angle at X that is part of the bearing. Subtract $44.63°$ from $70°$, which is an opposite interior angle of the $70°$ angle at H: $70° - 44.63° = 25.37°$.

Add to find the full bearing: $180° + 25.37° = 205.37°$

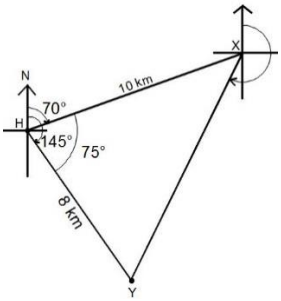
The bearing to the nearest degree is $205°$.

Diagrams:

a.



b.



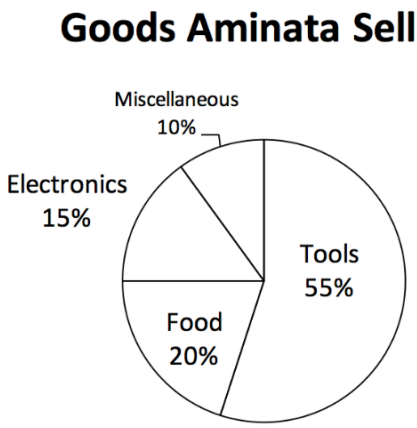
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L124 in the Pupil Handbook.

Lesson Title: Drawing pie charts	**Theme:** Statistics and Probability	
Lesson Number: M2-L125	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw pie charts from given data.	**Preparation** 1. Draw the pie chart in Opening on the board. 2. Bring a protractor to class (see lesson M2-L080 for making a protractor), a straight edge, and a calculator if available. Ask pupils to bring geometry sets if they already have them.	

Opening *(3 minutes)*

1. Draw the pie chart on the board:



2. Discuss:
    a. What do you know about pie charts? (Example answers: They represent 1 whole; each segment represents 1 part of the whole)
    b. What information can you learn from this pie chart? (Example answers: Aminata sells more tools than other items; 20% of her sales are food.)
3. Explain that today's lesson is on drawing pie charts. The next lesson is on interpreting pie charts and solving problems related to them.

Teaching and Learning *(25 minutes)*

1. Explain:
    a. A pie chart is a type of graph in which a circle is divided into sectors that each represent a portion of the whole.
    b. Each sector of the pie chart is a certain percentage of the whole, and the percentages in the chart add up to 100%.
    c. The pie chart on the board shows the percentages of different goods Aminata sells in the market.

2. Explain: We will create our own pie chart as a class. First, we need to collect data.
3. Draw the empty table on the board (use different foods if you prefer):

Favourite Fruit	Frequency	Percentage
Banana		
Mango		
Orange		
Pineapple		
TOTAL		

4. Ask pupils to raise their hands to vote for their favourite fruit from the selection in the table.
5. Fill out the “Frequency” column of the table on the board as pupils raise their hands to vote.
6. Find the total, which should be the same as the number of pupils in your class.
7. Ask pupils to work with seatmates to calculate the percentage of the class who prefers each fruit. You may ask different rows to calculate different percentages in order to do this quickly.
8. Invite volunteers to write the percentages in the table on the board. Make sure they add up to 100%.
   Example table:

Favourite Fruit	Frequency	Percentage
Banana	16	40%
Mango	10	25%
Orange	6	15%
Pineapple	8	20%
TOTAL	40	100%

9. Explain:
   a. To draw a pie chart accurately, we must use a protractor.
   b. Each fruit type is one part of the whole. We must find what part of the whole it is, and assign a degree to it. Then, we use a protractor to draw an angle inside the pie chart with that degree.
10. Discuss: How many degrees are in a whole circle? (Answer: 360 degrees)
11. Explain:
    a. To find the size of each segment, write the frequency for each fruit as a fraction of the whole. Multiply this fraction by 360°.
    b. This will result in the degree measure of the segment for that fruit.
12. Calculate the degree measure for each fruit on the board, explaining each step (use the values from your own table; you may use a calculator):

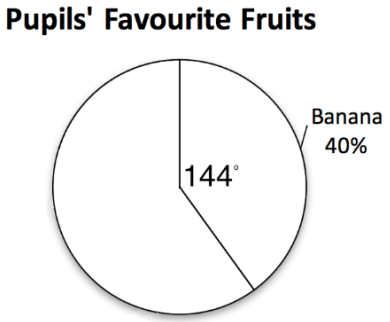
$$\text{Banana} = \frac{16}{40} \times 360° = 144°$$

$$\text{Mango} = \frac{10}{40} \times 360° = 90°$$

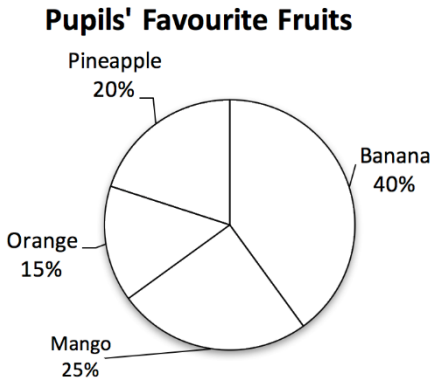
Orange $= \frac{6}{40} \times 360° = 54°$

Pineapple $= \frac{8}{40} \times 360° = 72°$

13. Draw an empty pie chart on the board (an empty circle and the heading “Pupils’ Favourite Fruits”).
14. Draw the segment for “Banana”:
    a. Place the centre of the protractor on the centre of the pie chart and place the bottom of the protractor exactly along one radius of the circle.
    b. Find the angle measurement for banana, 144°.
    c. Use a straight-edge to draw another radius from the centre at 144°.



15. Repeat these steps for each of the sectors until the pie chart is complete.
16. Label each sector of the pie chart as shown:



17. Write the following problem on the board: Create a pie chart for the information shown in the table below.

Favourite school subjects from a survey of 1000 pupils in Sierra Leone	
Subject	**Percentage**
Mathematics	60%
English	25%
Science	10%
Other	5%

18. Calculate the interior angle for Mathematics on the board:

Mathematics $= \frac{60}{100} \times 360° = 216°$

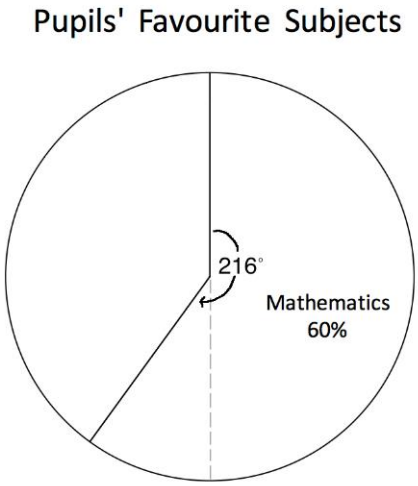
19. Ask pupils to work with seatmates to find the angles for the other subjects.
20. Invite volunteers to write the answers on the board.

English $= \frac{25}{100} \times 360° = 90°$

Science $= \frac{10}{100} \times 360° = 36°$

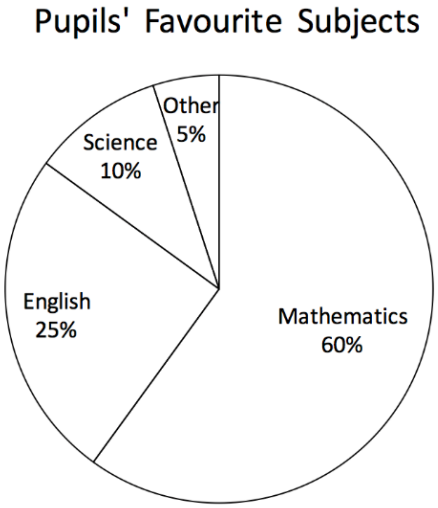
OIther $= \frac{5}{100} \times 360° = 18°$

21. Discuss:
    a. The angle for Maths, 216°, is larger than 180°, which is the degree of our protractor.
    b. First find 180°, which is the diameter of a circle. Then, find the difference between 216 and 180 (it is 36°), and extend the line for Maths this many degrees beyond the diameter.
22. Draw the segment for Maths on the board:



23. Ask pupils to work with seatmates to draw the complete pie chart in their exercise books.
24. Invite volunteers to come draw the remaining 3 segments on the board.

Answer:

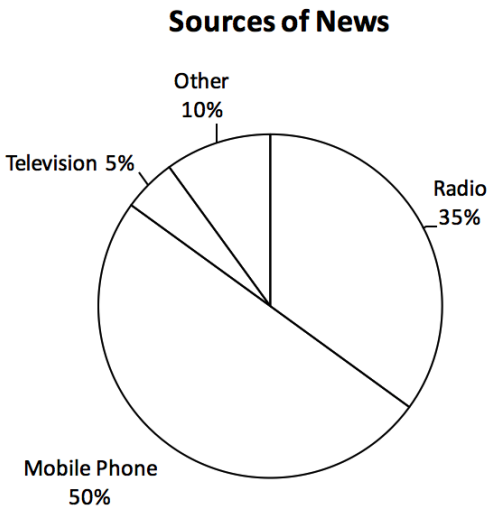


Practice *(10 minutes)*

1. Write a problem on the board: A survey was conducted of 1,000 people in Sierra Leone to find out their sources of news. Draw a pie chart for the information in the table:

Source	Percentage
Radio	35%
Mobile Phone	50%
Television	5%
Other	10%

2. Ask pupils to work with independently to solve the problem. Allow discussion with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to show their pie charts to the class and explain how they made them. All other pupils should check their own work.
   Solution:
   Angle calculations: Radio $= \frac{35}{100} \times 360° = 126°$; Mobile Phone $= \frac{50}{100} \times 360° = 180°$; Television $= \frac{5}{100} \times 360° = 18°$; Other $= \frac{10}{100} \times 360° = 36°$.



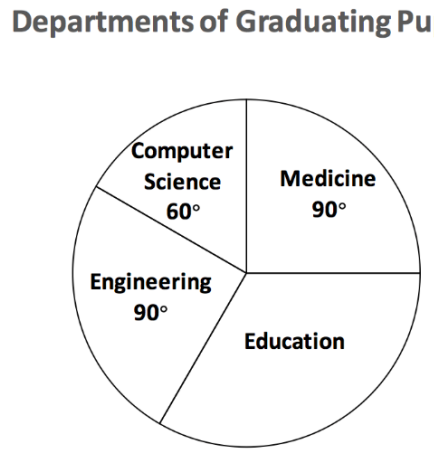
Closing *(2 minutes)*

1. Ask questions to check for understanding of the above pie chart:
   a. What is the most common source of news in Sierra Leone? (Answer: mobile phones)
   b. Do more people get their news from the radio or television? (Answer: radio)
2. For homework, have pupils do the practice activity PHM2-L125 in the Pupil Handbook.

Lesson Title: Interpretation of pie charts	**Theme:** Statistics and Probability	
Lesson Number: M2-L126	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to interpret and solve pie chart problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: This year, 1,000 pupils graduated from a certain university. The pie chart below shows the departments they graduated from. Use it to answer the questions below.



   a. How many pupils graduated from the education department?
   b. What percentage of the total graduated from the medicine department?
2. Discuss:
   a. Is there enough information to solve this problem? (Answer: yes)
   b. How would you solve part a.? (Example answer: Find the degree measure of the education segment and use it to find the number of pupils.)
   c. How would you solve part b.? (Example answer: Find those graduating from medicine as a proportion of the whole, and multiply by 100%.)
3. Explain that today's lesson is on interpreting pie charts and solving problems related to them.

Teaching and Learning *(24 minutes)*

1. Explain:
   a. To answer part a., we need to find the proportion of pupils who graduated from Education. We then multiply this by the total number of pupils, 1,000.
   b. To find the proportion, we first need to find the degree measure of the Education segment.
2. Solve part a. on the board, explaining each step:
   Step 1. Find the degree measure of Education:

$$\text{Education measure} = 360° - (90° + 60° + 90°) = 120°$$

Step 2. Multiply the proportion by the total number of pupils to calculate those studying Education:

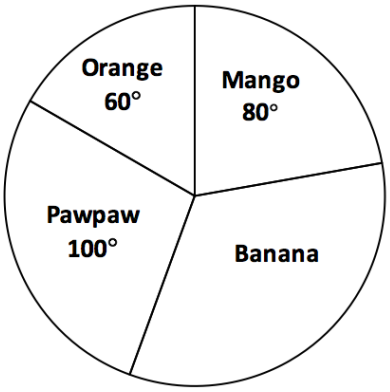
$$\begin{aligned} \text{Number in Education} &= \frac{120}{360} \times 1{,}000 \\ &= \frac{1}{3} \times 1{,}000 \\ &= 333.3 \\ &= 333 \qquad \text{Round to a whole number} \end{aligned}$$

Answer: 333 pupils are graduating from education.

3. Explain:
   a. We have all of the information we need to solve part b.
   b. Use the proportion of pupils studying medicine (in degrees) to find the percentage.
4. Solve part b. on the board and explain:
   Write medicine as a percentage of the whole using its degree measure:
   Pupils graduating from medicine $= \frac{90}{360} \times 100\% = 25\%$
5. Write the following problem on the board: Using the same pie chart, find:
   a. The percentage of pupils graduating from Computer Science.
   b. The number of pupils graduating from Computer Science.
6. Ask pupils to work with seatmates to find the answers.
7. Walk around to check for understanding and clear misconceptions.
8. Invite volunteers to write the solutions on the board.

   Solutions:
   a. Calculate the percentage using degree measure:
      Percentage graduating from Computer Science $= \frac{60}{360} \times 100\% = 16.7\%$
   b. Pupils may use either of the methods below; the proportions are the same.
      Method 1. Calculate the number of pupils using a proportion of degrees:
      Number graduating from Computer Science $= \frac{60}{360} \times 1{,}000 = 167$ pupils
      Method 2. Use the percentage calculated in part a.:
      Number graduating from Computer Science $= \frac{16.7}{100} \times 1{,}000 = 167$ pupils
4. Write the following problem on the board: The pie chart represents the pieces of fruit for sale in a market stand. If there are 60 mangoes, how many bananas are there?

Fruit in a Market Stand



9. Discuss: How can we solve this problem? What steps will we take?
10. Allow pupils to share their ideas, then explain:
    a. First find the angle measure of "Banana" by subtracting the known angles from 360°.
    b. Use the fact that there are 60 mangoes at the stand to find the total number of fruit at the stand.
    c. Calculate the number of bananas by multiplying the proportion of fruits that are bananas by the total number of fruits.
11. Solve the problem as a class. Ask pupils to describe each step before you solve it on the board. You may invite volunteers to come to the board to work some steps.

    Solution:

    Step 1. Find the angle measure of banana:

    Banana's measure: $360° - (100° + 60° + 80°) = 360° - 240°$

    $= 120°$

    Step 2. Use the fact that there are 60 mangoes to find the total number of fruit. Recall that we would have found 60 mangos by multiplying the proportion of fruits that are mangoes by the total number of fruits. Set up the equation:

    Number of mangoes $= 60 = \frac{80}{360} \times F$, where F is the total number of fruit.

    Solve for F:

60	$=$	$\frac{80}{360} \times F$	
60	$=$	$\frac{2}{9} \times F$	Simplify
540	$=$	$2 \times F$	Multiply throughout by 9
270	$=$	F	Divide throughout by 2

    There are 270 pieces of fruit in total.

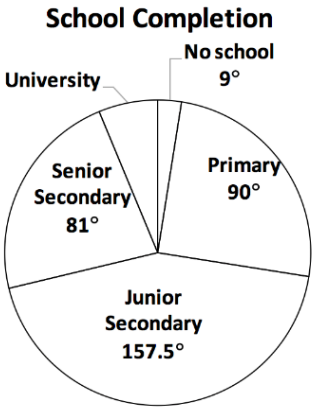
    Step 3. Find the number of bananas. Multiply the proportion that are bananas by the total number of fruit:

Number of bananas	$=$	$\frac{120}{360} \times F$	
	$=$	$\frac{1}{3} \times 270$	Simplify
	$=$	90	Multiply

Answer: There are 90 bananas in the market stand.

Practice *(12 minutes)*

1. Write the following problem on the board: The pie chart below shows the highest level of education achieved by 800 people in a village. Use it to answer the questions.



   a. How many people have received a university education?
   b. What percentage of people have a junior secondary education or higher?
2. Ask pupils to work independently to solve the problem. Allow discussion with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

   Solutions:

   a. Calculate the proportion of the pie chart that in the "University" segment, and multiply by 800 people.
   Degrees in segment $= 360° - (157.5° + 90° + 81° + 9°) = 22.5°$
   Number graduated from university:
   University $= \frac{22.5}{360} \times 800 = 50$ people

   b. Find the total degree of the segment covered by people with JSS or higher, and calculate percentage as a proportion of 360°.
   JSS or higher: $157.5° + 81° + 22.5° = 261°$
   Percentage of total: $\frac{261}{360} \times 100 = 72.5\%$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L126 in the Pupil Handbook.

Lesson Title: Drawing and interpretation of bar charts	**Theme:** Statistics and Probability	
Lesson Number: M2-L127	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw and interpret bar charts.	**Preparation** Draw the empty frequency table in Opening on the board.	

Opening *(4 minutes)*

1. Draw the frequency table on the board:

Favourite Fruits	
Mango	
Pawpaw	
Orange	
Banana	
Pineapple	

2. Explain: We will collect data from our class. Please choose your favourite fruit from among these 5. When I call your favourite fruit, raise your hand.
3. Call the name of each fruit in the table. Count the number of pupils who raise their hands for each. Record the numbers in the chart.
   Example:

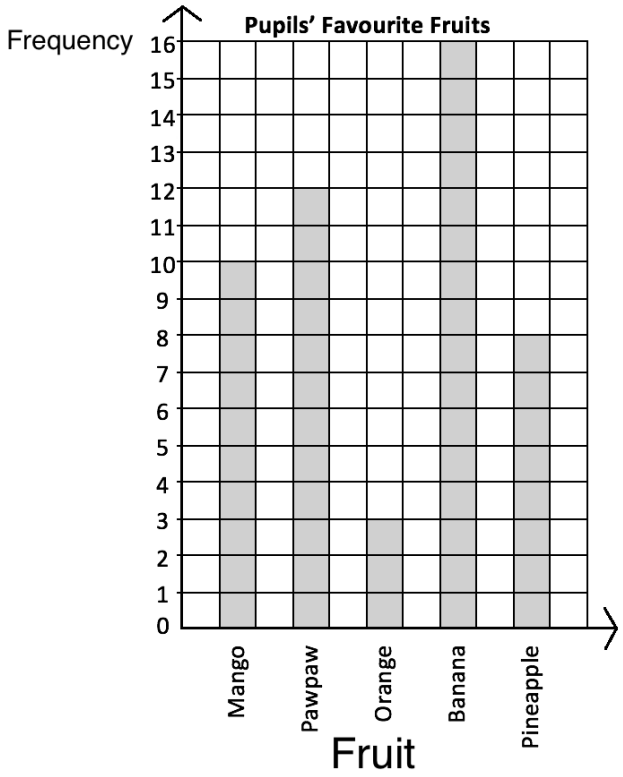
Favourite Fruits	
Mango	10
Pawpaw	12
Orange	3
Banana	16
Pineapple	8

4. Explain that today's lesson is on drawing and interpreting bar charts. We will make a bar chart of this data on favourite fruits.

Teaching and Learning *(23 minutes)*

1. Explain:
   a. Bar charts are used to compare different quantities.
   b. We can use a bar chart to compare the numbers of pupils who prefer each type of fruit.
2. Draw the axes on the board.
3. Mark an appropriate scale on the y-axis. For example, 5 cm to represent each pupil on the y-axis is a good scale to use on the board.

4. Mark an appropriate scale on the x-axis, for the width of each bar. 5 cm can be used again here.
5. Label the axes (as shown below).
6. Draw the bar for “mango”. Shade it in with chalk so it is clear.
7. Invite volunteers to come to the board and draw the bars for the other 4 fruit. Support them as needed.
   Example answer (bar height will depend on your pupils’ responses):



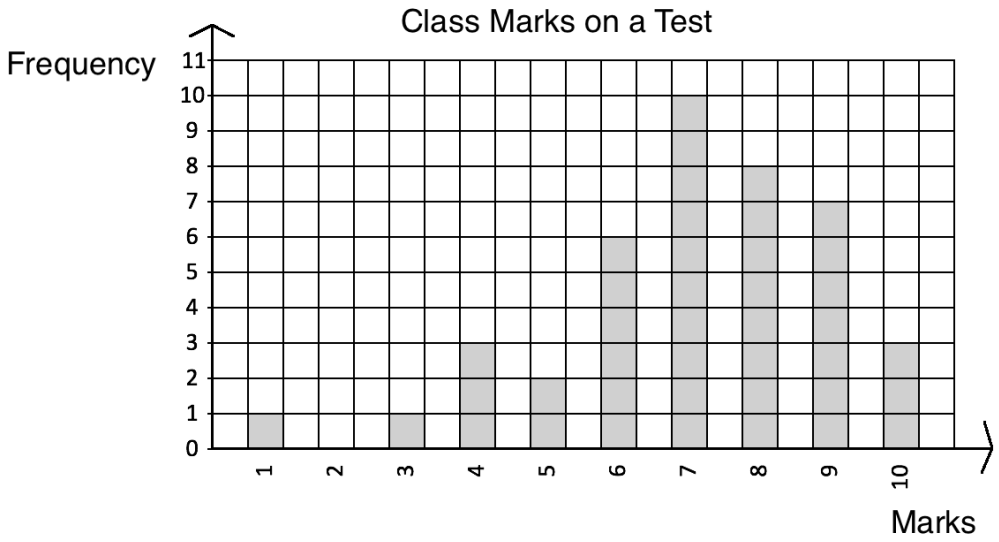
8. Write the following problem on the board: The table below shows the distribution of marks on a test that a certain class sat.
   a. Draw a bar chart for the distribution.
   b. If the pass mark is 6, how many pupils failed the test?
   c. The teacher awarded pupils with at least 9 marks with a new pencil. How many pencils did he award?

Marks	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	3	2	6	10	8	7	3

9. Discuss, and allow pupils to respond until they reach the correct answer:
   a. What will we label the x-axis? (Answer: Marks, 1-10)
   b. What will we label the y-axis? (Answer: Frequency)
10. Ask pupils to work with seatmates to solve the problem.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.

Solutions:

a. Bar chart:



b. If the pass mark is 6, all pupils scoring 1-5 failed. Add the frequencies: $1 + 0 + 1 + 3 + 2 = 7$ pupils failed.

c. Pupils scoring 9 or 10 marks received a pencil. Add the frequencies: $7 + 3 = 10$ pencils.

Practice *(12 minutes)*

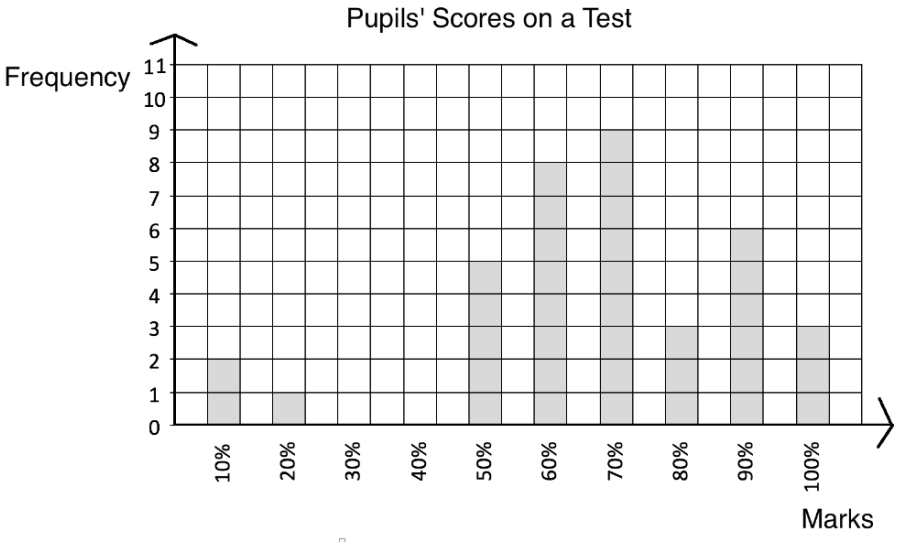
1. Write the following problem on the board: The table below shows the distribution of marks on a test given to a class of pupils. Draw a bar chart for the information, and use it to answer the questions below.

Marks	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Frequency	2	1	0	0	5	8	9	3	6	7

   a. How many pupils took the test?
   b. If 60% pass, how many pupils passed the test?
   c. What percentage of pupils passed the test?
   d. How many pupils scored 60% or 70%?

2. Ask pupils to work with independently to solve the problem. Allow discussion with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

Solution:



a. Add the frequencies: $2 + 1 + 5 + 8 + 9 + 3 + 6 + 7 = 41$
b. Add the frequencies that scored between $60\% - 100\%$: $8 + 9 + 3 + 6 + 7 = 33$ pupils.
c. Find the answer from part b. as a percentage of the answer from part a: $\frac{33}{41} \times 100\% = 80.5\%$.
d. Add the frequencies scoring 60% and 70%: $8 + 9 = 17$ pupils.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L127 in the Pupil Handbook.

Lesson Title: Mean, Median, and Mode	**Theme:** Statistics and Probability	
Lesson Number: M2-L128	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mean, median, and mode of a list of ungrouped data.	**Preparation** None	

Opening *(4 minutes)*

1. Discuss and allow pupils to describe each term in their own words:
    a. What is mean? (Example answer: It is the average of a set of numbers.)
    b. What is median? (Example answer: It is the number that falls in the middle when you list a set of numbers in ascending or descending order.)
    c. What is mode? (Example answer: It is the number that appears the greatest number of times in a set of data.)
2. Explain that today's lesson is on calculating the mean, median, and mode of a list of ungrouped data. Ungrouped data is information that is listed individually. We will work with grouped data in future lessons.

Teaching and Learning *(20 minutes)*

1. Write the following question on the board: 10 pupils received the following scores on their Maths exam: 87, 100, 76, 92, 90, 95, 85, 67, 99 and 95. Calculate the mean, median, and mode of the scores.
2. Explain:
    a. The **mean** is a number that can tell us where the middle of the data is. It is also commonly known as the "average". To find the mean of a set of data, add the numbers together and divide the total by the number of items. The quotient is the mean.
    b. The number in the middle is called the **median**. The median is another number that can tell us where the middle of the data is.
    c. The **mode** is the number that appears most often in a list.
3. Solve the problem on the board. First, calculate **mean:**
    Add the numbers: $87 + 100 + 76 + 92 + 90 + 95 + 85 + 67 + 99 + 95 = 886$
    Divide by the number of pupils: $886 \div 10 = 88.6$
    The mean score is 88.6.
4. Find **median** on the board:
    List the numbers in ascending order: 67, 76, 85, 87, 90, 92, 95, 95, 99, 100
    Identify the middle of the list: 90, 92
    Since there is not one number in the middle, find the mean of the 2 numbers in the middle. Add them together and divide by 2: median $= \frac{90+92}{2} = 91$

5. Find the mode: Note that 95 is the only number that occurs more than once, so it must be the mode.
6. Write the following problem on the board: On her exams, Fatu scored $x\%$ in Mathematics, 90% in English, 95% in Biology, and 80% in Chemistry. If her mean score for all subjects was 88%, what is the value of x?
7. Explain:
    a. We must work this problem backwards. We are given the mean, and we must solve for one of the numbers in the list.
    b. The first step is to set up an equation. Then we will solve for x using algebra.
8. Solve the problem on the board, explaining each step:

$$\begin{aligned} 88 &= \frac{x+90+95+80}{4} && \text{Set up the equation} \\ 88 &= \frac{x+265}{4} && \text{Simplify} \\ 4 \times 88 &= x + 265 && \text{Multiply throughout by 4} \\ 352 &= x + 265 \\ 352 - 265 &= x && \text{Subtract 265 from both sides} \\ 87 &= x \end{aligned}$$

   Her score on her Maths exam was 87%.
9. Write the following problems on the board:
    a. A group of 6 farmers harvested their cassava on the same day and sent it to the market in a big truck. The weight of each of their harvests were: 65.1 kg, 120 kg, 56.9 kg, 210.4 kg, 75.1 kg, and 84.5 kg. Find the mean, median, and mode of the weights.
    b. Fatu sells palm oil in the market. On Monday, she sold x litres of oil. On the other 4 days of the week, she sold 3.5 l, 10 l, 13.5 l, and 7 l. If she sold a mean of 9 litres that week, what is the value of x?
10. Ask pupils to work with seatmates to solve the problems.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write their solutions on the board and explain.

    Solutions:

    a. **Mean:**

    Add the numbers: $65.1 + 120 + 56.9 + 210.4 + 75.1 + 84.5 = 612 \text{ kg}$

    Divide by 6: $612 \div 6 = 102 \text{ kg}$

    Median:

    Write the numbers in order: $56.9, 65.1, 75.1, 84.5, 120, 210.4$

    Find the average of the 2 middle numbers: $\frac{75.1+84.5}{2} = \frac{159.6}{2} = 79.8 \text{ kg}$

    Mode: There is no mode, because each number appears only once.

    b.

$$\begin{aligned} 9 &= \frac{x+3.5+10+13.5+7}{5} && \text{Set up the equation} \\ 9 &= \frac{x+34}{5} && \text{Simplify} \\ 5 \times 9 &= x + 34 && \text{Multiply throughout by 5} \end{aligned}$$

$$
\begin{aligned}
45 &= x + 34 \\
45 - 34 &= x && \text{Subtract 34 from both sides} \\
11 &= x
\end{aligned}
$$

$x = 11\ l$; Fatu sold 11 litres of oil on Monday.

Practice *(15 minutes)*

1. Write the following problems on the board:
    a. 8 pupils measured the time that it takes them to walk to school. The times were 12, 5, 34, 20, 27, 5, 43, and 14 minutes. Find the mean, median, and mode.
    b. Ama has 5 chickens. Last week, she calculated the mean weight of her chickens as 0.8 kg. She knows that 4 of them weigh 0.5 kg, 0.9 kg, 0.7 kg, and 1 kg. What is the weight of the fifth chicken?
2. Ask pupils to work with independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

    Solutions:

    a. **Mean:**

    Add the numbers: $12 + 5 + 34 + 20 + 27 + 5 + 43 + 14 = 160$ minutes

    Divide by 8: $160 \div 8 = 20$ minutes

    Median:

    Write the numbers in order: $5, 5, 12, 14, 20, 27, 34, 43$

    Find the average of the 2 middle numbers: $\frac{14+20}{2} = \frac{34}{2} = 17$ minutes

    Mode: 5

    b.

$$
\begin{aligned}
0.8 &= \frac{x+0.5+0.9+0.7+1}{5} && \text{Set up the equation} \\
0.8 &= \frac{x+3.1}{5} && \text{Simplify} \\
5 \times 0.8 &= x + 3.1 && \text{Multiply throughout by 5} \\
4.0 &= x + 3.1 \\
4.0 - 3.1 &= x && \text{Subtract 3.1 from both sides} \\
0.9 &= x
\end{aligned}
$$

    The weight of the fifth chicken is 0.9 kg.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L128 in the Pupil Handbook.

Lesson Title: Mean, median, and mode from a table or chart	**Theme:** Statistics and Probability	
Lesson Number: M2-L129	**Class:** SSS 2	**Time:** 40 minutes
 **Learning Outcome** By the end of the lesson, pupils will be able to calculate mean, median, and mode from a frequency table or a bar chart.	 **Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Write a revision problem on the board: Calculate the mean, median, and mode of: 30, 21, 47, 35, 72, and 35.
2. Give pupils 2 minutes to solve the problem independently.
3. Invite volunteers to write the solutions on the board. All other pupils should check their work.

   Solutions:

   Mean: $\frac{30+21+47+35+72+35}{6} = 40$

   Median: Order the numbers: 21, 30, 35, 35, 47, 72. The median is 35.

   Mode: 35
4. Explain that today's lesson is on calculating the mean, median, and mode of data presented in a frequency table or bar chart.

Teaching and Learning *(22 minutes)*

1. Draw the chart shown below on the board: The table below shows the distribution of marks on an assignment that a class completed. No one scored below 6 marks. Find the mean, median, and mode of the scores.

Marks	6	7	8	9	10
Frequency	3	9	4	3	1

2. Discuss: How we can find the mean score on the assignment?
3. Allow pupils to share their ideas, then explain:
   a. We need to find the sum of the scores of all pupils in the class, then divide by the number of pupils.
   b. We can use multiplication to find the sum of numbers that are the same. In this case, we will use multiplication to find the sum within each mark. Then, we add all of the results together.
4. Solve for the mean on the board:

   Find the total marks:

   $6(3) + 7(9) + 8(4) + 9(3) + 10(1) = 18 + 63 + 32 + 27 + 10 = 150$

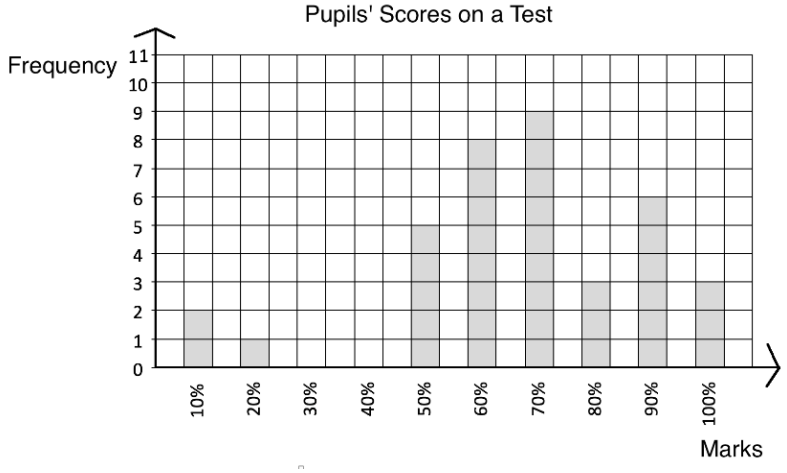
   Find the number of pupils in the class by adding the frequencies:

   $$3 + 9 + 4 + 3 + 1 = 20$$

   Divide the total marks by the number of pupils: $150 \div 20 = 7.5$

Mean = 7.5 marks

5. Discuss: How we can find the **median** score on the assignment?
6. Allow pupils to share their ideas, then explain:
    a. There are 20 pupils in the class. The median is the mean score of the 2 pupils in the middle, which are the 10^{th} and 11^{th} pupils.
    b. Find the scores of the 10^{th} and 11^{th} pupils by counting in the frequency table.
    c. The first 3 pupils scored 6, then the next 9 pupils scored 7 marks. That makes 12 pupils in total. Therefore, the 10^{th} and 11^{th} pupils scored 7 marks.
7. Write on the board: **median** = 7 marks
8. Discuss: How we can find the **mode** score on the assignment?
9. Allow pupils to share their ideas, then explain: The mode is the mark that appears most often. In other words, it has the highest frequency.
10. Ask a volunteer to give the mode, then write it on the board. (Answer: **mode** = 7).
11. Write the following problem on the board: The bar chart below shows marks that pupils achieved on a test, as percentages. Find the mean, median, and mode.



12. Explain:
    a. We will calculate **mean** in the same way as we did for the frequency tables. Use multiplication to find the total within each mark, then add them together to find the total marks. Divide the total marks by the number of pupils in the class.
    b. For the **median**, use the total number of pupils in the class to find the number who is in the middle. Locate that pupil's score (or 2 pupils if there is an even number of pupils in the class).
    c. The **mode** is the mark that appears most often, or the highest bar.
13. Solve the problem on the board, explaining each step:

    Calculate **mean:**

    Find the sum of the marks:

$$2(10) + 20 + 5(50) + 8(60) + 9(70) + 3(80) + 6(90) + 3(100)$$
$$= 20 + 20 + 250 + 480 + 630 + 240 + 540 + 300$$
$$= 2{,}480$$

Find the number of pupils: $2 + 1 + 5 + 8 + 9 + 3 + 6 + 3 = 37$

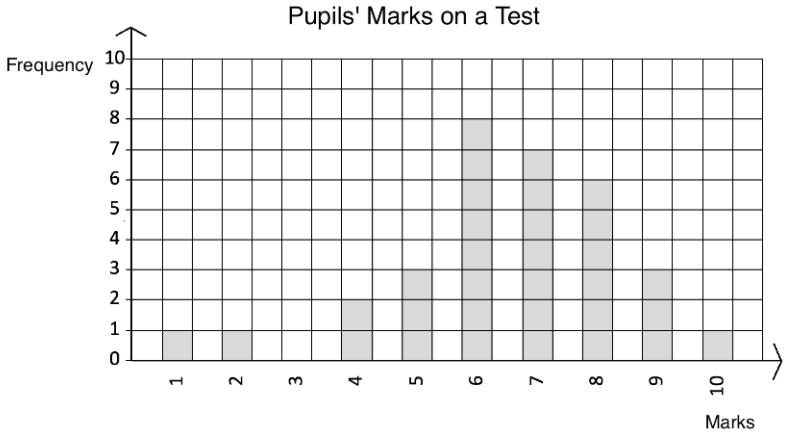
Divide: $2{,}480 \div 37 = 67.02\%$

Calculate **median:**

- Since there are 37 pupils in the class, the 19th pupil is in the middle. There are 18 pupils with a lower score, and 18 pupils with a higher score.
- Locate the 19th pupil in the bar chart by counting the bars, from least to greatest.
- The 19th pupil is within 70%. Therefore, the median = 70%

Observe **mode:** The highest bar is at 70%; therefore, the mode is 70%.

14. Write the following problem on the board: The bar chart below shows marks that pupils achieved on a test. Find the mean, median, and mode.



15. Ask pupils to work with seatmates to solve the problem.
16. Walk around to check for understanding and clear misconceptions.
17. Invite volunteers to write the solutions on the board and explain.

Solutions:

Mean:

Find the sum of the marks:

$$1 + 2 + 2(4) + 3(5) + 8(6) + 7(7) + 6(8) + 3(9) + 10$$
$$= 1 + 2 + 8 + 15 + 48 + 49 + 48 + 27 + 10$$
$$= 208$$

Find the number of pupils by addition or counting the bars: 32 pupils

Divide: $208 \div 32 = 6.5$ marks

Median: Since there are 32 pupils in the class, the 16th and 17th pupils are in the middle. Counting up on the chart, their scores are both 7, which is the median.

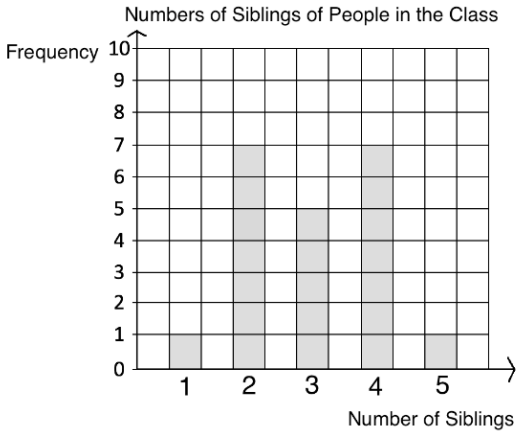
Mode: 6 marks, which has the highest bar.

Practice *(13 minutes)*

1. Write the following problems on the board:
   a. 10 pupils ran 1 kilometre in a race. Their finishing times are in the table below, to the nearest 30 seconds. Find the mean, median, and mode of their times.

Time (minutes)	4.0	4.5	5.0	5.5	6.0
Frequency	1	2	2	4	1

b. A class collected data on the number of siblings of each pupil, and created the bar chart below. Find the mean, median, and mode.



2. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

   Solutions:

   a. **Mean:**
   Find the sum of the times: $4.0 + 2(4.5) + 2(5.0) + 4(5.5) + 6.0 = 51$ minutes
   Divide by the number of pupils, 10: $51 \div 10 = 5.1$ minutes
   Median: Out of 10 pupils, the 5th and 6th pupils are in the middle. Count up to 5 and 6 in the table. The 5th pupil's time was 5.0, the 6th pupil's time was 5.5. To find the median, we need to add them together and divide by 2. Median = 5.25
   Mode: 5.5 minutes

   b. **Mean:**
   Find the sum of the siblings: $1(1) + 7(2) + 5(3) + 7(4) + 1(5) = 63$
   Find the number of pupils: $1 + 7 + 5 + 7 + 1 = 21$
   Divide: $63 \div 21 = 3$ siblings
   Median: The 11th pupil is in the middle of the 21 pupils. Count up the 11 in the bar chart. The 11th pupil has 3 siblings, which is the median.
   Mode: The bars for 2 siblings and 4 siblings both have a height of 7. Thus, there are 2 modes. Mode = 2, 4.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L129 in the Pupil Handbook.

Lesson Title: Grouped frequency tables	**Theme:** Statistics and Probability	
Lesson Number: M2-L130	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Present and interpret grouped data in frequency distribution tables. 2. Apply class intervals.	**Preparation** Write the list of numbers at the start of the Teaching and Learning section on the board.	

Opening *(3 minutes)*

1. Ask pupils the following questions and allow volunteers to answer:
   a. What does **frequency** mean in statistics? (Answer: the number of times something happens)
   b. What does the word **interval** mean? (Answer: the distance or time between parts)
2. Explain that today's lesson is on presenting grouped data in frequency distribution tables.

Teaching and Learning *(23 minutes)*

1. Write the list of numbers on the board: Lengths in centimetres: 23, 21, 22, 19, 23, 17, 18, 19, 21, 26, 25, 19, 22, 24, 28, 23, 18, 17, 30, 23.
2. Explain:
   - These are the lengths in cm of 20 pupils' feet, which they measured to buy new school shoes.
   - If we have a lot of data, sometimes it can be helpful to divide the data into groups. This is called "grouped data."
3. Discuss: How can we group, or divide, this data into smaller groups?
4. Allow pupils to share their ideas, then explain:
   - We need to divide the data into groups based on length.
   - The first thing we must do is order the data from smallest to largest.
5. Ask pupils to write the numbers in order in their exercise books, and invite a volunteer to come to the board rewrite the numbers in the correct order. (Answer: 17, 17, 18, 18, 19, 19, 19, 21, 21, 22, 22, 23, 23, 23, 23, 24, 25, 26, 28, 30.)
6. Ask volunteers to give the range of the data. (Answer: $30\ \text{cm.} - 17\ \text{cm.} = 13$)
7. **Explain:**
   - If we divide the range by the number of groups we want to put our data into, it can help us decide how large our groups will be.
   - Let's divide this data into 3 groups. We can also call our groups "class intervals."
   - The groups (or intervals) must always be equal in size, even though they will have different numbers of frequencies.
8. Write on the board and explain: $13 \div 3 \approx 4.33$, round up to 5.

9. Explain**:**
    - Now we can make our groups. Each group should have a range of 5.
    - The starting point should be smaller than or equal to the smallest number in the data set.
    - The ending point should be greater than the largest number in the set.
10. Draw the empty frequency table shown below on the board:

Measurement of Pupils' Feet	
Measurement	**Frequency**
16 cm – 21 cm	
22 cm – 26 cm	
27 cm – 32 cm	
Total	

11. Ask pupils to determine how many pupils fall into each category. Give them a moment to count the numbers and share their answers. Write the answers in the frequency table on the board:

Measurement of Pupils' Feet	
Measurement	**Frequency**
16 cm – 21 cm	9
22 cm – 26 cm	9
27 cm – 32 cm	2
Total	20

12. Discuss:
    - Which category does the greatest number of pupils fall into? (Answer: Two categories, 16-21 and 22-26, both have 9 pupils.)
    - Which category does the least number of pupils fall into? (Answer: 27-32 cm, which only contains 2 pupils.)
    - How many pupils have feet that are 22 cm or longer? (Answer: Add the pupils in the last 2 categories: $9 + 2 = 11$ pupils.)
13. Write the following problem on the board: The frequency table below shows pupils' scores on an exam. Use the table to answer the questions.

Pupils' Scores on an Exam	
Marks (%)	**Frequency**
0-10	2
11-20	0
21-30	3
31-40	1
41-50	0
51-60	9
61-70	8
71-80	12
81-90	15
91-100	7

    a. How many pupils scored 81-90%?
    b. How many pupils scored higher than 80%?
    c. If pupils must achieve more than 70% to pass, how many pupils failed?

14. Ask pupils to work with seatmates to answer the questions.
15. Invite volunteers to stand and give their answers and a short explanation. (Answers: a. 15 pupils, the frequency in the 81-90 interval; b. Add the frequencies in the last 2 intervals, which are greater than 80: $15 + 7 = 22$; c. Add the frequencies that are 70% or less: $2 + 0 + 3 + 1 + 0 + 9 + 8 = 23$.)

Practice *(13 minutes)*

1. Write the following problem on the board: The following are marks scored by 20 pupils in an examination: 87, 83, 73, 59, 48, 90, 93, 81, 87, 90, 39, 61, 54, 72, 79, 57, 98, 47, 93, 85.
    a. Draw a frequency table using class intervals 1-10, 11-20, 21-30, …
    b. Which interval does the greatest number of pupils fall into?
    c. If 61% or higher is passing, how many pupils passed?
    d. How many pupils scored 50% or lower?
2. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

    Solutions:

    a. Write the numbers in ascending order before counting and grouping them: 39, 47, 48, 54, 57, 59, 61, 72, 73, 79, 81, 83, 85, 87, 87, 90, 90, 93, 93, 98.

Pupils' Scores on an Exam	
Marks	**Frequency**
0-10	0
11-20	0
21-30	0
31-40	1
41-50	2
51-60	3
61-70	1
71-80	3
81-90	7
91-100	3

    b. The interval 81-90.
    c. Add the last 4 frequencies: $1 + 3 + 7 + 3 = 14$ pupils
    d. Add the intervals up to 50: $1 + 2 = 3$ pupils

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L130 in the Pupil Handbook.

Lesson Title: Drawing histograms	**Theme:** Statistics	
Lesson Number: M2-L131	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to present and interpret grouped data in histograms.	**Preparation** Draw the table at the start of Teaching and Learning on the board.	

Opening *(3 minutes)*

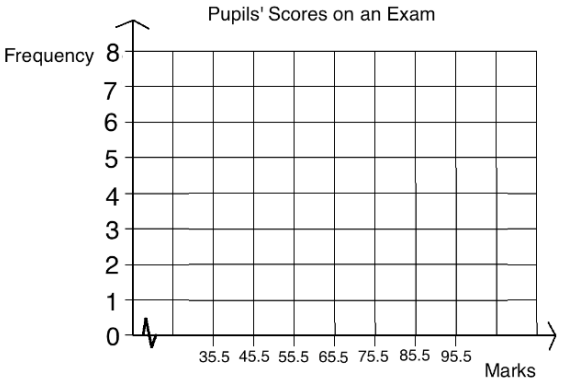
1. Ask pupils the following questions and allow volunteers to answer:
    a. What is grouped data? (Answer: Data that has been divided into groups called class intervals, based on size.)
    b. What is a class interval? (Answer: A class interval is a group of data from a set, with a certain range. Class intervals of a set of grouped data should all have the same range.)
2. Explain that today's lesson is on presenting grouped data in histograms.

Teaching and Learning *(23 minutes)*

1. Write the frequency table at right on the board:
2. Explain:
    - This is the frequency table that you made in the previous lesson. Today we will draw a histogram for it.
    - Histograms look like bar charts, but they are actually a different tool for representing data.

Pupils' Scores on an Exam	
Marks	**Frequency**
0-10	0
11-20	0
21-30	0
31-40	1
41-50	2
51-60	3
61-70	1
71-80	3
81-90	7
91-100	3

3. Draw and label the axes, as shown:
4. Explain:
    - Like a bar chart, a histogram consists of vertical bars. However, in histograms, the bar does not represent only 1 piece of data, but a range of data. Each bar represents a **class interval.** Recall that a class interval is a group, represented in 1 row of a grouped frequency table.
    - In histograms, each bar is centred on a **class mid-point** on the x-axis.

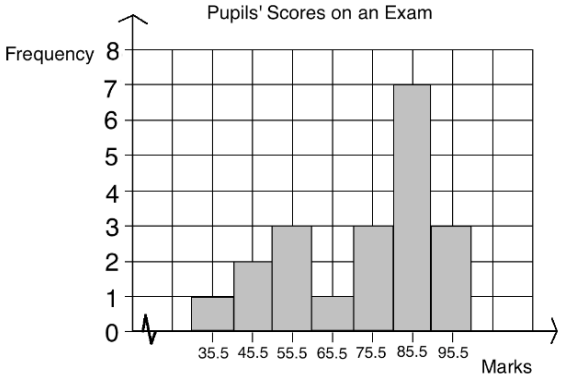


Class midpoints are the points that lie exactly in the middle of class intervals.

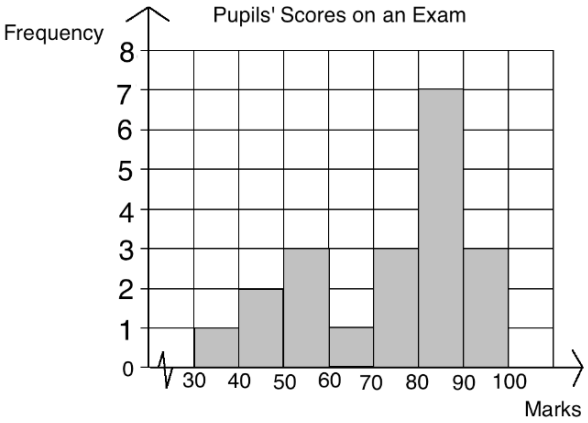
- Sometimes the class mid-point is labeled on the x-axis, and sometimes the high and low values of each class interval are labeled.
- The vertical axis is frequency, which is the same as with bar charts.

5. Draw the first bar, for the interval 31-40. The centre is at 35.5 (see below).
6. Ask a volunteer to explain how to draw the next bar. As they explain, draw it.
7. Explain:
    - The bars of a histogram touch each other, unlike bar charts. Histogram bars touch each other because they represent continuous intervals.
    - All of the possible values between 31 and 100 will be represented in our histogram, although the values fall within several different intervals.
8. Invite volunteers to come to the board and draw the other bars. Support them as needed.

    Answer:



9. Draw the histogram again with the class boundaries on the x-axis as shown:



10. Explain:
    - The bars still have the same class boundaries and are centred at the midpoint, but way it is drawn and labeled is different.
    - We can either draw our histograms with either midpoints or class boundaries labeled. You may see either form on the WASSCE exam.
11. Write another problem on the board: The table below gives the marks scored by 40 pupils on an assignment.

Marks scored	Frequency (f)	Class midpoints
1-5	2	
6-10	3	
11-15	7	
16-20	10	
21-25	12	
26-30	6	

a. Find the class midpoints, and write them in the table.
b. Draw a histogram for the distribution.
c. Which interval do the most pupils fall into?
d. How many pupils scored 10 or fewer marks?
e. How many pupils scored 16-30 marks?

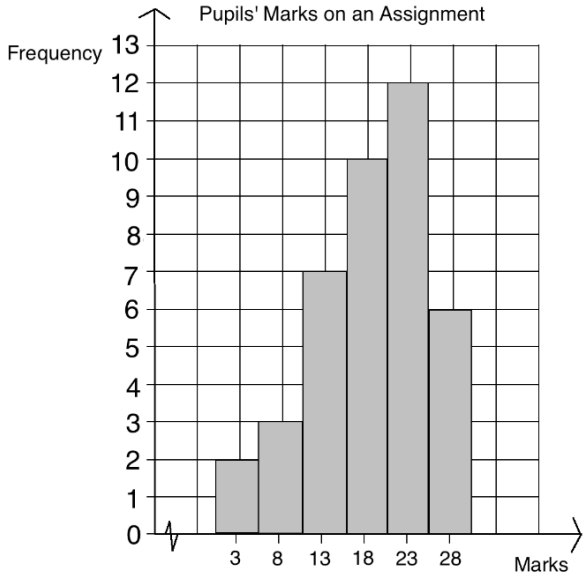
12. Ask volunteers to give the midpoints of the first 2-3 class intervals. As they give them, write them in the table on the board.
13. Ask pupils to complete the table and questions a.-c. with seatmates.
14. Walk around to check for understanding and clear misconceptions.
15. Invite volunteers to write the solutions on the board.

Solutions:

a. Completed table:

Marks scored	Frequency (f)	Class midpoints
1-5	2	3
6-10	3	8
11-15	7	13
16-20	10	18
21-25	12	23
26-30	6	28

b. Histogram:



c. Interval 21-25
d. Add the first 2 frequencies: $2 + 3 = 5$ pupils
e. Add the frequencies of all classes above 16: $10 + 12 + 6 = 28$ pupils

Practice *(13 minutes)*

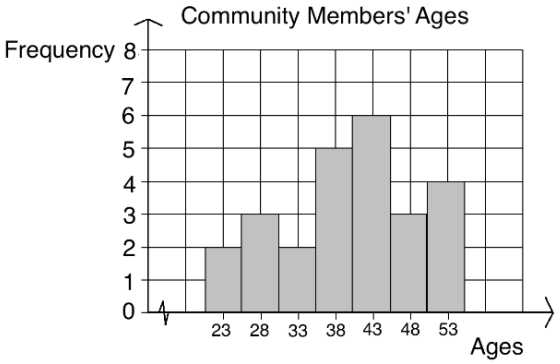
1. Write on the board: The ages of 25 community members are given: 25, 31, 45, 42, 36, 28, 43, 49, 52, 28, 24, 40, 44, 36, 48, 52, 41, 54, 32, 38, 39, 41, 54, 50, 28.
    a. Create a frequency distribution table using intervals 21-25, 26-30, 31-35, and so on.
    b. Draw the histogram of the distribution.
2. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

    Solutions:

    a. Frequency Table:

Marks scored	**Frequency (f)**	**Class midpoints**
21-25	2	23
26-30	3	28
31-35	2	33
36-40	5	38
41-45	6	43
46-50	3	48
51-55	4	53

    b. Histogram:



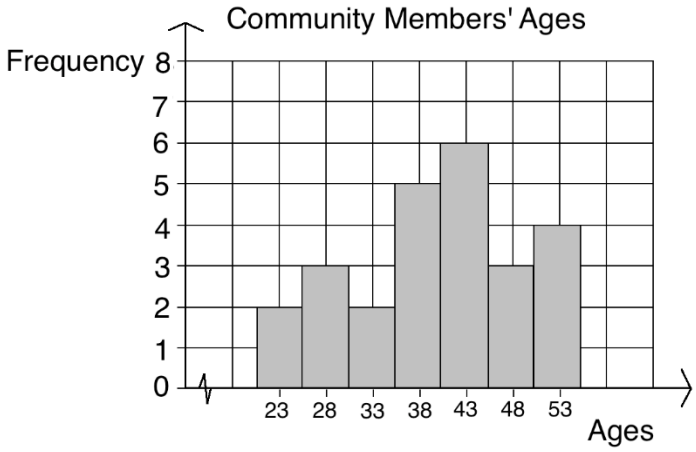
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L131 in the Pupil Handbook.

Lesson Title: Interpreting histograms	**Theme:** Statistics and Probability	
Lesson Number: M2-L132	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to interpret information in a histogram, including estimating mode.	**Preparation** Draw the histogram in Opening on the board.	

Opening *(4 minutes)*

1. Draw the histogram shown below on the board (from Practice in the previous lesson):

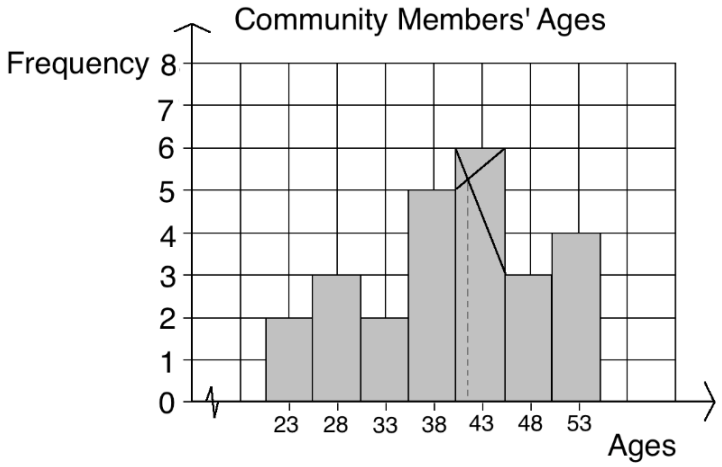


2. Ask questions to revise the previous lesson. Allow discussion.
    a. What is the class interval with the greatest number of pupils? (Answer: people aged 41-45)
    b. How many people are more than 35 years old? (Answer: Add the heights of the bars starting from 36: $5 + 6 + 3 + 4 = 18$ people)
    c. How many people are 30 years old or younger? (Answer: Add the heights of the first 2 bars: $2 + 3 = 5$ people)
3. Explain that this lesson is also on interpretation of histograms.

Teaching and Learning *(20 minutes)*

1. Discuss: Can you find the class interval that contains the median? How?
2. Allow discussion, then explain:
    - Recall that half of people fall below the median, and half fall above.
    - As with bar charts, we can count on the bars to find where the median lies.
    - Recall that there are 25 people in this data set. Therefore, the median age is that of the 13th person. There are 12 people younger and 12 people older.
    - Let's find the 13th person in this histogram.
3. Count the frequency up to 13, starting with the first bar. Identify that 13 falls into the class interval 41-45.

4. Discuss:
    a. What do you think the mode is?
    b. Which class do you think the mode falls into?
5. Allow discussion, then explain:
    a. It's impossible to know the exact mode of grouped data from a histogram alone.
    b. Using the histogram, we can **estimate** the mode.
    c. The class interval that the estimated mode lies in is called the **modal class.**
    d. The tallest bar in the histogram is the modal class. We estimate the mode using this bar.
6. On the board, draw intersecting lines using vertices of the tallest bar, and draw a vertical dotted line to the x-axis:



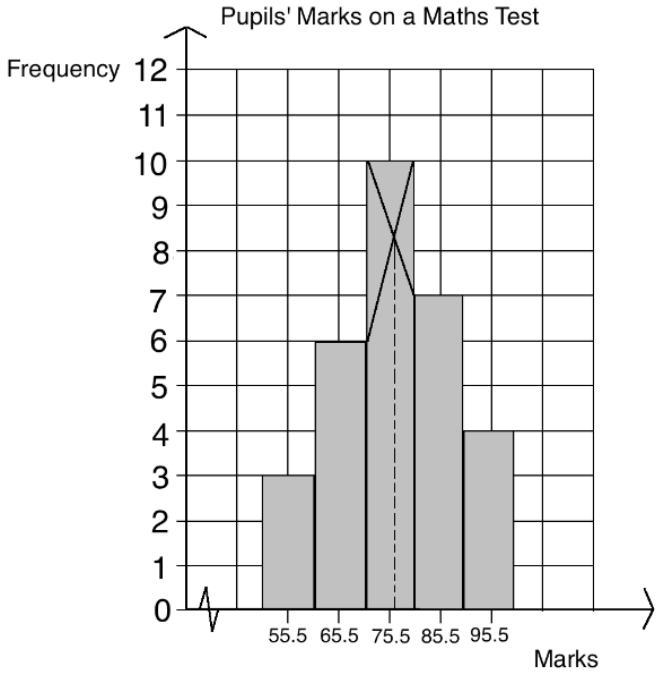
7. Explain: To estimate the mode, draw intersecting lines like this. The point where they intersect on the x-axis is the estimated mode.
8. Ask pupils to give the estimated mode. Write it on the board. (Answer: Accept realistic answers, for example in the range 41-42.)
9. Write the following problem on the board: The table below shows 30 pupils' marks on a Maths test.

Marks	51-60	61-70	71-80	81-90	91-100
Frequency	3	6	10	7	4

    a. Draw a histogram of the distribution.
    b. Which class interval does the median fall into?
    c. Use your histogram to estimate the mode.
10. Ask pupils to work with seatmates to solve the problem.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.

Solutions:

a. See the histogram below.
b. The median is the average mark of the 15th and 16th pupils. These pupils fall in the third bar, which is the class interval 71-80.
c. Check pupils' histograms (see below) and accept reasonable estimates of the mode. For example, 76-77.



Practice *(15 minutes)*

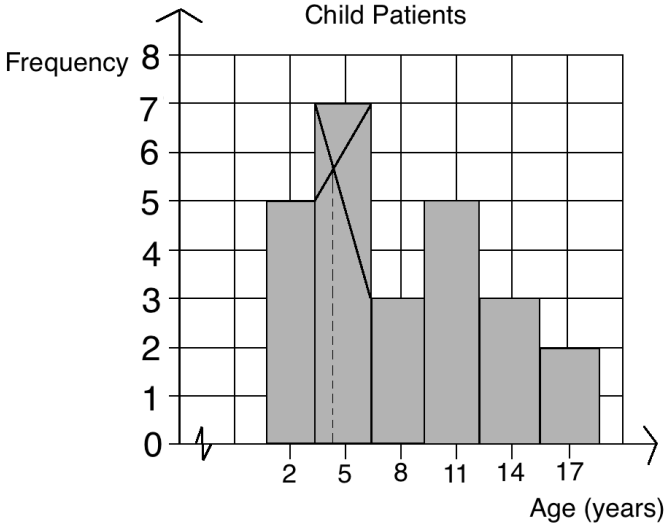
1. Write the following problem on the board: The children's ward of a hospital has 25 patients. Their ages in years are: 1, 13, 17, 5, 2, 6, 4, 2, 6, 12, 15, 3, 2, 16, 4, 14, 12, 10, 9, 5, 10, 8, 7, 5, 11.
    a. Draw a frequency table using class intervals 1-3, 4-6, 7-9, 10-12, 13-15, 16-18.
    b. Draw a histogram to display the data.
    c. One doctor is assigned to all patients under the age of 10. How many patients does she have?
    d. Which class interval contains the median?
    e. Which is the modal class?
    f. Use the histogram to estimate the mode.
2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

    Solutions:

a. Write the numbers in ascending order before counting and grouping them: 1, 2, 2, 2, 3, 4, 4, 5, 5, 5, 6, 6, 7, 8, 9, 10, 10, 11, 12, 12, 13, 14, 15, 16, 17

Child Patients	
Ages	**Frequency**
1-3	5
4-6	7
7-9	3
10-12	5
13-15	3
16-18	2

b. See the histogram below.
c. Add the heights of the first 3 columns, representing ages 1-9: $5 + 7 + 3 = 15$ patients
d. The median is the age of the 13th child. This falls in the 3rd bar, which is class interval 7-9.
e. The modal class is 4-6, because it has the tallest bar.
f. Check pupils' histograms (see below) and accept reasonable estimates of the mode. For example, in the range 4-4.5 years.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L132 in the Pupil Handbook.

Lesson Title: Frequency polygons	**Theme:** Statistics	
Lesson Number: M2-L133	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to present and interpret grouped data in frequency polygons.	**Preparation** Write the problem in Opening on the board.	

Opening *(5 minutes)*

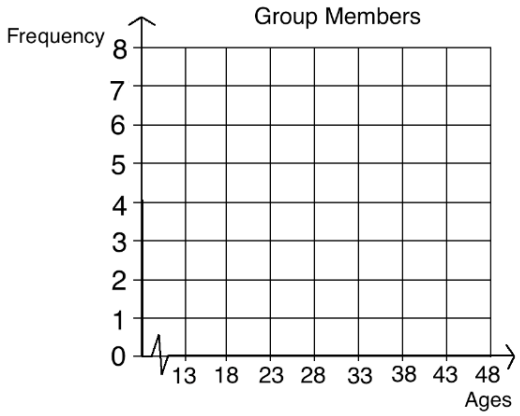
1. Write on the board: A certain women's group has 25 members. Their ages are 21, 42, 35, 26, 32, 19, 23, 27, 29, 38, 41, 42, 27, 35, 18, 30, 31, 26, 24, 41, 22, 35, 37, 23, 20. Draw a frequency table using class intervals 16-20, 21-25, 26-30, 31-35, 36-40, 41-45.
2. Ask pupils to draw the frequency table in their exercise books.
3. Invite a volunteer to write the solution on the board.
   Solution:
   Write the numbers in order: 18, 19, 20, 21, 22, 23, 23, 24, 26, 26, 27, 27, 29, 30, 31, 32, 35, 35, 35, 37, 38, 41, 41, 42, 42.
   Draw the table:

Group Members	
Ages	**Frequency**
16-20	3
21-25	5
26-30	6
31-35	5
36-40	2
41-45	4

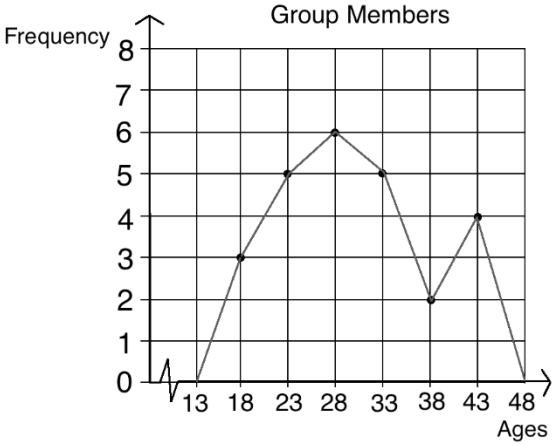
4. Explain that this lesson is on frequency polygons. This is another way to display grouped data.

Teaching and Learning *(19 minutes)*

1. Explain:
   - Recall line graphs, which are used to show ungrouped data. We create line graphs by plotting and connecting points.
   - Frequency polygons are similar to line graphs, in the same way that histograms are similar to bar charts.
   - Frequency polygons are used to display grouped data, which means that we plot class intervals.
2. Draw and label the axes, as shown below:



3. Explain:
    - We must find each class midpoint on the x-axis, and plot the frequency for the corresponding class interval.
    - Recall that for histograms we can use either the class midpoints or class boundaries to draw the bars. For frequency polygons, we must use the midpoints.
4. Plot the first point, centred at 18, which is the class midpoint for interval 16-20 (see frequency polygon below).
5. Ask volunteers to describe the other points on the frequency histogram. As they give the points, plot them.
6. Join the points together. Then, join the point at 18 to a frequency of 0 at 13. Join the point at 43 to a frequency of 0 at 48.



7. Explain:
    - Normally we extend the line of the frequency polygon to the midpoint of what would be the next interval, if that interval existed in the data.
    - In our data set, we don't have any women in the class intervals that contain 13 and 48. Therefore, we can extend the line down to zero.
8. Discuss:
    a. What is the modal class?
    b. What is the median class?
5. Allow discussion, then explain:

a. Recall that for histograms, the tallest bar is the modal class. Similarly, for frequency polygons, the highest point gives the modal class. The modal class contains 28, which is the class interval 26-30.
b. The median class contains the age that is in the middle. In a set of 25 women, the 13th woman has the median age. We can count up in the frequency polygon in the same way that we did for the histogram. The 13th woman is in the interval 26-30, so this is the median class.

9. Write the following problem on the board: The table below shows 32 pupils' marks on a Maths test.

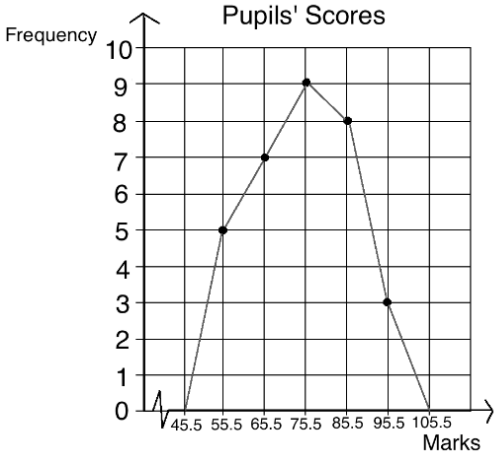
Marks	51-60	61-70	71-80	81-90	91-100
Frequency	5	7	9	8	3

a. Draw a frequency polygon of the distribution.
b. What is the modal class?
c. What is the median class?

10. Ask pupils to work with seatmates to solve the problem.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.

Solutions:

a. Frequency polygon:



b. The modal class is the one with the highest point or greatest frequency, which is 71-80.
c. The median class contains the average mark of the 16th and 17th pupils. These pupils fall into the class interval 71-80.

Practice *(15 minutes)*

1. Write the following problem on the board: The heights of 20 pupils in centimetres are: 179, 180, 161, 163, 170, 182, 168, 172, 175, 164, 168, 157, 158, 169, 159, 178, 164, 175, 167, 183.
   a. Draw a frequency table using class intervals 156-160, 161-165, 166-170, 171-175, 176-180, 181-185.
   b. Draw a frequency polygon to display the data.

c. What is the modal class?
d. What is the median class?
e. How many pupils are 170 cm or shorter?
f. What percentage of the pupils are taller than 175 cm?

2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain. All other pupils should check their own work.

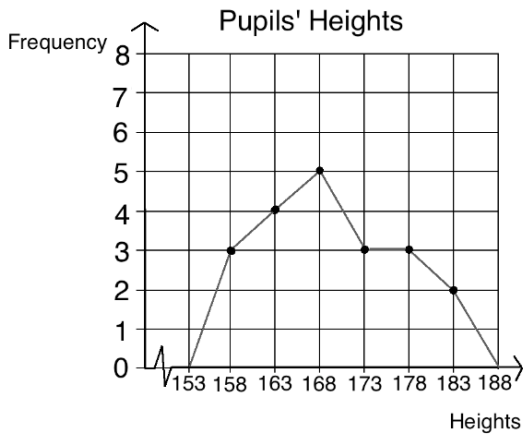
Solutions:

5. Write the numbers in ascending order before counting and grouping them: 157, 158, 159, 161, 163, 164, 164, 167, 168, 168, 169, 170, 172, 175, 175, 178, 179, 180, 182, 183.

a. Table:

Pupils' Heights	
Heights	**Frequency**
156-160	3
161-165	4
166-170	5
171-175	3
176-180	3
181-185	2

b. Frequency polygon:



c. The modal class is 166-170.
d. The median class is where the 10th and 11th pupils fall, which is 166-170.
e. Add the frequencies of the first 3 class intervals: $3 + 4 + 5 = 12$ pupils.
f. Find the number of pupils taller than 175 cm as a percentage of 20. Pupils taller than 175: $3 + 3 = 5$. As a percentage of 20: $\frac{5}{20} \times 100\% = 25\%$ of pupils.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L133 in the Pupil Handbook.

Lesson Title: Mean of grouped data	**Theme:** Statistics	
Lesson Number: M2-L134	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate and interpret the estimated mean of grouped data.	**Preparation** None	

Opening *(2 minutes)*

1. Discuss:
    a. How do you find the mean of ungrouped data? (Answer: Add up all the values in a data set and divide by the number of items.)
    b. What is grouped data? (Answer: Data arranged into equal intervals according to size.)
    c. Do you think it's possible to find the mean of grouped data? How would you do it? (Allow pupils to discuss and share answers.)
2. Explain that this lesson is on estimating the mean of grouped data.

Teaching and Learning *(24 minutes)*

1. Explain:
    - When data is divided into groups, we cannot determine the value of each piece of data in the set. Therefore, we cannot determine the exact mean.
    - We can **estimate** the mean of grouped data using a formula.
2. Write the following problem on the board: In one village, 15 farmers have just harvested their peppers. The table below shows the amount of pepper they harvested in kilogrammes. Estimate how much pepper each farmer harvested on average.

Farmers' Harvests	
Peppers (kg)	**Frequency**
0 – 4	2
5 – 9	5
10 – 14	4
15 – 19	3
20 – 24	1
Total	15

3. Write on the board: estimated mean $= \bar{x} = \frac{\sum fx}{\sum f}$, where f is frequency, and x is the corresponding class midpoint.
4. Explain:
    - This is the formula for estimating the mean.
    - Recall that the sigma symbol (Σ) tells us to find the sum.

- This formula can be read as “The estimated mean equals sigma (Σ) frequency (f) times the class midpoint (x) all divided by sigma (Σ) frequency (f)”.
- The numerator tells us to find the sum of each frequency multiplied by each corresponding class midpoint.
- The denominator tells us to find the sum of the frequencies.

5. Ask volunteers to give the class midpoint for each interval. Add a column to the table on the board, and write the midpoints:

Farmers’ Harvests		
Pepper (kg)	**Frequency**	**Midpoint**
0 – 4	2	2
5 – 9	5	7
10 – 14	4	12
15 – 19	3	17
20 – 24	1	22
Total	15	

6. Solve the problem on the board, explaining each step:
   Step 1. In the numerator, find the sum of the products of each frequency and midpoint. In the denominator, add the frequencies:
   $\bar{x} = \frac{\sum fx}{\sum f} = \frac{(2\times2)+(5\times7)+(4\times12)+(3\times17)+(1\times22)}{2+5+4+3+1}$
   Step 2. Simplify the result:
   $= \frac{4+35+48+51+22}{15}$
   $= \frac{160}{15}$
   $= 10.67$ to 2 d.p.
7. Write on the board: The scores of 25 pupils on an exam are: 58, 93, 86, 59, 99, 87, 64, 72, 67, 69, 92, 57, 91, 88, 76, 79, 83, 88, 90, 92, 74, 65, 89, 78, 91.
   a. Create a frequency table for the data using class intervals 51-60, 61-70, 71-80, 81-90, 91-100.
   b. Add a column to the frequency table for the class midpoints.
   c. Find the estimated mean score for the class.
8. Ask pupils to work with seatmates to complete parts a. and b.
9. Invite a volunteer to write the completed table on the board.
   Solution:
   First, write the numbers in ascending order: 57, 58, 59, 64, 65, 67, 69, 72, 74, 76, 78, 79, 83, 86, 87, 88, 88, 89, 90, 91, 91, 92, 92, 93, 99.

Pupils' Scores		
Marks	**Frequency**	**Midpoint**
51 – 60	3	55.5
61 – 70	4	65.5
71 – 80	5	75.5
81 – 90	7	85.5
91 – 100	6	95.5
Total	25	

10. Ask pupils to work with seatmates to complete part c.
11. Walk around to check for understanding and clear misconceptions.
12. Invite a volunteer to write the solution on the board. Support them as needed.

Solution:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(3\times55.5)+(4\times65.5)+(5\times75.5)+(7\times85.5)+(6\times95.5)}{3+4+5+7+6}$$ Substitute frequencies and midpoints

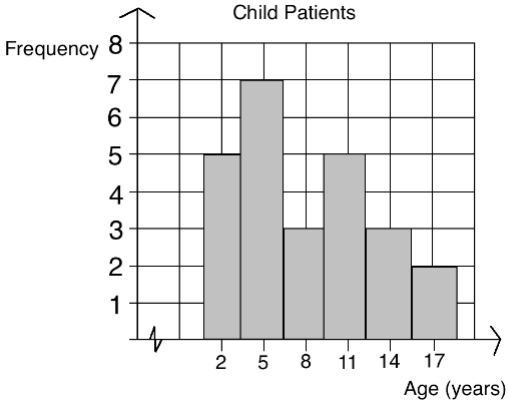
$$= \frac{166.5+262+377.5+598.5+573}{25}$$ Simplify

$$= \frac{1977.5}{25}$$

$$= 79.1$$

Practice *(13 minutes)*

1. Write the following problem on the board: The histogram below shows the ages of 25 patients in a hospital. Use the histogram to estimate the mean age of the patients.



2. Discuss: How can you find the estimated mean using the histogram? (Answer: To find the estimated mean we only need the frequency and midpoint for each class interval; we can observe these in the histogram and substitute them into the formula.)
3. Ask pupils to work independently to solve the problem. Allow discussion with seatmates if needed.
4. Walk around to check for understanding and clear misconceptions.
5. Invite a volunteer to write the solution on the board and explain. All other pupils should check their own work.

Solution:

$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(5\times2)+(7\times5)+(3\times8)+(5\times11)+(3\times14)+(2\times17)}{5+7+3+5+3+2}$ Substitute frequencies and mid-points

$= \frac{10+35+24+55+42+34}{25}$ Simplify

$= \frac{200}{25}$

$=$ 8 years old

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L134 in the Pupil Handbook.

Lesson Title: Median of grouped data	**Theme:** Statistics and Probability	
Lesson Number: M2-L135	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate and interpret the estimated median of grouped data.	**Preparation** None	

Opening *(2 minutes)*

1. Discuss:
    a. How do you find the median of ungrouped data? (Answer: Write the data in ascending or descending order, and identify the value in the middle.)
    b. Do you think it's possible to find the median of grouped data? How would you do it? (Allow pupils to discuss and share answers.)
2. Explain that this lesson is on estimating the median of grouped data.

Teaching and Learning *(22 minutes)*

1. Explain:
    - We have found the median class for grouped data in previous lessons. When we know the position of the median, we can find which class it falls in.
    - However, if we do not know each piece of data, we cannot determine the exact median.
    - We can **estimate** the median of grouped data using a formula.
2. Write the following problem on the board: In one village, 17 farmers have just harvested cassava. The table below shows the amount of cassava they harvested in kilograms. Estimate the median amount of cassava that was harvested.

Farmers' Harvests	
Cassava (kg.)	**Frequency**
10 – 14	1
15 – 19	3
20 – 24	6
25 – 29	5
30 – 34	2
Total	17

3. Discuss: How can we find which class the median falls into? (Answer: Since there are 17 farmers, the 9th farmer has the median harvest. 8 farmers harvested more, and 8 farmers harvested less. The 9th farmer is in interval 20-24)
4. Write on the board: estimated median $= L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c$
5. Read the parts of formula out loud and write definitions on the board:

- L is the lower class boundary of the group containing the median.
- n is the total frequency of the data (in the example, 17)
- $(\sum f)_L$ is the total frequency for the groups **before** the median group.
- f_m is the frequency of the median group.
- c is the group width.

6. Ask pupils to determine the value of each part of the formula in the example on the board.
7. Discuss each part as needed, and write them on the board as pupils identify them: $L = 20$; $n = 17$; $(\sum f)_L = 1 + 3 = 4$; $f_m = 6$; $c = 5$, because there are 5 values in the group: 20, 21, 22, 23, 24
8. Solve on the board, explaining each step:

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c = 20 + \left[\frac{\frac{17}{2} - (1+3)}{6}\right] \times 5 \quad \text{Substitute values}$$

$$= 20 + \left[\frac{8.5-4}{6}\right] \times 5 \quad \text{Simplify}$$

$$= 20 + \left[\frac{4.5}{6}\right] \times 5$$

$$= 20 + 0.75 \times 5$$

$$= 20 + 3.75$$

$$= 23.75$$

9. Write the following problem on the board: The scores of 30 pupils on an exam are shown in the table below. Calculate the estimated median.

Pupils' Scores	
Marks	**Frequency**
51 – 60	5
61 – 70	4
71 – 80	7
81 – 90	10
91 – 100	4
Total	30

10. Ask pupils to find the class interval that contains the median.
11. Allow a volunteer to share the answer and explain. (Answer: The median is the mean score of pupils 15 and 16. These are in interval 71-80.)
12. Ask pupils to work with seatmates to solve the problem.
13. Walk around to check for understanding and clear misconceptions. Remind pupils of the items in the formula as needed.
14. Invite a volunteer to write the solution on the board. Support them as needed.

Solution:

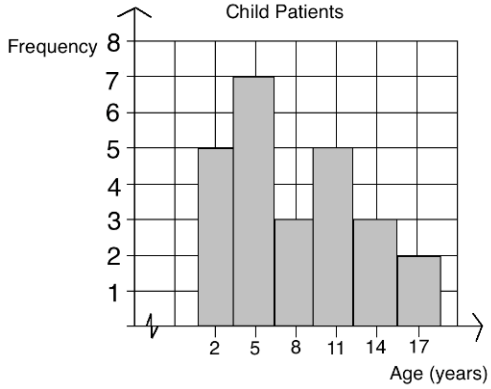
$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c = 71 + \left[\frac{\frac{30}{2} - (5+4)}{7}\right] \times 10 \quad \text{Substitute values}$$

$$= 71 + \left[\frac{15-9}{7}\right] \times 10 \quad \text{Simplify}$$

$$= \quad 71 + \left[\frac{6}{7}\right] \times 10$$
$$= \quad 71 + \frac{60}{7}$$
$$= \quad 71 + 8.57$$
$$= \quad 79.57 \text{ to 2 d.p.}$$

Practice *(14 minutes)*

1. Write the following problem on the board: The histogram below shows the age of 25 patients in a hospital. Use the histogram to estimate the median age of the patients.



2. Discuss: How can you find the estimated median using the histogram? (Answer: We need to find the value of each item in the formula.)
3. Ask pupils to find the class interval that contains the median. Allow them to discuss with seatmates
4. Ask a volunteer to give the answer, and write it on the board. (Answer: The interval containing the median is ages 7-9.)
5. Write the intervals on the board if needed, to help pupils identify them: 1-3, 4-6, 7-9, …
6. Ask pupils to work independently to solve the problem. Allow discussion with seatmates if needed.
7. Walk around to check for understanding and clear misconceptions.
8. Invite a volunteer to write the solution on the board and explain. All other pupils should check their own work.

   Solution:

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c \quad = \quad 7 + \left[\frac{\frac{25}{2} - (5+7)}{3}\right] \times 3 \qquad \text{Substitute values}$$
$$= \quad 7 + \left[\frac{12.5 - 12}{3}\right] \times 3 \qquad \text{Simplify}$$
$$= \quad 7 + \left[\frac{0.5}{3}\right] \times 3$$
$$= \quad 7 + 0.5$$
$$= \quad 7.5 \text{ years old}$$

Closing *(2 minutes)*

1. Remind pupils that in the previous class, they estimated the mean age of the children in the hospital.
2. Ask pupils to check their notes and give the mean age of the children. (Answer: 8 years old)
3. Explain:
    a. Recall that mean and median are measures of central tendency. That means they tell us approximately where the centre of the data is.
    b. The mean and median are usually close in value, as in this case.
4. For homework, have pupils do the practice activity PHM2-L135 in the Pupil Handbook.

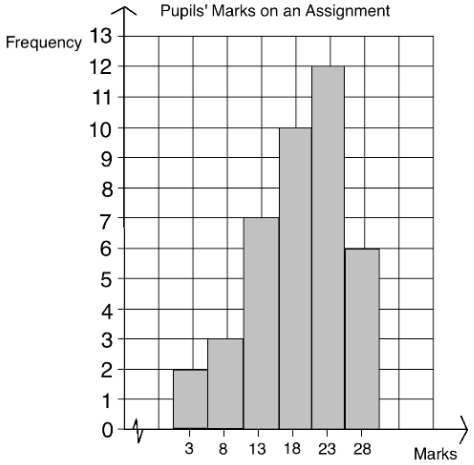
Lesson Title: Practice with mean, median, and mode of grouped data	**Theme:** Statistics and Probability	
Lesson Number: M2-L136	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to estimate the mean, median, and mode of grouped data.	**Preparation** None	

Opening *(3 minutes)*

1. Discuss:
    a. How do you find the mean of grouped data? (Answer: Use the formula.)
    b. How do you find the median of grouped data? (Answer: Use the formula.)
    c. How do you find the mode of grouped data? (Answer: Draw a histogram and use the modal class to estimate it.)
2. Invite volunteers to write the formulae for finding the mean and median of grouped data on the board. (Answers: mean $= \bar{x} = \frac{\sum fx}{\sum f}$; median $= L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c$)
3. Explain that this lesson is on solving problems on finding the mean, median and mode of grouped data. The style of the questions that pupils will see today is similar to those on the WASSCE exam.

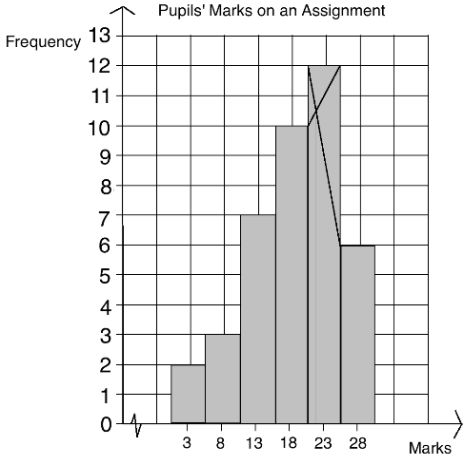
Teaching and Learning *(22 minutes)*

1. Write the following problem on the board: The histogram below shows the marks 40 pupils received on an assignment.



    a. Estimate the mode of the distribution.
    b. Estimate the mean.
    c. What is the median class?
    d. Estimate the median.

2. Ask pupils to copy the histogram in their exercise books and work with seatmates to solve the problem.
3. Encourage pupils to look at their notes and Pupil Handbook from previous lessons if needed.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to write the solutions on the board and explain.
   Solution:
   a. Check pupils' drawings on their histograms. Accept 22, or other reasonable estimates.



   b. Estimated mean:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(2\times3)+(3\times8)+(7\times13)+(10\times18)+(12\times23)+(6\times28)}{2+3+7+10+12+6}$$ Substitute frequencies and midpoints

$$= \frac{6+24+91+180+276+168}{40}$$ Simplify

$$= \frac{745}{40}$$

$$= 18.63$$

   c. The median is the mean of the 20th and 21st scores. Pupils 20 and 21 fall into the interval 16-20, which is the median class.
   d. Estimated median:

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c = 16 + \left[\frac{\frac{40}{2} - (2+3+7)}{10}\right] \times 5$$ Substitute values

$$= 16 + \left[\frac{20-12}{10}\right] \times 5$$ Simplify

$$= 16 + \left[\frac{8}{10}\right] \times 5$$

$$= 16 + 4$$

$$= 20$$

6. Write the following problem on the board: The table shows the distribution of the scores of some pupils on a test. Calculate the mean score.

Scores	0-4	5-9	10-14	15-19	20-24
Frequency	2	4	7	6	3

7. Ask pupils to work with seatmates to find the midpoint of each interval and write it in another row of the table.
8. Invite volunteers to come to the board and write the midpoints.
   Solution:

Scores	0-4	5-9	10-14	15-19	20-24
Frequency	2	4	7	6	3
Mid-point	2	7	12	17	22

9. Ask pupils to work with seatmates to calculate the estimated mean.
10. Invite a volunteer to write the solution on the board.

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(2\times2)+(4\times7)+(7\times12)+(6\times17)+(3\times22)}{2+4+7+6+3}$$

Substitute frequencies and mid-points

$$= \frac{4+28+84+102+66}{22}$$

Simplify

$$= \frac{284}{22}$$

$$= 12.91$$

Practice *(14 minutes)*

1. Write the following problem on the board: The table below shows the ages of teachers from a certain school. Calculate, correct to 2 decimal places:
   a. The mean
   b. The median

Teachers' Ages	
Years	**Frequency**
25 – 29	1
30 – 34	2
35 – 39	4
40 – 44	5
45 – 49	3
50 – 54	3
55 – 59	2
Total	20

2. Ask pupils to work independently to solve the problem. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solution on the board and explain. All other pupils should check their own work.
   Solution:
   a. Estimated mean:
      Step 1. Find the class midpoints. Draw another column in the table:

Teachers' Ages		
Years	**Frequency**	**Mid-point**
25 – 29	1	27
30 – 34	2	32
35 – 39	4	37
40 – 44	5	42
45 – 49	3	47
50 – 54	3	52
55 – 59	2	57
Total	20	

Step 2. Use the formula:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(1\times27)+(2\times32)+(4\times37)+(5\times42)+(3\times47)+(3\times52)+(2\times57)}{1+2+4+5+3+3+2}$$ Substitute frequencies and mid-points

$$= \frac{27+64+148+210+141+156+114}{20}$$ Simplify

$$= \frac{860}{20}$$

$$= 43 \text{ years}$$

b. Estimated median:
The median age is the average age of teachers 10 and 11. They fall into the class interval 40 – 44.

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c = 40 + \left[\frac{\frac{20}{2} - (1+2+4)}{5}\right] \times 5$$ Substitute values

$$= 40 + \left[\frac{10-7}{5}\right] \times 5$$ Simplify

$$= 40 + \left[\frac{3}{5}\right] \times 5$$

$$= 40 + 3$$

$$= 43 \text{ years}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L136 in the Pupil Handbook.

Lesson Title: Cumulative frequency tables	**Theme:** Statistics	
Lesson Number: M2-L137	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to construct cumulative frequency tables.	**Preparation** None	

Opening *(3 minutes)*

1. Discuss: Have you heard the word “cumulative” before? What does it mean? (Example answer: It is used when the quantity of something continuously grows or increases.)
2. Allow pupils to share ideas, then explain: When something is “cumulative”, it increases in quantity by successive addition. The quantity grows.
3. Explain that this lesson is on constructing cumulative frequency tables. This is another type of frequency table.

Teaching and Learning *(22 minutes)*

1. Write the following problem on the board: The table below shows the peppers harvested by 20 farmers in kilograms. Use it to construct a cumulative frequency table.

Farmers’ Harvests	
Pepper (kg)	**Frequency**
0 – 4	2
5 – 9	6
10 – 14	7
15 – 19	4
20 – 24	1
Total	20

2. Explain: This is a normal frequency table. You have seen a similar table in a previous lesson. We will use it to create a cumulative frequency table.
3. Draw a third column on the table, and label it “Cumulative Frequency”.
4. Fill the first 3 rows, explaining as you write them on the board (see complete table below).
5. Explain: To find the cumulative frequency for a row, add the frequency for that row to the cumulative frequency of the rows above it.
6. Ask volunteers to tell you the cumulative frequencies for each of the remaining 2 rows. As they explain, write the cumulative frequencies in the table.

Solution:

Farmers' Harvests		
Pepper (kg)	**Frequency**	**Cumulative Frequency**
0 – 4	2	2
5 – 9	6	6 + 2 = 8
10 – 14	7	7 + 8 = 15
15 – 19	4	4 + 15 = 19
20 – 24	1	1 + 19 = 20
Total	20	

7. Explain:
    a. The cumulative frequency for the last class interval should be equal to the total frequency. In this example, we see that the cumulative frequency is the total 20 farmers.
    b. Drawing a cumulative frequency table helps us to answer certain types of questions more easily.
8. Write on the board:
    a. How many farmers harvested 14 kg or less?
    b. How many farmers harvested 19 kg or less?
9. Ask pupils to discuss the questions with seatmates and try to arrive at the answer using the cumulative frequency table.
10. Ask volunteers to share their answers with the class. Guide them to the correct answers. (Answers: a. 15; b. 19)
11. Explain: Cumulative frequency tables are useful for answering "less than" questions like these.
12. Write the following problem on the board: The scores of 20 pupils on a Maths test are: 91, 82, 75, 72, 68, 90, 85, 75, 67, 59, 74, 55, 92, 64, 99, 94, 83, 87, 75, 81. Draw a cumulative frequency table for the data, using class intervals 51-60, 61-70, 71-80, 81-90, 91-100.
13. Discuss: You have made a frequency table for data similar to this before. How would you make a **cumulative** frequency table? (Example answer: Draw a normal frequency table and use a third column to write the cumulative frequency.)
14. Explain: When drawing a cumulative frequency table, you will have 2 columns for frequency: a column for the frequency of each row, and a column for cumulative frequency.
15. Ask pupils to work with seatmates to draw the cumulative frequency table. Remind pupils to write the data in ascending order first.
16. Walk around to check for understanding and clear misconceptions.
17. Invite a volunteer to draw the cumulative frequency table on the board. All other pupils should check their work.

    Solution:

    Step 1. Write the numbers in ascending order: 55, 59, 64, 67, 68, 72, 74, 75, 75, 75, 81, 82, 83, 85, 87, 90, 91, 92, 94, 99.

Step 2. Draw the cumulative frequency table:

Pupils' Scores		
Marks	**Frequency**	**Cumulative Frequency**
51 – 60	2	2
61 – 70	3	3 + 2 = 5
71 – 80	5	5 + 5 = 10
81 – 90	6	6 + 10 = 16
91 – 100	4	4 + 16 = 20
Total	20	

18. Ask questions verbally. Allow pupils to think and discuss with seatmates before giving the answer:
    a. How many pupils scored 80 or fewer marks? (Answer: 10 pupils)
    b. If 71 marks is passing, how many pupils failed? (Answer: 5 pupils)

Practice *(14 minutes)*

1. Write the following problem on the board: The height of 25 pupils in centimetres is: 157, 175, 182, 158, 180, 183, 160, 168, 164, 180, 175, 167, 178, 159, 161, 164, 176, 169, 170, 172, 168, 179, 181, 163, 170.
    a. Draw a cumulative frequency table using class intervals 156-160, 161-165, 166-170, 171-175, 176-180, 181-185.
    b. How many pupils are 175 cm or shorter?
2. Ask pupils to work independently to solve the problem. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solution on the board and explain. All other pupils should check their own work.

   Solution:

   a.

   Step 1. Write the numbers in ascending order: 157, 158, 159, 160, 161, 163, 164, 164, 167, 168, 168, 169, 170, 170, 172, 175, 175, 176, 178, 179, 180, 180, 181, 182, 183.

Step 2. Draw the table:

Pupils' Heights		
Height	**Frequency**	**Cumulative Frequency**
156-160	4	4
161-165	4	4 + 4 = 8
166-170	6	6 + 8 = 14
171-175	3	3 + 14 = 17
176-180	5	5 + 17 = 22
181-185	3	3 + 22 = 25
Total	25	

b. From the cumulative frequency of the row for 171-175, 17 pupils are 175 cm or shorter.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L137 in the Pupil Handbook.

Lesson Title: Cumulative frequency curves	**Theme:** Statistics	
Lesson Number: M2-L138	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to construct cumulative frequency curves and estimate the median.	**Preparation** Write the problem in Opening on the board.	

Opening *(5 minutes)*

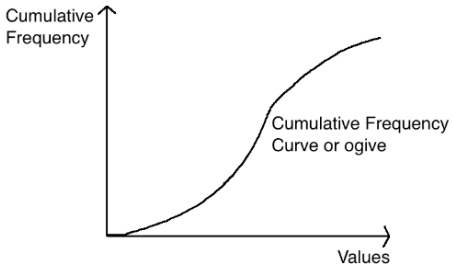
1. Fill the cumulative frequency column in the table:

Pupils' Scores on a Maths Test		
Marks	**Frequency**	**Cumulative Frequency**
51 – 60	5	
61 – 70	4	
71 – 80	7	
81 – 90	10	
91 – 100	4	
Total	30	

2. Ask pupils to copy and fill the table in their exercise books.
3. Invite a volunteer to fill the table on the board. (Answer: "Cumulative Frequency" column should contain 5, 9, 16, 26, 30)
4. Explain that this lesson is on constructing cumulative frequency curves. This is a graph that shows cumulative frequency.

Teaching and Learning *(20 minutes)*

1. Explain:
    a. A **cumulative frequency curve** can be graphed in a similar way to line graphs and frequency polygons.
    b. Cumulative frequency curves can also be called "**ogive**".
2. Draw a simple sketch of an ogive on the board (it does not need to be accurate to the data):



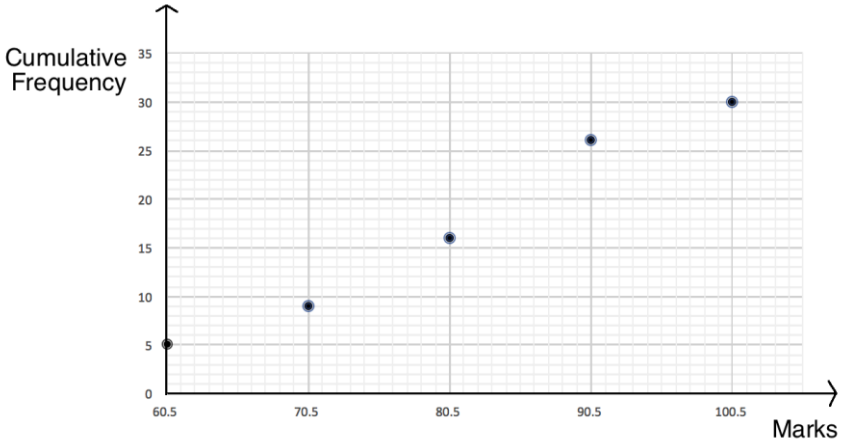
3. Explain:
    a. Remember that cumulative frequency only increases, or grows. It never decreases.

b. A cumulative frequency curve increases as you move in the positive direction along the x-axis.

4. Explain:
   a. For the x-values, we will plot the upper class boundary of each class interval. This is the highest data point in each class interval.
   b. Notice that there is a space of 1 unit between each interval. The first class interval ends at 60, and the second class interval begins at 61.
   c. For the purpose of graphing, we will take the point in the middle of the class intervals. For example, we will plot the value 60.5.
   d. For the y-value, we will plot the cumulative frequency from the table.
5. Draw another column in the table on the board, and write the upper class boundary for each class interval. Make sure pupils understand.

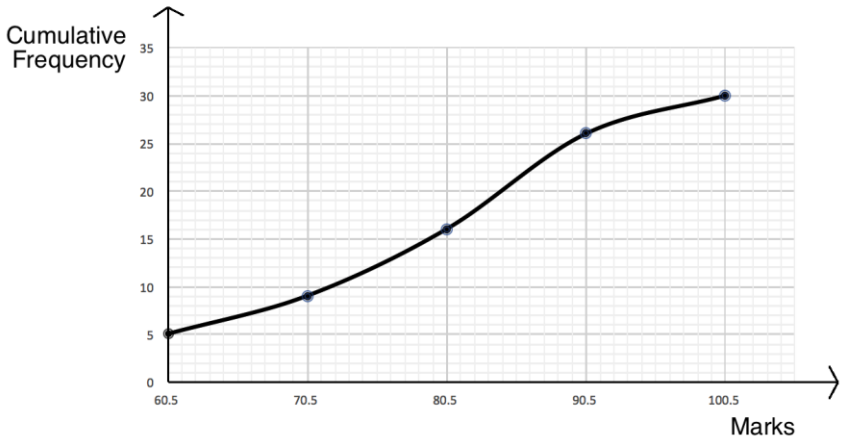
Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Cumulative Frequency**	**Upper Class Interval**
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5
91 – 100	4	4 + 26 = 30	100.5
Total	30		

6. Draw the axes on the board, and plot each point from the table:

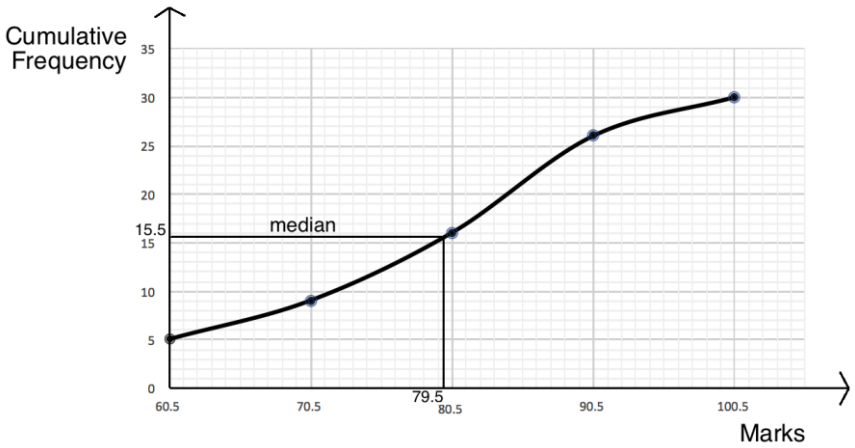


Note that for the sake of time, it is not necessary to draw each minor gridline on the graph. Pupils have the same ogive printed in the Pupil Handbook.

7. Explain:
   a. When we connect the points, we should use a smooth curve. We do not connect the points with straight lines as we did for frequency polygons.
   b. If you do use straight lines to connect the points, it is not a cumulative frequency curve. It would be a cumulative frequency polygon.
8. Connect the points on the board with a smooth curve:



9. Explain:
    a. We can estimate the median using the cumulative frequency curve.
    b. Remember that we cannot find the exact median from grouped data, so our result will only be an estimate.
    c. Recall that the median mark is scored by the pupil in the middle. There are 30 pupils in the data set, so the median is the mean of the scores of the 15^{th} and 16^{th} pupils.
    d. To find the estimated median using the curve, find the mark that corresponds to the 15.5^{th} pupil.
10. Draw a horizontal line on the board 15.5 on the y-axis (see below).
11. Draw a vertical line connecting this point on the curve to the x-axis. Identify the number of marks given at this point.



12. Explain: We use the cumulative frequency 15.5, and find the corresponding number of marks on the curve. In this case, the estimated median is 79.5.

Practice *(14 minutes)*

1. Write the following problem on the board: The table below gives the cassava harvests of 17 farmers.
    a. Fill the empty columns.
    b. Draw the cumulative frequency curve.

c. Use the curve to estimate the median harvest of the distribution.

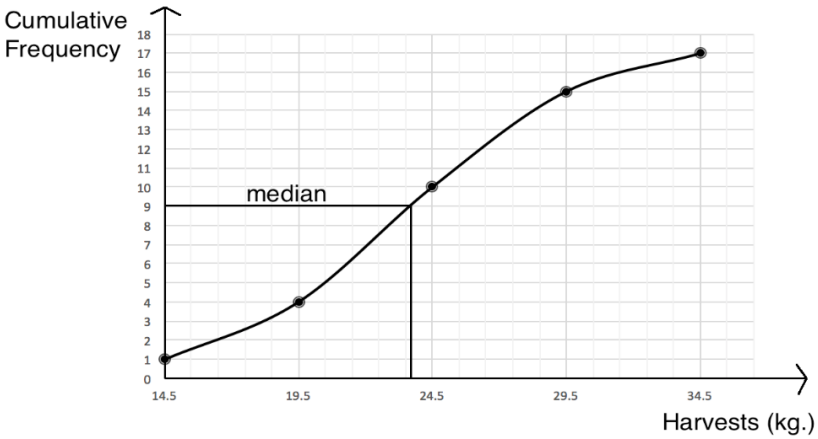
Farmers' Harvests			
Cassava (kg)	**Frequency**	**Upper Class Interval**	**Cumulative Frequency**
10 – 14	1		
15 – 19	3		
20 – 24	6		
25 – 29	5		
30 – 34	2		
Total	17		

2. Ask pupils to work with seatmates to complete part a., the table.
3. Invite volunteers to come to the board and fill the table:

Farmers' Harvests			
Cassava (kg)	**Frequency**	**Upper Class Interval**	**Cumulative Frequency**
10 – 14	1	14.5	1
15 – 19	3	19.5	3+1=4
20 – 24	6	24.5	6+4=10
25 – 29	5	29.5	5+10=15
30 – 34	2	34.5	2+15=17
Total	17		

4. Ask pupils to work with seatmates to complete parts b and c.
5. Walk around to check for understanding and clear misconceptions.
6. Invite volunteers to draw the curve on the board, and use it to estimate the median.

Solution:



To find median, note that the 9th pupil is in the middle. Accept reasonable estimates for median, such as 23.5 or 23.6.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L138 in the Pupil Handbook.

Lesson Title: Quartiles	**Theme:** Statistics and Probability	
Lesson Number: M2-L139	**Class:** SSS 2	**Time:** 40 minutes
 **Learning Outcomes** By the end of the lesson, pupils will be able to: 1. Estimate quartiles using a cumulative frequency curve. 2. Calculate the interquartile range. 3. Calculate the semi-interquartile range.	 **Preparation** None	

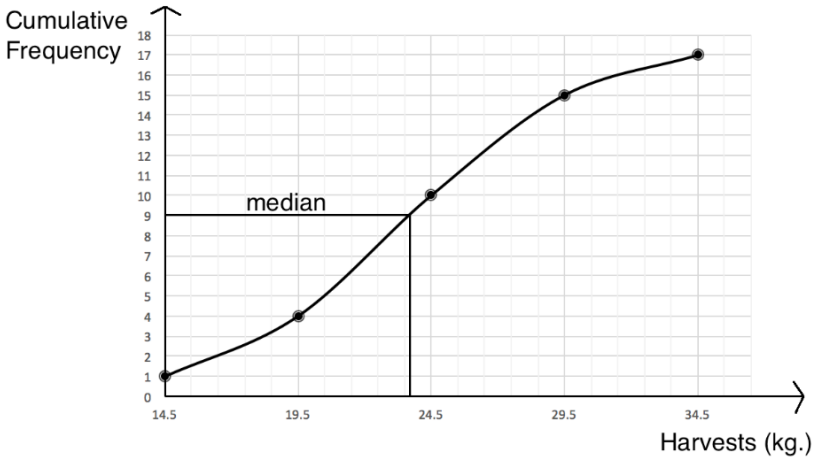
Opening *(3 minutes)*

1. Discuss:
    - What is a median? (Example answers: A measure of central tendency; the middle value in a data set.)
    - What are some methods you have used to calculate a median? (Answers: For ungrouped data, we have listed data and found the middle value, and identified the middle value in a table or bar chart; for grouped data, we have found the median using the formula and cumulative frequency table.)
2. Explain that this lesson is on quartiles. Quartiles are related to the median.

Teaching and Learning *(20 minutes)*

1. Explain:
    a. Quartiles are found by dividing a data set into 4 equal parts. The word "quartile" is related to "quarter", which means fourths.
    b. The lower quartile is one-quarter of the way from the bottom of the data.
    c. The upper quartile is one-quarter of the way from the top of the data set.
    d. The median is the second quartile, or the middle quartile.
2. Write on the board: Q_1: lower quartile; Q_2: second quartile, or median; Q_3: upper quartile
3. Explain:
    a. We use formulae to find the placement of the quartiles.
    b. Recall that to find the place of the median, find the value in the middle of the dataset.
    c. After finding the placement of the quartiles in the dataset, we can use the cumulative frequency curve to find the value of each quartile. This process is very similar to the one we used in the previous lesson to estimate median.
4. Write on the board: The upper and lower quartiles are given by the formulae, where n is the total frequency: $Q_1: \frac{1}{4}(n+1)$ and $Q_3: \frac{3}{4}(n+1)$
5. Make sure pupils understand that the formulae tell us the placement of the quartiles, and not their values.

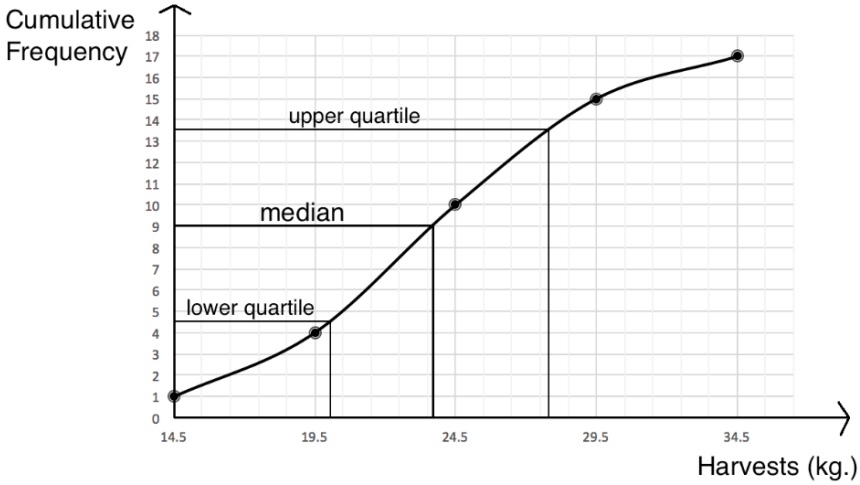
6. Draw the cumulative frequency curve from the practice section of the previous lesson on the board, with the median labeled:



7. Remind pupils that we used this curve to estimate the median. Now we will use it to estimate the quartiles.
8. Ask a volunteer to give the total frequency of this data set. (Answer: 17 farmers)
9. Use the formulae to find the place of each quartile on the board:

$$Q_1: \frac{1}{4}(n+1) = \frac{1}{4}(17+1) = \frac{1}{4}(18) = \frac{18}{4} = 4\frac{1}{2}$$
$$Q_3: \frac{3}{4}(n+1) = \frac{3}{4}(17+1) = \frac{3}{4}(18) = \frac{54}{4} = 13\frac{1}{2}$$

10. Explain:
    a. To find the lower quartile, we need to identify the $4\frac{1}{2}$th farmer on the y-axis.
    b. To find the upper quartile, we need to identify the $13\frac{1}{2}$th farmer.
11. Draw lines to identify both the upper and lower quartiles:



12. Ask volunteers to identify the values of the upper and lower quartiles. Accept reasonable estimates, and write them on the board. (Approximate answers: $Q_1 =$ 20.1 kg, $Q_3 = 27.7$ kg)
13. Discuss: What is range? (Answer: The difference between the minimum and maximum values in a data set.)
14. Explain:

a. Just as we can calculate the range of a data set, we can calculate the interquartile range.
b. The interquartile range can be found by subtracting the lower quartile from the upper quartile.
c. The interquartile range represents how spread out the middle half of the data is.

15. Calculate the interquartile range on the board: $Q_3 - Q_1 = 27.7 - 20.1 = 6.6$ kg.
16. Explain:
    a. The farmers who produced harvests in the middle half of the data set have harvests that range from 20.1 kg to 27.7 kg.
    b. The interquartile range tells us how spread out these harvests are. They are spread out over 6.6 kg.
17. Discuss: We can also find the semi-interquartile range. What is the meaning of the prefix "semi"? (Answer: "semi" means half, as in semi-circle.)
18. Explain:
    a. The inter-quartile range tells us about half of the data set. Therefore, the semi-interquartile range tells us about half of half of the data set, or one quarter of the data set.
    b. The semi-interquartile range tells us the range of one quartile.
19. Write the equation for the semi-interquartile range on the board: $Q = \frac{Q_3 - Q_1}{2}$
20. Calculate the semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{27.7 - 20.1}{2} = \frac{6.6}{2} = 3.3$ kg
21. Explain: This shows that about half of the farmers grew a harvest within 3.3 kg of the median.

Practice *(16 minutes)*

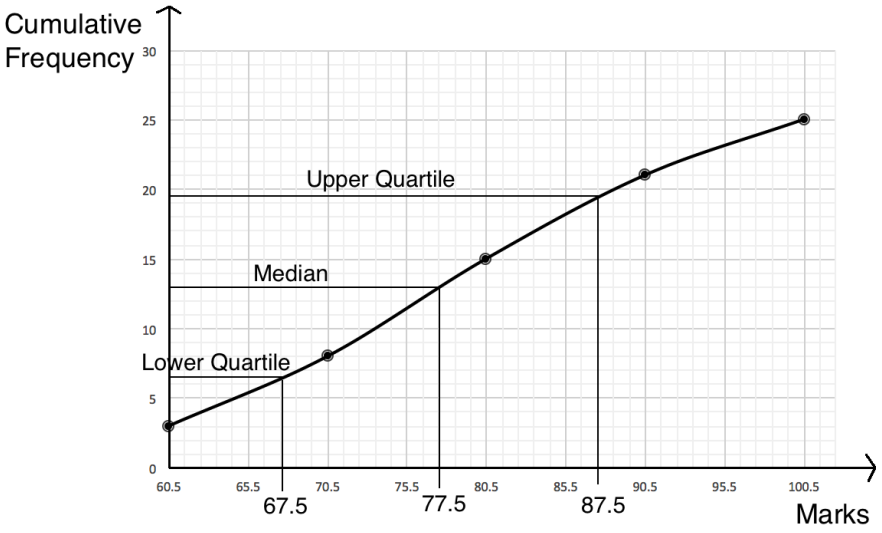
1. Write the following problem on the board: The table below gives the marks of 25 pupils on a Maths test.
    a. Fill the empty columns.
    b. Draw the cumulative frequency curve.
    c. Use the curve to estimate the median mark.
    d. Use the curve to estimate the upper and lower quartiles.
    e. Calculate the interquartile range.
    f. Calculate the semi-interquartile range.

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Upper Class Interval**	**Cumulative Frequency**
51 – 60	3		
61 – 70	5		
71 – 80	7		
81 – 90	6		
91 – 100	4		
Total	25		

2. Ask pupils to work with seatmates to solve the problem. Work the steps as a whole class if pupils have difficulty.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solution on the board.
   Solutions:
   a. Completed table:

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Upper Class Interval**	**Cumulative Frequency**
51 – 60	3	60.5	3
61 – 70	5	70.5	5 + 3 = 8
71 – 80	7	80.5	7 + 8 = 15
81 – 90	6	90.5	15 + 6 = 21
91 – 100	4	100.5	21 + 4 = 25
Total	25		

   b. See below.
   c. Estimated median: Pupil 13
   d. Estimated Q_1: $\frac{1}{4}(n+1) = \frac{1}{4}(25+1) = \frac{1}{4}(26) = \frac{26}{4} = 6\frac{1}{2}$
   Estimated Q_3: $\frac{3}{4}(n+1) = \frac{3}{4}(25+1) = \frac{3}{4}(26) = \frac{78}{4} = 19\frac{1}{2}$



   e. Interquartile range: $Q_3 - Q_1 = 87.5 - 67.5 = 20$ marks
   f. $Q = \frac{Q_3 - Q_1}{2} = \frac{87.5 - 67.5}{2} = \frac{20}{2} = 10$ marks
5. Explain: Remember that a semi-interquartile range of 10 marks means that half of the pupils are within 10 marks of the median.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L139 in the Pupil Handbook.

Lesson Title: Practice with cumulative frequency	**Theme:** Statistics and Probability	
Lesson Number: M2-L140	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to construct cumulative frequency tables and curves, and use them to estimate the median, quartiles, interquartile range, and semi-interquartile range**.**	**Preparation** None	

Opening *(3 minutes)*

1. Discuss:
    - What are quartiles? (Answer: Quartiles are values that divide a data set into 4 equal parts.)
    - What is interquartile range? (Answer: Interquartile range is the range between the upper and lower quartiles. It tells us the spread of the middle half of the data.)
    - What is semi-interquartile range? (Answer: Semi-interquartile range is half of the interquartile range. It is a measure of the spread of the distribution which tells us that half of all the data lies within a certain distance from the median.)
2. Explain that this lesson is on solving problems on cumulative frequency curves. The problems in today's lesson are in the same style as WASSCE problems.

Teaching and Learning *(18 minutes)*

1. Write the following problem on the board: The table below shows the weight of 100 football players:

Weight (kg)	40-49	50-59	60-69	70-79	80-89
Football players	5	12	30	35	18

    a. Construct the cumulative frequency table.
    b. Draw the cumulative frequency curve.
    c. From the curve, estimate the: i. Median ii. Semi-interquartile range
2. Discuss: How can we construct the cumulative frequency table? (Answer: Add rows to the table for upper class interval and cumulative frequency.)
3. Explain that, as with frequency tables, cumulative frequency tables can be arranged either vertically or horizontally. Data may be displayed in columns or rows.
4. Draw the next 2 rows on the board, and ask volunteers to fill them with the correct values.

Solution:

a. Cumulative frequency table:

Weight (kg)	40-49	50-59	60-69	70-79	80-89
Football players	5	12	30	35	18
Upper class interval	49.5	59.5	69.5	79.5	89.5
Cumulative frequency	5	5+12=17	30+17=47	35+47=82	18+82=100

5. Ask pupils to work with seatmates to complete the other parts of the problem.
6. Walk around to check for understanding and clear misconceptions.
7. Invite volunteers to write the solution on the board.

Solutions:

b. See cumulative frequency curve below.

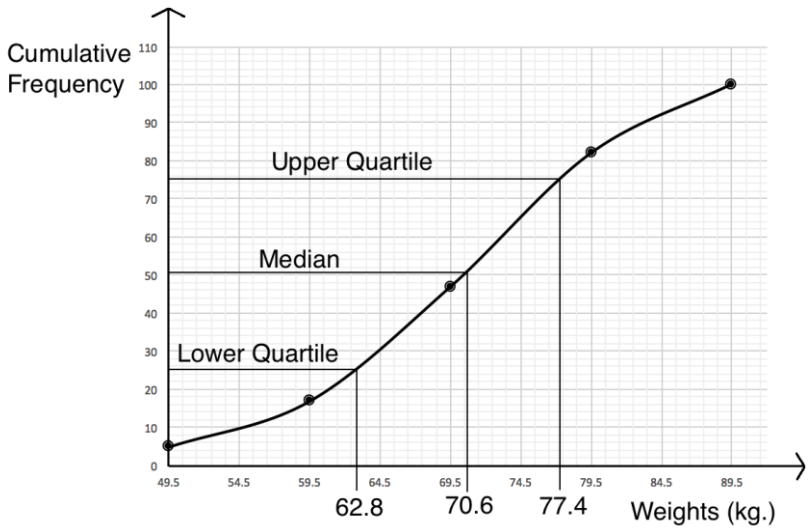
c. The median is at 50.5, which corresponds to approximately 70.6 kg on the x-axis.

d. Estimated Q_1 placement: $\frac{1}{4}(n+1) = \frac{1}{4}(100+1) = \frac{1}{4}(101) = \frac{101}{4} = 25\frac{1}{4}$

Estimated Q_3placement: $\frac{3}{4}(n+1) = \frac{3}{4}(100+1) = \frac{3}{4}(101) = \frac{303}{4} = 75\frac{3}{4}$

Using the cumulative frequency curve, these correspond to approximately $Q_1 = 62.8$ and $Q_3 = 77.4$

Semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{77.4-62.8}{2} = \frac{14.6}{2} = 7.3$ marks



Practice *(18 minutes)*

1. Write the following problem on the board: The table below gives the number of eggs produced in one year by 60 chickens on a farm.

No. of Eggs	50-59	60-69	70-79	80-89	90-99
Chickens	8	10	17	14	11

a. Draw a cumulative frequency curve of the distribution.

b. Use your curve to estimate the median.

c. Use your curve to find the interquartile range.
d. Write a sentence to describe what the interquartile range tells you.
e. Calculate the semi-interquartile range.
f. Write a sentence to describe what the semi-interquartile range tells you.

2. Ask pupils to work independently to solve the problem. Allow discussion with seatmates if needed.
3. Remind pupils that they need to draw the cumulative frequency table before drawing the cumulative frequency curve, even though the question does not state this.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to write the solution on the board.

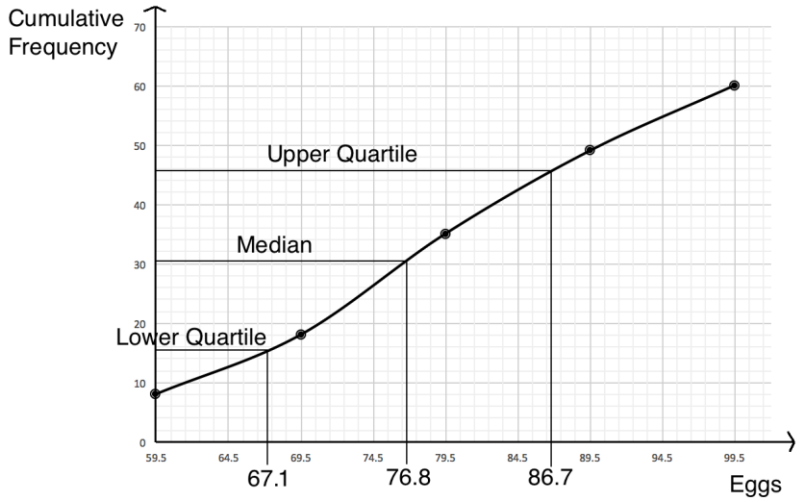
Solutions:

a. See curve below. The table used is:

No. of Eggs	50-59	60-69	70-79	80-89	90-99
Chickens	8	10	17	14	11
Upper class interval	59.5	69.5	79.5	89.5	99.5
Cumulative frequency	8	10+8=18	17+18=35	14+35=49	11+49=60

b. The median is at 30.5, which corresponds to approximately 76.8.

c. Estimated Q_1 placement: $\frac{1}{4}(n+1) = \frac{1}{4}(60+1) = \frac{1}{4}(61) = \frac{61}{4} = 15\frac{1}{4}$
Estimated Q_3placement: $\frac{3}{4}(n+1) = \frac{3}{4}(60+1) = \frac{3}{4}(61) = \frac{183}{4} = 45\frac{3}{4}$
Using the cumulative frequency curve, these correspond to approximately $Q_1 = 67.1$ and $Q_3 = 86.7$
Interquartile range $= Q_3 - Q_1 = 86.7 - 67.1 = 19.6$ eggs



d. Accept sentences similar to: The interquartile range says that the spread for the middle half of the chickens is 19.6 eggs per year.

e. Semi-interquartile range $= Q = \frac{Q_3 - Q_1}{2} = \frac{86.7-67.1}{2} = \frac{19.6}{2} = 9.8$ eggs

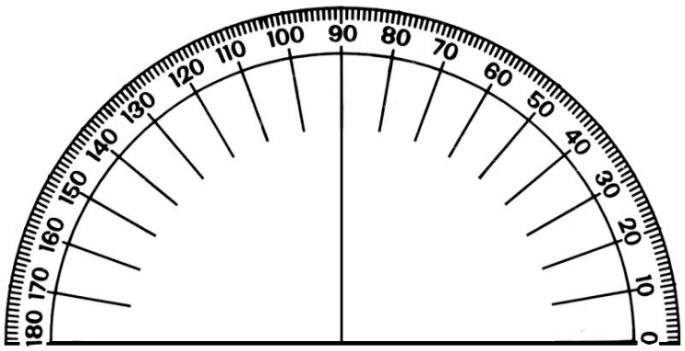
f. Accept sentences similar to: The semi-interquartile range tells us that half of all chickens at the farm produce within 9.8 eggs of the median per year.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L140 in the Pupil Handbook.

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



Appendix II: Sines of Angles

Sines of Angles (*x* in degrees) $x \rightarrow \sin x$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	8	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	8	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	4	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	3	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	3	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

x	-0	-1	-2	-3	-4	-5	-6	-7	-8	-9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995	0	0	0	0	0	0	1	1	1
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	1	1	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	3	3	4	4	5	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	5	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	14
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16

Appendix IV: Tangents of Angles

$x \rightarrow \tan x$

Tangents of Angles (x in degrees)

GOVERNMENT OF SIERRA LEONE

FUNDED BY

IN PARTNERSHIP WITH